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Deflection of a round turbulent jet in a cross-wind

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A MODEL for the round jet in a cross-wind describes the jet in terms of its lateral spread, its cross-section and its velocity, which is taken constant across every section. New empirical relations for the entrainment by the jet and for the development of the cross-sectional area are proposed. The solution of the balance-equations for mass and momentum, which form a system of ordinary differential equations, is constructed as a Taylor expansion of the cross-wind number σ, which is defined as the ratio of the cross-wind velocity to the initial jet velocity. Analytical expressions are given for the coefficients in the expansion up to second-order terms. The free empirical parameters of the entrainment, the cross-sectional area and the length of the potential core are determined in such a way that the theoretical results fit the values obtained experimentally for the locus of the centre line, the lateral spread of the jet and the mean axial velocity of the jet at any cross-section. It is shown that the empirical parameters can be chosen so, that the theory is in good agreement with the experiments for cross-wind numbers σ between 1/8 and 1/4. However, for $\sigma = 1/20$, the theoretically determined deflections of the jet are greater than the experimental values.

Model okrągłego strumienia w przepływie poprzecznym opisuje strumień w funkcji jego rozszerzenia przekroju i prędkości, która w każdym przekroju jest przyjęta jako stała. Zaproponowano nowe zależności empiryczne dla "załadowania" (entrainment) oraz dla wzrostu przekroju poprzecznego. Uzyskano rozwiązanie równań bilansu masy i pędu, stanowiących układ równań różniczkowych zwyczajnych, w postaci taylorowskiego rozwinięcia dla liczby przepływu poprzecznego σ, zdefiniowanej jako stosunek prędkości przepływu poprzecznego do początkowej prędkości strumienia. Podano wyrażenia analityczne na współczynniki rozwinięcia do drugiego rzędu włącznie. Swobodne parametry empiryczne: "załadowanie", pole przekroju poprzecznego oraz długość rdzenia potencjalnego są określane w taki sposób, że wyniki teoretyczne zgadzają się z wartościami doświadczalnymi dla położenia linii centralnej, rozszerzenia strumienia oraz średniej prędkości osiowej strumienia we wszystkich przekrojach. Pokazano, że parametry empiryczne mogą być dobrane w taki sposób, że teoria zgadza się dobrze z doświadczeniem dla obu liczb przepływu poprzecznego σ pomiędzy 1/8 i 1/4. Dla $\sigma = 1/20$ odchylenia strumienia określone teoretycznie są większe od wartości doświadczalnych.

Модель кругового потока в поперечном течении описывает поток в функции его расширения сечения и скорости, которая в каждом сечении принимается постоянной. Предложены новые эмпирические зависимости для „загрузки'' (entrainment) и для роста поперечного сечения. Получено решение уравнений баланса массы и импульса, составляющих систему обыкновенных дифференциальных уравнений, в виде тейлоровского разложения для числа поперечного течения σ, определенного как отношение скорости поперечного течения к начальной скорости потока. Даются аналитические выражения для коэффициентов разложения ко второму порядку включительно. Свободные эмпирические параметры: „загрузка'', поле поперечного сечения и длина потенциального сердечника определены таким образом, чтобы теоретические результаты совпадали с экспериментальными значениями для положения центральной линии, расширения потока и средней осевой скорости потока во всех сечениях. Показано, что эмпирические параметры могут быть подобраны таким образом, чтобы теория хорошо согласовалась с экспериментом для обоих чисел поперечного течения σ между 1/8 и 1/4. Для $\sigma = 1/20$ отклонения потока определенные теоретически больше чем экспериментальные значения.

1. Introduction

THE TURBULENT structure of plane and of round jets, which are injected into a medium at rest, is known from the theories of TOLLMIEN [13], GÖRTLER [5] and SCHLICHTING [11, p. 699] and from the measurements of REICHARDT [9]. RICOU and SPALDING [10] used a new method to determine the entrainment coefficient of the jet directly.

If the jet is injected into a cross-wind, the turbulence pattern of the jet is substantially modified by its own turbulent wake. Also, the deflection of the jet gives rise to a secondary flow similar to that in a curved pipe.

As long as no full theoretical description of all these details exists, the jet flow is approximated by models. The model of WOOLER, BURGHART and GALLAGHER [14] describes the jet by the centre line, the cross-sectional area, the lateral spread and the mean velocity of every cross-section. This model takes into account the interaction between the jet and the cross-wind due to the drag of the jet in the cross-wind and due to the entrainment of fluid by the jet. In the above theory, empirical equations are used for the entrainment by the jet and for the cross-sectional area.

In the following new empirical equations for these two quantities are proposed. The empirical constants in these equations are determined in such a way that the theory does not only fit the measurements of the centre line by KEFFER and BAINES [8] and by JORDINSON [7], as does the theory of WOOLER, BURGHART and GALLAGHER, but also fits the measurements of the lateral spread and the mean velocity by KEFFER and BAINES.

In many practical cases, the initial jet flow is strong in comparison with the cross-flow; the downward directed jet of a VTOL plane in the transition phase is an example. In such cases, the cross-wind number — that is the ratio of the velocity of the cross-wind to the initial velocity of the jet is small. Therefore, a Taylor series expansion for small cross-wind numbers is proposed here. Contrary to the method of WOOLER, BURGHART and GALLAGHER this leads to analytical solutions.

Le GRIVÈS and BENOIT [6] and ABRAMOVICH [1, p. 549] also give models for the round jet in a cross-wind; but they neglect the effect of entrainment. This effect, however, is considered to be the only effect of the interaction between the jet and the cross-wind as given in the model by BRAUN and McALLISTER [2].

2. Basic equations

A jet is formed by the flow of a fluid through a circular orifice of diameter D, as shown in Fig. 1. The initial direction of flow in the jet is perpendicular to the plane of a plate in which the orifice lies. In the orifice, the jet has the constant velocity U_{j0}^* over the entire cross-section. A Cartesian system of spatial coordinates (x^*, z^*) is introduced with the origin in the centre of the orifice. The x^*-axis has the direction of the cross-wind; this cross-wind is parallel to the plate and has the velocity U_∞^*. The z^*-axis is perpendicular to the plate and has the direction of the initial jet velocity. The centre line M of the jet is defined as the line which passes through the points of maximum velocity of the various cross-sections; the origin of the coordinate system also lies on the centre line. Along

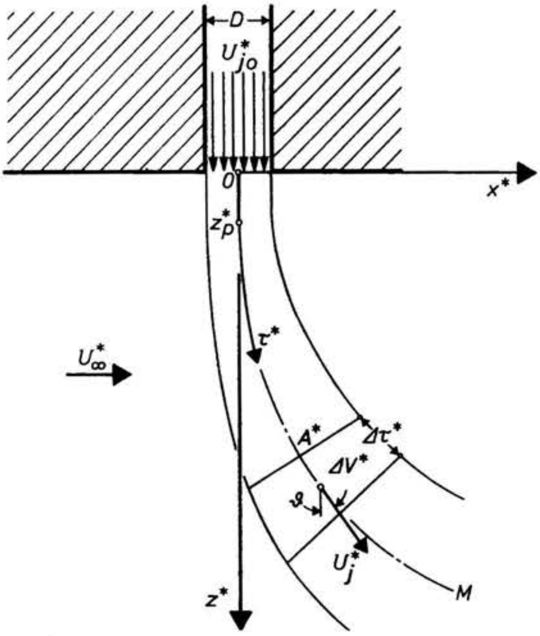

FIG. 1. Model of a round turbulent jet in a cross-wind.

this line the arc length τ^* is introduced. The angle ϑ between the centre line and the z^*-axis is the angle of deflection of the jet. The area of the cross-section of the jet is denoted by A^*.

Since the turbulent structure of the jet in the cross-wind is very complicated, the following model is taken as a first approximation for the jet flow:

(i) It is assumed that the jet flow up to $z^* = z_p^*$ is not influenced by the cross-wind; that is, between $z^* = 0$ and $z^* = z_p^*$ the flow is a uniform potential flow.

(ii) Also, it is assumed that the jet velocity has a constant value U_j^* across every cross-section perpendicular to the centre line. The direction of this velocity is assumed to be parallel to the centre line.

2.1. Balance equations

With the above assumptions, the balance equations for mass and momentum are formulated for a volume element ΔV^*. In the limit $\Delta\tau^* \to 0$ the balance of mass is given by:

$$\varrho^* \frac{d}{d\tau^*}(A^* U_j^*) = \varrho^* e^*, \tag{2.1}$$

where ϱ^* is the density of the fluid. The left-hand side of this equation represents the variation of the mass flux from one cross-section to the next one; this variation is caused by the fact that due to the turbulent frictional forces, fluid is entrained through the surface of the jet; e^* is the entrained volume of fluid per length of the jet and per time.

The momentum balance in the direction of the centre line and perpendicular to it is represented by the following equations:

$$\varrho^* \frac{d}{d\tau^*}(A^* U_j^{*2}) = \varrho^* e^* U_\infty^* \sin\vartheta, \tag{2.2}$$

$$\varrho^* A^* \frac{U_j^{*2}}{R^*} = \varrho^* e^* U_\infty^* \cos\vartheta + c_D b^* \frac{\varrho^*}{2} U_\infty^{*2} \cos^2\vartheta. \tag{2.3}$$

Here, R^* is the radius of the curvature of the centre line; the drag coefficient is denoted by c_D and the lateral spread of the jet by b^*. The variation of the momentum flux of the jet is caused by the fact that the entrained mass $\varrho^* e^*$ transfers its momentum to the jet, and by the frictional force of the cross-wind. The influence of pressure gradients is neglected.

In this model, the frictional forces of the cross-wind are estimated as follows: If the jet would not entrain fluid from the cross-wind, the relative velocity between the jet and the cross-wind would be responsible for the frictional forces. But since part of the cross-wind flow is entrained by the jet, the frictional forces are smaller. In order to take this effect into account, only the component $U_\infty^* \cos\vartheta$ of the relative velocity is used to calculate the frictional forces. The other component of the relative velocity determines the amount of cross-wind, which is entrained.

These balance equations were given by WOOLER, BURGHART and GALLAGHER [14] for the first time.

2.2. Empirical equations

In the balance Eqs. (2.1), (2.2) and (2.3), the density ϱ^* cancels. The centre line of the jet may be described by a function $x^* = x^*(z^*)$, and the arc length τ^* and the radius R^* of the curvature of the centre line may be expressed by this function. Then the balance equations are three equations for the five unknown functions of z^*, namely A^*, U_j^*, e^*, b^* and x^*. Therefore, two further equations are needed in order to determine all unknown functions. Empirical equations may be used for the mass e^* entrained by the jet and for the area A^* of the cross-section of the jet. Since both quantities e^* and A^* are known for the case of vanishing cross-wind, these known equations are extended to the case with non-vanishing cross-wind by introduction of empirical functions.

Following the theories of TOLLMIEN [13] and SCHLICHTING [11, p. 699] for zero cross-wind — that is, for $U_\infty^* = 0$ — the value of $e^*/(U_j^* b^*)$ is constant. In the case with no cross-wind, the quantity U_j^* must be replaced by the corresponding quantity $(U_j^* - U_\infty^* \sin\vartheta)$. Furthermore, the value of $e^*/\{(U_j^* - U_\infty^* \sin\vartheta) b^*\}$ is no longer constant; it is assumed, that it depends only on the ratio of the two components of the relative velocity between the cross-wind and the jet:

$$\frac{e^*}{(U_j^* - U_\infty^* \sin\vartheta) b^*} = f\left(\frac{U_\infty^* \cos\vartheta}{U_j^* - U_\infty^* \sin\vartheta}\right). \tag{2.4}$$

The area A^* of the cross-section is the area of a circle with the diameter b^* for a jet without cross-wind. In the case with cross-wind a correction factor, g, is used; this factor takes into account the deformation of the area of the cross-section by the cross-wind;

it is assumed that this function depends only on the ratio of the distance z^* from the plate and the diameter D of the orifice, and on the ratio of the velocities U_∞^* of the cross-wind and U_{j0}^* of the jet in the orifice:

$$A^* = \frac{\pi}{4} b^{*2} g\left(\frac{z^*}{D}, \frac{U_\infty^*}{U_{j0}^*}\right). \tag{2.5}$$

The Eqs. (2.4) and (2.5) proposed here are different from those given by WOOLER, BURGHART and GALLAGHER [14].

2.3. Boundary conditions

Since it is assumed that the jet flow is a uniform potential flow between $z^* = 0$ and $z^* = z_P^*$, the boundary conditions at the end of the potential flow — i.e., at the beginning of the turbulent flow — are as follows:

$$U_j^*(z_P^*) = U_{j0}^*, \quad b^*(z_P^*) = D, \quad x^*(z_P^*) = 0, \quad \left.\frac{dx^*}{dz^*}\right|_{z^*=z_P^*} = 0. \tag{2.6}$$

3. Taylor expansion for small cross-wind numbers

The ratio of the velocities U_∞^* of the cross-wind and U_{j0}^* of the jet in the orifice is called the cross-wind number:

$$\sigma = \frac{U_\infty^*}{U_{j0}^*}. \tag{3.1}$$

This number is a measure of the influence of the cross-wind on the jet and vice versa.

First, the limit of a cross-wind number $\sigma = 0$ may be considered. In this case, the jet is injected into a medium at rest. Therefore, only the momentum transferred from the jet to the medium at rest gives rise to a force on the plate. Since this momentum is directed downward (Fig. 1), the force on the plate is directed upward.

Secondly, the opposite limit may be considered, in which the cross-wind number tends to infinity. This occurs when the cross-wind has the finite velocity U_∞^*, while the fluid is initially at rest in the orifice. Then this fluid is entrained by the cross-wind. Due to the friction of this entrained fluid at the wall in front of the orifice, a downward directed force acts on the plate.

For finite values of σ, these two opposite effects are superimposed. It follows that only for moderate cross-wind numbers σ, the force on the plate is directed upwards; from experiments, it is known that σ should not exceed a value of one third (see [8], e.g.).

For small cross-wind numbers,

$$\sigma \ll 1 \tag{3.2}$$

it is not necessary to solve the whole system of differential equations. Every unknown function $\Phi(z, \sigma)$ is expanded into a Taylor series with respect to σ:

$$\Phi(z, \sigma) = \Phi^{(0)}(z) + \sigma\Phi^{(1)}(z) + \sigma^2\Phi^{(2)}(z) + \ldots. \tag{3.3}$$

In this expression, as well as in the following, non-dimensional quantities are used. That is, all lengths are divided by D and all velocities by U_{J0}^*. These non-dimensional quantities are denoted by the same symbols as the corresponding dimensioned quantities, but without the asterisk. The beginning of the z-axis, $z = 0$, is taken at the end of the potential flow — that is, at $z^* = z_P^*$.

The series expansions of the empirical functions g for the cross-sectional area and f for the entrained mass will be explained in detail:

Since the cross-sectional area of a jet without cross-wind is a circular area,

$$g^{(0)}(z) \equiv 1. \tag{3.4}$$

Also, at the end of the potential flow of the jet the cross-section is circular; therefore, $g^{(i)}(0) = 0$. From the experiments of SHANDOROV [12], JORDINSON [7] and KEFFER and BAINES [8] it is known that between $z = 0$ and $z = 1$ the shape of the cross-section changes rapidly from a circular one to one similar to a horseshoe; for $z > 1$ this shape varies only slightly, although the cross-sectional area increases. The least complex functions, which have these desired properties, are:

$$g^{(i)}(z) = \left\{\begin{matrix} (-1)^i a^{(i)} z & \text{for} & 0 \leqslant z \leqslant 1 \\ (-1)^i a^{(i)} & \text{for} & z > 1 \end{matrix}\right\} i = 1, 2 \tag{3.5}$$

with constant $a^{(i)}$.

The argument of the empirical function f, which will be called η, is small for small cross-wind numbers; therefore f is expanded into a Taylor series with respect to this argument:

$$f(\eta) = e^{(0)} + e_1\eta + e_2\eta^2 + \ldots; \quad \eta = \frac{\sigma}{U_J\sqrt{1+\left(\frac{dx}{dz}\right)^2} - \sigma\frac{dx}{dz}}. \tag{3.6}$$

The entrainment coefficient $e^{(0)}$ for a jet without cross-wind has been measured by RICOU and SPALDING [10] using a direct method:

$$e^{(0)} = 0.251. \tag{3.7}$$

The theory of SCHLICHTING [11] and the measurements of REICHARDT [9] for the velocity distribution lead to $e^{(0)} = 0.266$.

3.1. Zeroth-order solution

The zeroth-order solution describes a jet without cross-wind:

$$x^{(0)}(z) \equiv 0, \quad b^{(0)}(z) = 1 + \frac{4e^{(0)}}{\pi} z, \quad U_J^{(0)}(z) = (b^{(0)}(z))^{-1}. \tag{3.8}$$

Since in this case the jet is not deflected, the equation for its centre line is $x^{(0)} = 0$. The lateral spread $b^{(0)}$ of the jet increases in proportionally to z beginning with the value one at the end of the potential flow. This increase is determined by the entrainment coefficient $e^{(0)}$.

Since the forces caused by pressure gradients are neglected, no force acts upon the jet in the z-direction. Hence the total momentum flux through a cross-section of the jet is independent of z, as TOLLMIEN has already stated [13]. But the momentum flux is proportional to the cross-sectional area of the jet — that is, proportional to the square of the diameter $b^{(0)}$; moreover, the momentum flux is proportional to the square of the jet velocity $U_j^{(0)}$. Hence $b^{(0)}U_j^{(0)}$ is independent of z.

This simple model of a jet without cross-wind gives a lateral spreading of the jet with $db^{(0)}/dz = 0.319$. The more detailed model of SCHLICHTING [11] leads to the value of 0.170.

3.2. First-order solution

In the first-order solution, the cross-wind causes a parabolic deflection of the centre line of the jet:

$$x^{(1)}(z) = \frac{2e^{(0)}}{\pi} z^2. \tag{3.9}$$

The total deflection $x(z)$ is proportional to the cross-wind number σ and to the entrainment coefficient $e^{(0)}$ of the jet without cross-wind. The drag coefficient c_D is not involved in this equation. This leads to the conclusion that the entrainment of momentum has a larger influence on the deflection than the action of the frictional forces between the cross-wind and the jet.

The equations for the lateral spread

$$b^{(1)}(z) = -\frac{1}{2} g^{(1)}b^{(0)} + \frac{e_1}{2e^{(0)}}(b^{(0)2}-1) - \frac{2e^{(0)}}{\pi} G^{(1)}(z) \tag{3.10}$$

with

$$G^{(1)}(z) \equiv \int\limits_0^z g^{(1)}(z')\,dz' \tag{3.11}$$

and for the velocity

$$U_j^{(1)}(z) = -U_j^{(0)2}b^{(1)} - \frac{1}{2} U_j^{(0)}g^{(1)} \tag{3.12}$$

contain not only the entrainment coefficient $e^{(0)}$, but also e_1; furthermore, the function $g^{(1)}(z)$ — i.e., the deviation of the cross-sectional area from the circular form — is involved here. The influence of the frictional forces given by c_D does not yet appear in this order of the solution.

3.3. Second-order solution

In contrast with the first-order coefficient of deflection of the centre line $x^{(1)}$, the corresponding second-order coefficient

$$x^2(z) = \frac{2}{\pi}\left(e_1 + \frac{c_D}{2}\right)\left(1 + \frac{4e^{(0)}}{3\pi} z\right) z^2 - \frac{2e^{(0)}}{\pi} \int\limits_0^z G^{(1)}(z')\,dz' \tag{3.13}$$

contains not only $e^{(0)}$ but also $e_1, g^{(1)}(z)$ and c_D. The coefficient of the velocity

$$(3.14)\quad U_j^{(2)}(z) = b^{(0)}U_j^{(1)^2} + \frac{7}{12}b^{(0)} - \frac{3}{4} - \frac{1}{4}\frac{1}{b^{(0)}} + \frac{5}{12b^{(0)^2}} + \frac{2e^{(0)}}{\pi b^{(0)^2}}\int_0^z \left(g^{(2)}(z') - \frac{3}{4}g^{(1)^2}(z')\right)dz' + \frac{2e_1}{\pi b^{(0)}}G^{(1)} + \frac{e_1^2}{6e^{(0)^2}}\left(-b^{(0)} + \frac{3}{b^{(0)}} - \frac{2}{b^{(0)^2}}\right) - \frac{e_2}{3e^{(0)}}\left(b^{(0)} - \frac{1}{b^{(0)^2}}\right),$$

and of the lateral spread of the jet

$$(3.15)\quad b^{(2)}(z) = -b^{(0)^2}U_j^{(2)} + b^{(0)^3}U_j^{(1)^2} + \frac{1}{2}g^{(1)}b^{(0)^2}U_j^{(1)} + b^{(0)}\left\{\frac{1}{4}(b^{(0)}-1)^2 + \frac{3}{8}g^{(1)^2} - \frac{1}{2}g^{(2)}\right\}$$

even in this order are not influenced by the frictional forces between the cross-wind and the jet, but only by the entrainment of momentum.

3.4. Third-order solution for the centre line

Since contrary to the other quantities the centre line is trivial in the zeroth order [see $(3.8)_1$] an additional coefficient of this quantity is calculated. Indeed, this is possible without knowledge of the other third-order coefficients:

$$(3.16)\quad x^{(3)}(z) = \frac{4}{\pi}\int_0^z dz'\left(\int_0^{z'} H(z'')dz''\right),$$

$$H(z) \equiv 2b^{(0)}U_j^{(2)}\left\{e^{(0)} + \frac{e^{(0)}}{2}g^{(1)} - \left(e_1 + \frac{c_D}{2}\right)b^{(0)}\right\} - \frac{e^{(0)}}{2}(b^{(0)}-1)^2 + \frac{1}{4}(b^{(0)^2} - 6b^{(0)} + 5) - \frac{1}{2}g^{(2)} + \frac{3}{8}g^{(1)^2} + \left(e_1 + \frac{c_D}{2}\right)b^{(1)} + e_2 b^{(0)^2}.$$

4. Determination of the empirical parameters by comparison with experimental results

The empirical parameters have been determined so, that the theory fits the experimental results of KEFFER and BAINES [8] and those of JORDINSON [7]. The centre line, the lateral spread and the mean velocity of the jet, which have been measured by KEFFER and BAINES for the cross-wind numbers $\sigma = 1/8$ and $\sigma = 1/4$, have been used for this comparison; furthermore, the centre line given by the measurements of JORDINSON for $\sigma = 1/8.1$ and $\sigma = 1/4.3$ has also been used. In the present theory, the value of $c_D = 1.8$ has been taken, which gives the drag coefficient of an elliptical cylinder with the axis of the cylinder and the larger axis of the cross-section perpendicular to the flow.

In order to compare the experimental results of KEFFER and BAINES with the present theory, at a given cross-section the distance between those two lateral points, in which the velocity excess above the external undisturbed flow, $(U^*-U^*_\infty)$, is half the maximum excess at this cross-section, has been taken as lateral spread b^*. Further, the velocity U^*_j has been identified with the mean velocity $\overline{U}^*$ of the cross-section, which has been taken as defined by $(\overline{U}^*-U^*_\infty) = \frac{3}{4}(U^*_m-U^*_\infty)$; here, U^*_m is the maximum velocity of the cross-section.

Close agreement between theory and the above cited experiments is obtained if the parameters take the following numerical values:

$$e_1 = -0.2, \quad e_2 = 1.7, \quad a^{(1)} = 1, \quad a^{(2)} = 4, \quad z_P(\sigma) = \frac{1}{15}\frac{1}{\sigma\sqrt{\sigma}}. \tag{4.1}$$

Furthermore, experiments of GERTSBERG [4], who measured the centre line for a cross-wind number $\sigma = 1/20$, have been taken into account to determine the function $z_P(\sigma)$. Since a weak potential jet is destroyed instantaneously by a strong cross-wind by means of the turbulent frictional forces, z_P tends to zero for large cross-wind numbers σ. The limit of zero cross-wind number $\sigma = 0$ is not given correctly by $(4.1)_5$; for, following this formula, $z_P(0)$ has no finite value — that is, the jet flow would remain a potential flow.

In the following figures the experimental results of different authors are compared with the present theory, the empirical parameters of which have been determined by the method outlined above.

In Fig. 2 the centre line of the jet is plotted for a cross-wind number σ of approximately a quarter. The experimental results of ENDO and NAKAMURA [3] and those of KEFFER

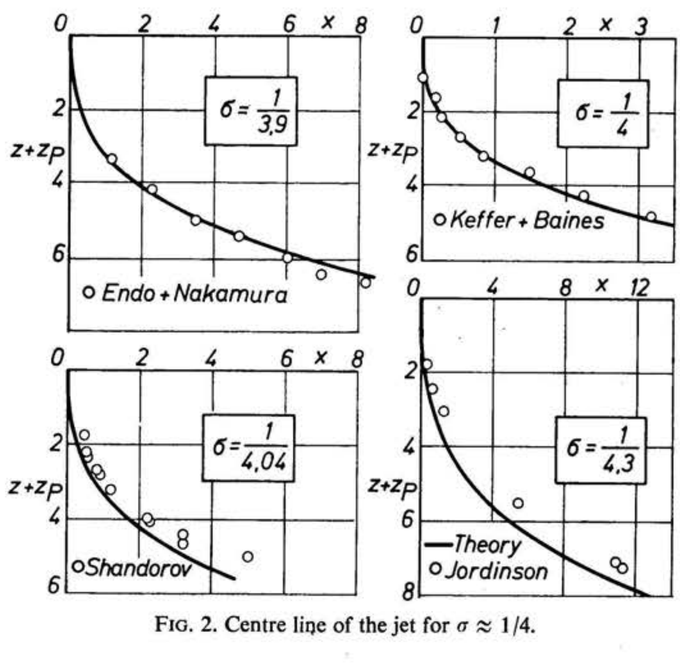

FIG. 2. Centre line of the jet for $\sigma \approx 1/4$.

and BAINES compare well with this theory. On the other hand, the theory gives somewhat smaller deflections of the jet than the experiments of SHANDOROV [12] and those of JORDINSON. Perhaps this may be caused by the fact that the experimental results of different authors are obtained on different conditions; these should be described by additional parameters, which are not considered in the theory.

Figure 3 shows good agreement of the centre line given by the theory with the measurements by JORDINSON for the cross-wind number $\sigma = 1/8.1$. For the very small cross-wind

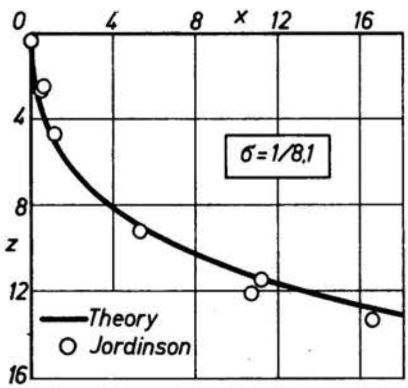

FIG. 3. Centre line of the jet for $\sigma = 1/8.1$.

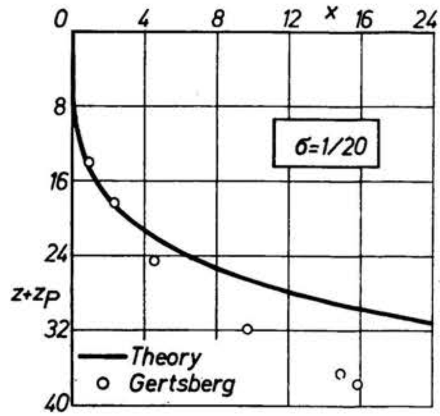

FIG. 4. Centre line of the jet for $\sigma = 1/20$.

number $\sigma = 1/20$, the theoretical results are compared with the experiments of GERTSBERG [4] in Fig. 4. It can be seen that the theory describes the measurements well up to a distance from the plate $z+z_P = 18$, but it gives deflection values which are too large for greater distances. Since the local cross-wind number U_∞^*/U_j^* tends to one for large distances from the plate, the quality of the theory decreases with increasing z. The given Taylor series expansions with respect to σ are not justified in this case, because

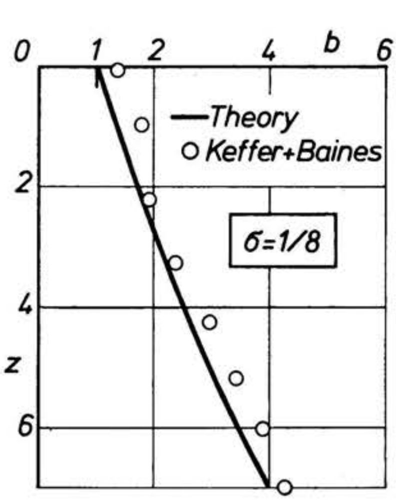

FIG. 5. Lateral spread of the jet.

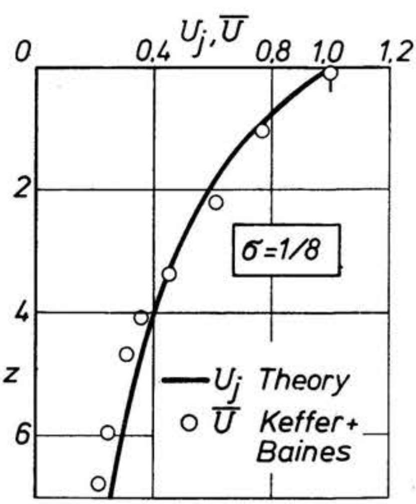

FIG. 6. Velocity of the jet.

the jet velocity U_j has the order of σ, while in the series expansions it is assumed that U_j has the order of one. Another reason for the breakdown of the theory at large distances from the orifice is the fact that the real flow has a strong vorticity, which is not taken into consideration by the present theory.

Figures 5 and 6 show the good agreement of the lateral spread b and of the velocity U_j with the experimental results of KEFFER and BAINES for the mean velocity for $\sigma = 1/8$. It can be seen that the lateral spread increases monotonically with increasing z; at $z = 7$ it has already quadrupled. The velocity decreases monotonically with increasing z.

References

1. G. N. ABRAMOVICH, *The theory of turbulent jets*, MIT Press, Cambridge, Mass, 1963.
2. G. W. BRAUN and J. D. MCALLISTER, *Cross-wind effects on trajectory and cross-sections of turbulent jets*, in: Analysis of a jet in a subsonic cross-wind, Symp. Hampton, Virginia NASA SP-218, 141, 1969.
3. H. ENDO and M. NAKAMURA, *Bending and development of a three-dimensional turbulent jet in a cross-wind* [in Japanese], NAL TR 216, 1968.
4. M. B. GERTSBERG, *Trajectory of a liquid jet injected into a subsonic flow*, IZV Vyss. Uc. Zav. Aviacionnaja Technika, 99, 1970.
5. H. GÖRTLER, *Berechnung von Aufgaben der freier Turbulenz auf Grund eines neuen Näherungsansatzes*, ZAMM, **22**, 244, 1942.
6. É. LE GRIVÈS et A. BENOIT, *Nouveau modèle théorique d'analyse de la pénétration d'un jet dans un écoulement subsonique*, Rech. Aérosp., no. 148, 133, 1972.
7. R. JORDINSON, *Flow in a jet directed normal to the wind*, ARC, RM 3074, 1958.
8. J. F. KEFFER and W. D. BAINES, *The round turbulent jet in a cross-wind*, J. Fluid Mech., **15**, 481, 1963.
9. H. REICHARDT, *Gesetzmäßigkeiten der freien Turbulenz*, VDI-Forschungsheft, 414, 1942.
10. F. P. RICOU and D. B. SPALDING, *Measurements of entrainment by axisymmetrical turbulent jets*, J. Fluid Mech., **11**, 21, 1961.
11. H. SCHLICHTING, *Boundary-layer theory*, 6th ed. McGraw-Hill, New York 1968.
12. G. S. SHANDOROV, *Flow from a Channel into stationary and moving media*, Zh. Tekhn. Fiz., **37**, 1, 1957
13. W. TOLLMIEN, *Berechnung turbulenter Ausbreitungsvorgänge*, ZAMM, **6**, 468, 1926.
14. P. T. WOOLER, G. H. BURGHART and J. T. GALLAGHER, *Pressure distribution on a rectangular wing with a jet exhausting normally into an airstream*, J. Aircraft, **4**, 537, 1967.

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