Archives of Mechanics • Archiwum Mechaniki Stosowanej • 26, 5 pp. 861-869, Warszawa 1974

Self-similar problems of the one-dimensional, unsteady motion of viscous, heat-conducting gas

V. P. SHIDLOVSKY (MOSCOW)

A CERTAIN class of self-similar solutions of the complete system of Navier-Stokes equations for compressible fluid is considered. These solutions correspond to unsteady, one-dimensional motion under condition that the perturbed region is separated from the unperturbed one by the weak discontinuity surface, called the perturbation front. The equations of self-similar motions, belonging to this class, are derived, and the behaviour of integral curves in vicinity of the perturbation front is investigated.

Rozważono pewną klasę samopodobnych rozwiązań pełnego układu równań Naviera-Stokesa dla płynu lepkiego. Rozwiązania te opisują jednowymiarowy ruch nieustalony pod warunkiem, że obszar zaburzony oddzielony jest od obszaru niezaburzonego powierzchnią słabej nieciągłości, zwaną czołem zaburzenia. Wyprowadzono równania ruchów samopodobnych należących do tej klasy, jak również przeanalizowano zachowanie się krzywych całkowych w pobliżu czoła zaburzenia.

Рассматривается некоторый класс автомодельных решений полной системы уравнений Навье-Стокса для сжимаемой жидкости. Эти решения соответствуют неустановившемуся, одномерному движению при условии, что возмущенная область отделяется от невозмущенной поверхностью слабого разрыва, называемой фронтом возмущений. Выводятся уравнения автомодельных движений, принадлежащих этому классу и исследуется поведение интегральных кривых в окрестности фронта возмущений.

THE THEORY of nonstationary one-dimensional flow of a perfect (nonviscous) gas has found relatively wide development. Its explanation and a survey of relevant research works can be found in the monograph by K. P. STANYUKOVICH [1]. Among problems of one-dimensional nonstationary motion of a gas, a particular role is played by self-similar problems, the solution of which can be obtained by simple and reliable numerical methods. In some cases, they can also be represented in an analytic form. A systematic explanation of the theory of a nonstationary one-dimensional motion of gas as applied to self-similar problems appears in a monograph by L. I. SEDOV [2] who considers, above all, adiabatic gas flows with shock waves, but the general assumptions of his theory remain valid also for any other self-similar problems, with special reference to viscosity and heat conduction, in particular. Below we shall consider a certain class of self-symilar problems of a nonstationary one-dimensional motion of a viscous heat-conducting gas satisfying such conditions when the region of perturbed motion is separated from the unperturbed gas by a weak discontinuity surface, referred to as a perturbation front. We derive equations of a self-similar motion belonging to that class and investigate the behaviour of the integral curves in the neighbourhood of the perturbation front.

1

Let us consider the general case of a nonstationary one-dimensional motion of a viscous heat-conducting gas. The term of one-dimensional motion will be used to denote a motion having symmetry of any type, all the properties depending on a unique argument having a dimension of length. The gas will be considered to be perfect, the specific heats and Prandtl number to be constant, and the viscosity and heat conduction to be expressed as power functions of the temperature.

With the above assumptions, the motion of the gas and the thermodynamic processes occurring in it are governed by the Navier-Stokes equations, which have the form:

$$
\begin{gathered}
\frac{\partial \varrho}{\partial t}+v\frac{\partial \varrho}{\partial r}+\varrho\left(\frac{\partial v}{\partial r}+\frac{\nu-1}{r}v\right)=0,\\
\varrho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}\right)+\frac{\partial p}{\partial r}=\frac{4}{3}\frac{\partial}{\partial r}\left[\mu\left(\frac{\partial v}{\partial r}-\frac{\nu-1}{2}\frac{v}{r}\right)\right]+2(\nu-1)\frac{\mu}{r}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right),\\
\varrho\left(\frac{\partial \varepsilon}{\partial t}+v\frac{\partial \varepsilon}{\partial r}\right)+p\left(\frac{\partial v}{\partial r}+\frac{\nu-1}{r}v\right)=\frac{\varkappa}{\sigma}r^{1-\nu}\frac{\partial}{\partial r}\left(\mu r^{\nu-1}\frac{\partial \varepsilon}{\partial r}\right)\\
+2\mu\left\{\left(\frac{\partial v}{\partial r}\right)^2+(\nu-1)\frac{v^2}{r^2}-\frac{1}{3}\left[r^{1-\nu}\frac{\partial}{\partial r}(r^{\nu-1}v)\right]^2\right\},\\
p=(\varkappa-1)\varrho\varepsilon,\qquad \mu=A\varepsilon^n,\qquad A,n=\mathrm{const},
\end{gathered}
\tag{1.1}
$$

where $\varkappa$ is the ratio of specific heats, σ — Prandtl number, t — time, r — spatial coordinate, v — gas velocity, ϱ — density, p — pressure and $\varepsilon \equiv c_v T$ — internal energy. The dimensionless parameter ν describes the symmetry type and may become 1, 2 and 3 for plane, axial and central symmetry, respectively.

The mathematical statement of a particular physical problem should enable the establishment of, in addition to the Eqs. (1.1), the initial and boundary conditions, and also any other conditions additionally imposed on the solution. Dimensional analysis of all the determining parameters of the problem involved in the Eqs. (1.1) and the conditions referred to, enables the establishment of the conditions of self-similarity and, if these are satisfied, the passage to nondimensional variables transforming (1.1) into a system of ordinary differential equations. The method of such a transformation, which is explained in the book of L. I. SEDOV [2] as applied to adiabatic flows of a gas with shock waves, can be generalized to the case of dissipative motions considered.

We must now observe that the defining parameters of one-dimensional adiabatic flow almost always include parameters of state of the non-perturbed gas — for instance, the density ϱ_1 and the pressure p_1. If the quantity p_1 is sufficiently small to be assumed to be zero, the resulting reduction in the number of determining parameters results in an essential extension of the class of self-similar motions. The condition

$$p_1 = 0 \tag{1.2}$$

and the condition $\varepsilon_1 = 0$ which follows from (1.2) for a finite ϱ_1 will be considered to be valid also in the case of flow with dissipation, which is considered. Let us observe that if the condition (1.2) is satisfied, the region of perturbed state of the viscous and heat-conducting gas is separated from the region of rest by a surface of weak discontinuity, which will be referred to in what follows as a perturbation front. The necessity of occurrence of such a wave front under the conditions considered was observed by a number of authors [3 to 5] and is connected with the fact that for a power relation between viscosity and temperature ($n > 0$) the viscosity coefficient becomes zero together with the temperature.

The density of the non-perturbed gas ϱ_1 is assumed, in the general case, to be determined by the law

$$\varrho_1 = Br^{-\omega}, \tag{1.3}$$

where ω is a positive constant, which may, in particular, be zero.

It will be assumed that the problem of one-dimensional motion of a viscous, heat-conducting gas enables us to show, within the perturbed region, a certain characteristic surface moving according to a prescribed law

$$r_1 = r_1(t), \tag{1.4}$$

the function $r_1(t)$ involving a fixed dimensionless parameter δ. By way of example, let us mention the problem of a piston moving according to the law $r_p = r_0 t^{\delta}$, in which it is essential to set $r_1 \equiv r_p$. The conditions of other problems, such as the problem of explosion, do not directly involve the function $r_1(t)$, but its form can be established by means of dimensional analysis.

Assuming that the function $r_1(t)$ is differentiable, it is also easy to show the characteristic velocity of the problem

$$U(t) = dr_1/dt. \tag{1.5}$$

Assuming that all the conditions of self-similarity are satisfied (one of them, connected with the property of viscosity, will be discussed somewhat later), we shall replace the two independent variables r and t by a new dimensionless variable

$$\eta = \frac{ar}{r_1(t)}, \tag{1.6}$$

where a is a dimensionless factor enabling us to set $\eta_f = 1$ independently of the real value of the coordinate r_f of the perturbation front.

The transformation of the Eqs. (1.1) is performed according to the equations

$$\begin{gathered} v = \frac{U(t)}{a\delta} V(\eta), \quad \varrho = B\left(\frac{r_1}{a}\right)^{-\omega} R(\eta), \quad p = B\left(\frac{r_1}{a}\right)^{-\omega}\left(\frac{U}{a\delta}\right)^2 P(\eta), \\ \varepsilon = \frac{1}{\varkappa-1}\left(\frac{U}{a\delta}\right)^2 N(\eta), \quad \mu = \chi B\left(\frac{r_1}{a}\right)^{1-\omega}\frac{U}{a\delta} N^n(\eta). \end{gathered} \tag{1.7}$$

The structure of the latter equation, which transforms the coefficient of viscosity μ, is determined by the dimension of that coefficient. By setting equal the right-hand members of that formula and the last of the Eqs. (1.1), it is easy to obtain the following expression of the dimensionless parameter:

$$\chi = \frac{A}{(\varkappa-1)^n}\frac{(U/a\delta)^{2n-1}}{B(r_1/a)^{1-\omega}}, \tag{1.8}$$

characterizing the influence of viscosity and heat conduction.

The right-hand member of the formula (1.8) involves prescribed functions of time $U(t)$ and $r_1(t)$, so that by selecting the parameters n, ω and δ in an arbitrary manner we shall have $\chi = \chi(t)$. However, it will be seen in what follows that the quantity χ is essentially involved in the transform of the equation (1.1), and its dependence on time would affect the self-similarity assumption introduced above. Thus it is clear that it is essential to require that $\chi = \text{const}$ which implies, in turn, a relation between n, ω and δ ensuring the self-similarrity. By the term self-similarity condition we shall in what follows understand this particular relation, because self-similar of the corresponding problem of adiabatic flow is assumed to be ensured beforehand. As regards the additional self-similarity condition for flows of viscous and heat conducting gases, it can in many cases be obtained by means of dimensional analysis only, the approach described above is, however, more general, because it is applicable to a wider class of functions $r_1(t)$.

By substituting (1.7) into (1.1) we obtain a set of ordinary differential equations of a self-similar motion of a viscous, heat conducting gas in the presence of a perturbation front. This set of equations has the form (the prime denoting differentiation with respect to η)

$$\omega\delta R+\delta\eta R'-VR'-RV'-(\nu-1)\eta^{-1}RV = 0,$$

$$R(GV-\delta\eta V'+VV')+P' = \frac{4}{3}\chi\left[N^n\left(V'-\frac{\nu-1}{2\eta}V\right)\right]'+\frac{2(\nu-1)}{\eta}\chi N^n\left(V'-\frac{V}{\eta}\right),$$

$$(1.9)\qquad R(2GN-\delta\eta N'+VN')+(\varkappa-1)P[V'+(\nu-1)\eta^{-1}V] = \chi\frac{\varkappa}{\sigma}\left[(N^nN')'+\frac{\nu-1}{\eta}N^nN'\right]$$
$$+2(\varkappa-1)\chi N^n\left[V'^2+\frac{\nu-1}{\eta^2}V^2-\frac{1}{3}\left(V'+\frac{\nu-1}{\eta}V\right)^2\right],\qquad P = RN.$$

The Eqs. (1.9) constitute a non-linear set of order five involving a number of dimensionless parameters, one of which was introduced above and corresponds to the equation (the dot denoting differentiation with respect to time)

$$(1.10)\qquad G = \delta\frac{\dot{U}/U}{\dot{r}_1/r_1} = \delta\frac{\dot{U}r_1}{U^2}.$$

The condition of $G = \text{const}$ is assumed to be satisfied by appropriate selection of the function $r_1(t)$.

If we consider a particular problem, it is important to study the influence of the parameter χ on the solution, since it is connected with the fundamental effects of viscosity and heat conduction and the Eqs. (1.9) are degenerated for $\chi = 0$ into equations of adiabatic motion.

The boundary conditions for the Eqs. (1.9) cannot be given in a universal form and they depend on the character of the problem. However, as has already been observed, one of the boundaries of the flow region will always be a perturbation front. The conditions at that front — that is, for $\eta = 1$ — are those of smooth passage to a non-perturbed motion and, for the dimensionless functions introduced according to (1.7), take the form:

$$(1.11)\qquad V(1) = N(1) = 0,\qquad R(1) = 1.$$

Assuming that the region of perturbed motion is contained within the interval $\eta_0 \leqslant \eta \leqslant 1$, we must have as a rule two more conditions for $\eta = \eta_0$. The norming factor a involved in the transformation formulae (1.6) and (1.7) gives an additional degree of freedom for the satisfaction of one of the latter conditions. Thus the boundary-value problem can be reduced to a Cauchy problem with conditions at the point $\eta = 1$, one of which varies in order to satisfy the condition for $\eta = \eta_0$. Bearing in mind the singular character of the behaviour of the integral curves near the perturbation front, the neighbourhood of that point requires a special research.

2

It will be assumed that in the neighbourhood of the perturbation front all the functions sought-for can be expressed in the form of a power series of $z = 1-\eta$. If this assumption is impossible, these cases will be discussed separately.

Let us write the first terms of the expansions, in which we are interested:

$$(2.1) \qquad V = A_v z^{\alpha} + B_v z^{\alpha_1} + \dots, \qquad N = A_N z^{\beta} + B_N z^{\beta_1} + \dots, \qquad R = 1 + A_R z^{\gamma} + B_R z^{\gamma_1} + \dots$$

The form of the expansion (2.1) has been selected to satisfy the conditions (1.11). The constants A_v, A_N, A_R are positive real numbers. The exponents α, β, γ etc. are also positive and the remaining expansion coefficients may take any real values. It is also assumed that $\alpha_1 > \alpha$, etc.

The principal part of the solution of problems of the class considered is obtained by numerical integration, therefore we must confine ourselves to a minimum number of terms in the expansion (2.1) enabling us to move away from the singular point $\eta = 1$. On the other hand, in order to ensure solution of the boundary-value problem, these expansions should contain, as a minimum, one term with an arbitrary parameter, which determines the number of terms used in practice.

On substituting the expansions (2.1) into the Eqs. (1.9), our aim being to determine the constants, we must require that the influence of viscosity and heat conduction should already be manifested in the principal terms of those expansions. In the opposite case, the perturbation front would be transformed into a strong discontinuity surface. The second essential requirement is that the results obtained should not contradict the assumptions. If these requirements are satisfied, the third equation of the set (1.9) (the equation of energy) enables us to determine two constants:

$$(2.2) \qquad \beta = \frac{1}{n}, \quad A_N = \left(\frac{n\delta\sigma}{\varkappa\chi}\right)^{1/n}.$$

These expressions are universal, because they are related with only the essential assumption of $n > 0$ and the less logical but acceptable limitation of $\delta > 0$.

Next, if we assume that $\alpha < \beta$, we can find from the second equation of the set (1.9) (the equation of momentum)

$$\alpha = 3\varkappa(4n\sigma)^{-1}$$

for any value of A_v. By confronting this result with the first of the Eqs. (2.2), we see that its being consistent depends on the value of the parameter:

$$\zeta = \frac{3\varkappa}{4\sigma}. \tag{2.3}$$

If $\zeta < 1$, the assumption of $\alpha < \beta$ is not contradictory and the principal term of the expansion of the function $V(\eta)$ has the form:

$$A_v z^{\zeta/n}, \tag{2.4}$$

where A_v is any positive number.

It is evident that the value of the parameter ζ obtained according to (2.3) and connected with the physical properties of the gas essentially influences the constants in the expansions (2.1) and the minimum number of terms involving the arbitrary constant. In particular cases it is not only the values of the constants that depend on ζ, but also the form of the approximate representation of the function in the neighbourhood of the point $\eta = 1$. Thus for instance, for $\zeta = 1$, the principal term of the expansion of $V(\eta)$ cannot be represented in the power form (2.4). The latter must be replaced by the following more complicated representation

$$A_v z^{1/n} - (A_N/n\delta) z^{1/n} \ln z, \tag{2.4a}$$

where A_v is again an arbitrary constant. It will be seen that the term involving the arbitrary constant in the expansion of the function $V(\eta)$ has everywhere a form analogous to (2.4) but this term is principal for $\zeta < 1$ only. For $\zeta > 1$ its ordinal number will depend on the prescribed values of ζ and n. In addition, for certain values of ζ, a component of the same order of magnitude is added to the term just mentioned, with a logarithmic factor [see (2.4a)] and a constant coefficient.

Considering again the expansions (2.1) for $\zeta > 1$, we find:

$$\alpha = \beta = \frac{1}{n}, \quad A_v = \frac{\zeta}{\delta(\zeta-1)} A_N. \tag{2.5}$$

By means of the first of the Eqs. (1.9) (equation of continuity) we can also determine A_R and γ. Three cases can be distinguished:

$$\gamma = 1, \quad A_R = \omega \quad \text{for} \quad \alpha > 1, \omega > 0 \tag{2.6a}$$

$$\gamma = \alpha, \quad A_R = A_v/\delta \quad \text{for} \quad \alpha < 1 \text{ or any } \alpha \text{ while } \omega = 0 \tag{2.6b}$$

$$\gamma = 1, \quad A_R = A_v/\delta + \omega \quad \text{for} \quad \alpha = 1. \tag{2.6c}$$

The formulae (2.6) are valid for any $\zeta \neq 1$. In agreement with (2.4a), these formulae must be modified for $\zeta = 1$ by replacing A_v by the expression:

$$A_v - (A_N/n\delta)\ln z.$$

As already observed, the second approximation terms in the expansions (2.1) are of no practical value for $\zeta \leqslant 1$, but are necessary for $\zeta > 1$, for which α and A_v are determined by the formulae (2.5). Under the additional condition of $n > 1$ or $\alpha < 1$, we find from the third equation of the set (1.9)

$$\beta_1 = 2\alpha = \frac{2}{n}, \quad B_N = -\frac{\varkappa - 1}{2(1+n)} A_v^2. \tag{2.7a}$$

If the condition $\zeta < 2$ is satisfied, it follows from the second equation of the same set of equations that the second-approximation term in the expansion of the function $V(\eta)$ takes the form (2.4), for which it is reasonable to denote the arbitrary coefficient by the symbol B_v in order to preserve the notations (2.1). For $\zeta = 2$, a component with a logarithmic factor and must be added to this term with a coefficient, which can easily be determined by substitution into the fundamental equation. Finally, for $\zeta > 2$, we obtain $\alpha_1 = \beta_1 = 2$, and the constant B_v takes a fixed value. In agreement with the general principle, it is necessary to include higher approximation terms.

If $n < 1$, that is $\alpha > 1$, we obtain, instead of (2.7a),

$$\beta_1 = \alpha+1 = \frac{1+n}{n}, \quad B_N = \frac{1}{2(1+n)} A_N \left(\frac{2}{\delta} G + \frac{\omega-1}{n} + \nu - 1\right). \tag{2.7b}$$

Under the condition of $\zeta < 1+n$, the second-approximation term in the expansion of $V(\eta)$ preserves the form

$$B_v z^{\zeta/n},$$

with an arbitrary coefficient B_v. A logarithmic term is added for $\zeta = 1+n$. For $\zeta > 1+n$, we must set $\alpha_1 = \beta_1 = (1+n)/n$, find the fixed value of B_v, and construct the third approximation.

The above considerations remain valid in the case of $n = \alpha = 1$, but instead of (2.7b) we have now:

$$\beta_1 = 2, \quad B_N = \frac{1}{4} A_v \{[2G+\delta\omega+\delta(\nu-2)](1-\zeta^{-1}) - (\varkappa-1)A_v\}. \tag{2.7c}$$

From the first equation of the system (1.9) we can, if necessary, find the second-approximation terms in the expansion $R(\eta)$. The following cases are possible:

(2.8a) $\gamma_1 = 1, B_R = \omega (\text{if } \zeta < n)$ for $\frac{1}{2} < \alpha < 1, \omega \neq 0,$

$B_R = \omega + B_v/\delta (\text{if } \zeta = n);$

(2.8b) $\gamma_1 = \alpha_1 = \zeta/n, B_R = B_v/\delta$ for $\omega = 0, n < \zeta < 2 (n \geqslant 1),$ for $\omega = 0,$ $\zeta < 1+n (n < 1)$ for $\omega \geqslant 0, \alpha < 1, \zeta < n;$

(2.8c) $\gamma_1 = \alpha, B_R = A_v/\delta (\text{if } \alpha < 2),$ for $\alpha > 1, \omega \neq 0,$

$B_R = A_v/\delta + \frac{1}{2}\omega(\omega+1) \text{ if } \alpha = 2);$

(2.8d) $\gamma_1 = 2, B_R = \frac{1}{2}\omega(\omega+1)$ for $\alpha = 2, \omega \neq 0.$

There are also three cases when these coefficients are not constant:

(2.9a) $\gamma_1 = \alpha_1 = \zeta/n$ for $\alpha < 1, \omega = 0, \zeta = 2,$

$$B_R = \frac{A_v^2}{\delta^2} + \frac{1}{\delta}\left(B_v - \frac{3-\varkappa}{\delta n} A_v^2 \ln z\right);$$

(2.9b) $\quad \gamma_1 = \alpha_1 = 2 \quad$ for $\quad \alpha = 1, \zeta = 2,$

$$B_R = \frac{1}{2}\,\omega(\omega+1) + \frac{A_v}{2\delta}\left[3\omega-(\nu-2)+\frac{2A_v}{\delta}\right]+\frac{1}{\delta}\left[B_v - A_v\left(\frac{3-\varkappa}{\delta}A_v+\frac{3\omega+\nu}{2}\right)\ln z\right];$$

(2.9c) $\quad \gamma_1 = \alpha_1 = 1+n \quad$ for $\quad \alpha > 1, \omega = 0, \zeta = 1+n,$

$$B_R = -\frac{n}{n+1}(\nu-2)\,\frac{A_v}{\delta}+\frac{1}{\delta}\left\{B_v - \frac{A_v}{1+n}\left[(1-n)\,\frac{G}{\delta}+\nu\right]\ln z\right\}.$$

If it is necessary to construct a third approximation this can be done in an analogous manner. However, in many problems of practical importance we can make direct use of the results obtained above for the first two approximations. To elucidate this, let us take $n = 1/2$, which is often used in practice. With such a value of n, two approximations are sufficient if

$$\zeta \leqslant 1+n \quad \text{or} \quad \sigma \geqslant \varkappa/2.$$

For $\varkappa = 7/5$, this reduces to the limitation $\sigma \geqslant 0.7$, applicable for most gases. In the case of $\varkappa = 5/3$, we obtain a somewhat stronger limitation ($\sigma \geqslant 5/6$), but the Prandtl numbers thus obtained do not differ much from the real values.

The investigation carried out enables the following fundamental conclusion. The form of the approximate representation and the number of the necessary expansion terms of the dimensionless equivalents of the hydrodynamical functions of viscous gas in the neighbourhood of the perturbation front depend essentially on the parameter ζ, which is determined by the formula (2.3) and is related with the physical properties of the gas. For definite values of that parameter in the power expansions, logarithmic terms occur and the ordinal number i of the expansion term of $V(\eta)$, containing an arbitrary constant is, in general, higher for greater values of ζ and for smaller values of n. If we confine ourselves to the interval $0 < n < 1$, which is the most interesting for practice, it is easy to obtain the following relation for i

$$(2.10) \qquad (i-2)n < \zeta-1 \leqslant (i-1)n.$$

Thus, for instance, if $n = 1/6$, $\varkappa = 5/3$, $\sigma = 15/22 \approx 0.68$, the Eq. (2.3) yields $\zeta = 11/6$ and from the relation (2.10), we obtain $i = 6$, that is, the arbitrary constant, which is necessary for the solution of a boundary-value problem, occurs only (for such values of the physical parameters) in the sixth term of expansion, in the neighbourhood of the point $\eta = 1$.

Finally, let us observe that the above principles were used by the present author in Ref. [6] for the establishing the equations of self-similar motion, and the investigation and construction of a solution applicable to the particular case of explosion in a viscous, heat conducting gas. The gas considered has such physical parameters that for complete numerical solution of the problem it suffices to know the principal terms of the expansion of the dimensionless functions sought-for in the neighbourhood of the perturbation front. One variant of the piston problem under conditions such that it is necessary to know the first two expansion terms, was considered earlier by V. V. SYCHEV [7], to whom the present author is most grateful for interest and fruitful discussion.

References

1. К. П. Станюкович, *Неустановившиеся движения сплошной среды* [Non-stationary motions of a continuum, in Russian], ГИТТЛ, Москва 1955.
2. Л. И. Седов, *Методы подобия и размерности в механике* [Similarity and dimensional methods in mechanics, in Russian] „Наука", Москва 1972.
3. Л. Д. Ландау, Е. М. Лифшиц, *Механика сплошных сред* [Mechanics of continua, in Russian], Гостехиздат, Москва 1954.
4. Я. Б. Зельдович, Ю. П. Райзер, *Физика ударных волн и высокотемпературных гидродинамических явлений* [Physical problems of shock waves and high temperature hydrodynamic phenomena, in Russian], Физматгиз, Москва 1963.
5. В. Е. Неуважаев, *Распространение сферической волны в теплопроводном газе* [Propagation of a spherical explosion wave in a heat conducting gas, in Russian], ПММ, **26**, вып. 6, 1962.
6. В. П. Шидловский, *Автомодельные движения вязкого и теплопроводного газа при внезапном выделении энергии* [Self-similar motions of a viscous, heat conducting gas under sudden release of energy, in Russian], Изв. АН СССР, Механика жидкости и газа, № 3, 1972.
7. В. В. Сычев, Н. С. Аванесова, *О равноускоренном движении плоской пластины в вязком сжимаемом газе* [On the uniformly accelerated motion of a plane plate in a compressible viscous gas, in Russian], Журн. вычисл. мат. и матем. физики, 3, 6, 1963.

COMPUTING CENTER ACADEMY OF SCI. U.S.S.R.

Received December 8, 1973.