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Aerodynamic interference for the system of two spheres moving in free-molecular medium

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IN THE PAPER the classical problems of the drag and heat exchange for the system of two spheres, moving in free-molecular medium, are solved. The diffusion model of gas-wall interaction is assumed and the effects of the interaction and the screening one sphere by another are taking into account. The problem reduces to the solving of the impermeability conditions which forms the system of integral equations of Fredholm of the second kind. The solving of the problem in general form analytically as well as numerically is impossible. For this reason we solved only the physically and geometrically simplest cases; for example the case of the system moving along the axis connecting the centres of the spheres in approximation of their great distances (in comparison to the radii of the spheres) and small or great velocities of the system in comparison to the thermal velocity of the gas medium). For the spheres resting in the medium we receive the regularity — the spheres hotter than the gas of the medium repulse themselves, the colder ones — attract themselves; the interaction forces at the great distances are inversely proportional to the square of distance. For the system moving in the medium the interaction forces at small distances are comparable with the drag. The interaction heat exchange for the system of two spheres with equal temperatures, moving with arbitrary velocity, is completely determined by the screening of one sphere by another.

W pracy rozwiązano klasyczne problemy oporu i wymiany ciepła dla układu dwu kul, poruszającego się w ośrodku swobodnie-molekularnym, z uwzględnieniem efektów interakcji i zasłaniania jednej kuli przez drugą. W pracy przyjęto dyfuzyjny model oddziaływania gazu z powierzchnią. Problem sprowadza się do rozwiązania warunków nieprzenikalności powierzchni, które tworzą układ równań całkowych Fredholma II rodzaju. Ponieważ rozwiązanie problemu w postaci ogólnej zarówno analitycznie jak i numerycznie jest niemożliwe, problem rozwiązano jedynie w przypadkach najprostszych fizycznie i geometrycznie np. w przypadkach układu poruszającego się wzdłuż osi łączącej środki kul przy zastosowaniu przybliżenia dużych odległości (w porównaniu do promieni kul) i przybliżenia małej lub dużej prędkości układu (w porównaniu do prędkości termicznej gazu ośrodka). Dla kul spoczywających w ośrodku otrzymujemy regułę oddziaływania — kule gorętsze niż gaz ośrodka odpychają się, zimniejsze — przyciągają się; siły interakcji na dużych odległościach są odwrotnie proporcjonalne do kwadratu odległości. Dla układu poruszającego się w ośrodku siły interakcji na małych odległościach są porównywalne z oporem. Interakcyjna wymiana ciepła dla układu dwu kul o równych temperaturach, poruszającego się z dowolną prędkością, jest określona zupełnie przez zasłanianie jednej kuli przez drugą.

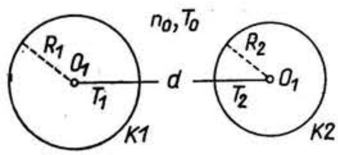
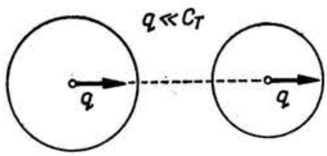
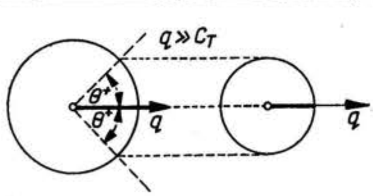
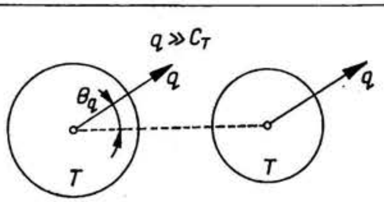
В работе решены классические задачи сопротивления и теплообмена для системы двух сфер, движущейся в свободно-молекулярной среде с учетом эффектов взаимодействия и закрытия одной сферы второй. В работе принята диффузная модель взаимодействия газа с поверхностью. Задача сводится к решению условий непроницаемости поверхности, которые образуют систему интегральных уравнений Фредгольма II рода. Т. к. решение задачи в общем виде, так аналитически, как и численно, невозможно задача решена только для самых простых физически и геометрически случаев, например для случаев системы движущейся вдоль оси соединяющей центры сфер, при применении приближения больших расстояний (по сравнению с радиусами сфер) и приближения малой и большой скорости системы (по сравнению с тепловой скоростью газа среды). Для сфер неподвижных в среде получаем следующее правило взаимодействия: сферы более нагретые чем газ среды — отталкиваются, более холодные — притягиваются; силы взаимодействия на больших расстояниях обратно пропорциональны к вадрату расстояния. Для системы движущейся в среде силы взаимодействия на малых расстояниях сравнимы с сопротивлением. Взаимный теплообмен для системы двух сфер с равными температурами, движущейся с произвольной скоростью, определен полностью закрытием одной сферы второй.

If two or more bodies move in the medium, then the presence of other bodies influences the aerodynamic characteristics of the body considered.

The problem of the influence of the presence of some bodies on the aerodynamic characteristics of other bodies is called the problem of aerodynamic interference.

In the present paper we deal with the problem of the interference for the system of two spheres moving in free-molecular medium. The free-molecular medium is such, in which

Table 1. Solved

Problem	Subject	Solution of integral equations
	Interaction forces	exact analytic
	Drag and interaction forces -	exact analytic in great distances approximation
	Drag and interaction forces	exact analytic in great distances approximation
	Heat exchange	eliminated

where m — mass of particle, k — Boltzmann constant, $A_z^{(i)} = n_0 \left(\frac{2\pi k T_0}{m}\right)^{-\frac{3}{2}}$, $\mathbf{r} = 0_1 0_2$ n_0 — density $A_z^{*(i)} = A_z^{(i)} e^{-B_z^{(i)} q^2}$, $k_1 = \frac{R_1}{d}$, $k_2 = \frac{R_2}{d}$, $k_1 = 0_1 0_2/\mathrm{d}$, $\sin\Theta^+ = R_2/R_1$ C_T — thermal velocity of the gas the mean free path of the gas particles is much greater than the dimensions of the bodies and their mutual distances.

We consider a system of two spheres K_1, K_2 with radii R_1, R_2, temperatures T_1, T_2, resting or moving with uniform velocity $\mathbf{q}$ in free-molecular medium with number n_0, temperature T_0. Fig. 1 (p. 226).

problems and results

Calculation of 5-fold integrals of interaction	Results
exact analytic and numerical	$\mathbf{F}_{2\to 1} = \frac{3}{8}\sqrt{\pi}\, m A_z^{(i)}\left(\frac{2kT_0}{m}\right)^2\left[\left(\frac{2kT_2}{m}\right)^{\frac{1}{2}} - \left(\frac{2kT_0}{m}\right)^{\frac{1}{2}}\right]\bar{g}\frac{\mathbf{r}}{r}$ $\mathbf{F}_{1\to 2} = \mathbf{F}_{2\to 1}(1 \rightleftarrows 2)$
exact analytic in great distances approximation	$\mathbf{F}_1 = -2mA_z^{*(i)}\left(\frac{2kT_0}{m}\right)^{\frac{3}{2}}\pi^2 R_1^2\left\{\frac{1}{9}q\left[5\left(\frac{2kT_0}{m}\right)^{\frac{1}{2}} + \frac{3}{4}\pi\left(\frac{2kT_1}{m}\right)^{\frac{1}{2}}\right.\right.$ $+k_2^2\left\{\frac{3}{16}\sqrt{\pi}\left(\frac{2kT_0}{m}\right)^{\frac{1}{2}}\left[\left(\frac{2kT_2}{m}\right)^{\frac{1}{2}} - \left(\frac{2kT_0}{m}\right)^{\frac{1}{2}}\right] - q\left[\left(\frac{2kT_0}{m}\right)^{\frac{1}{2}}\right.\right.$ $\left.\left.\left.\left. + \frac{1}{18}\left(\frac{2kT_2}{m}\right)^{\frac{1}{2}} + \frac{13}{72}\pi^2\left(\frac{2kT_1}{m}\right)^{\frac{1}{2}}\right]\right\}\right\}\mathbf{k}_1$ $\mathbf{F}_2 = \mathbf{F}_1(1 \rightleftarrows 2,\ q \to -q)$
exact analytic in great distances approximation	$\mathbf{F}_1 = -m\pi R_1^2 q^2 n_0 \cos^2\Theta^+ \mathbf{k}_1 - \frac{4}{9} m\pi^2 R_1^2 q n_0 \cos^3\Theta^+ \times$ $\times\left[\frac{1}{3}k_1^2 k_2^2\sqrt{\frac{2kT_2}{m\pi}} + \left(1 + \frac{1}{9}k_2^1 k_2^2\right)\frac{3}{4}\sqrt{\frac{2kT_1}{m\pi}}\right]\mathbf{k}_1,$ $\mathbf{F}_2 = -m\pi R_2^2 q n_0\left(q + \frac{4}{9}\pi\lambda_4/\lambda_2\right)\vec{k}_1 + \frac{8}{27}m\pi^2 R_2^2 q n_0 k_1^2 \cos^3\Theta^+(\lambda_4/\lambda_2 + \lambda_3/\lambda_1)\mathbf{k}_1$
exact analytic	$E_1 = mqn_0 R_2^2\left(q^2 - \frac{4kT}{m}\right)\left[\frac{\pi}{2} - I_g\right]$ $E_2 = mqn_0 R_2^2\left(q^2 - \frac{4kT}{m}\right)\frac{\pi}{2}$

of the medium, T_0, T_1, T_2 — temperatures of the medium and spheres $\bar{g}$, I_g — geometrical factors,

medium, $\Theta_q = \sphericalangle\,(q, 0_1 0_2)$, d — distance of the spheres

At infinity, the medium is in the state of the global thermodynamic equilibrium, described by the Maxwell-Boltzmann velocity distribution function (v.d.f) $f^{(i)}$:

$$f^{(i)} = A^{(i)} \exp[-B^{(i)} c^{(i)2}],$$

$$A^{(i)} = n_0 \left(\frac{2\pi k T_0}{m}\right)^{-3/2}, \quad B^{(i)} = \frac{m}{2kT_0},$$

where m — mass of the gas particles, k — Boltzmann constant, $c^{(i)}$ — velocity of the gas particle.

The particles of the gas medium, impinging on the bodies and reflected from the bodies, transmit the momentum and energy. The aerodynamic characteristics depends on the way of the interaction of the gas particles with the surfaces of the bodies. We assume in our work the diffusion model of gas-surface interaction.

The forces **F** and the energy exchange E are expressed by five-fold integrals, containing the v.d.f. of the molecules coming from the medium — $f^{(i)}$ and v.d.f. of the molecules reflected from the surfaces of the spheres — $f_1^{(r)}, f_2^{(r)}$:

$$FVE \sim \int\limits_{\Sigma_{1V2}} \left[\int\limits_{\Omega_c} \left(\mathbf{c} V \frac{c^2}{2} \right) (\mathbf{cn}) (f^{(i)} V f_{1V2}^{(r)}) \, d^3\mathbf{c} \right] d\Sigma,$$

n is the normal in the considered point of the surfaces of the spheres K_1, K_2, V — the sign of alternative.

The five-fold integrals correspond to the three-fold integrating over the space velocity Ω^c and two-fold-over the surfaces of the spheres $\sum_{1V2}$. According to the assumed diffusion model of gas-surface interaction, the functions $f_{1V2}^{(r)}$ have the following form:

$$f_{1V2}^{(r)} = A_{1V2}^{(r)} \exp(-B_{1V2} c^{(r)2}),$$

where

$$B_{1V2} = m/(kT_{1V2}).$$

The quantities $A_{1V2}^{(r)}$ are unknown.

We may obtain them from the conditions of the impermeability of the walls. These conditions form the system of two integral equations of Fredholm of the second kind.

In this way the algorytm of the problem of the interference of two spheres moving in free-molecular medium reduces to solving of the system of integral equations and next to calculation of five-fold integrals, expressing the forces **F** or the heat exchange E.

The solution of the problem in the general form is impossible analytically as well as numerically.

The main difficulty are complicated domains of the integration in the velocity space, connected with the screening of one sphere by another. For this reason, the problem was effectively solved only in the physically and geometrically simplest, cases [1, 2, 3, 4], namely:

1) the resting spheres;

2) the system of the spheres moving along the axis connecting the centres in approximation of small and great velocities of the system (in comparison to the thermal velocity of the gas medium);

3) the system moving in arbitrary direction, but only then, when the temperatures of the spheres are equal.

The problems solved in these cases and obtained results are illustrated in Table 1 (p. 220).

From these results we may imply the following conclusions:

1. *Resting spheres*

If the temperature of the body is higher than the temperature of surrounding medium, then the body exerts the repulsing action on another body; if lower — the attracting action. Consequence of this is, that the bodies hotter than the medium repulse themselves and colder ones — attract themselves. The magnitudes of the forces acting on the bodies are proportional to the geometrical factor g. This factor was calculated in mixed ways, analytically and numerically. At great distances the interaction forces are inversely proportional to square of the distance, at small distances the factor g may reach values close to unity. It is interesting that the force acting on the body does not depend on its temperature.

2. *The system moving along the axis connecting the centres in approximation of small velocities and great distances*

Apart from the obvious drag (the first expression in the brackett [...], which would appear, when the sphere moves in the medium alone) appears the interaction force, evoked by the presence of the second sphere (the term in the bracket {...}, proportional to $(2kT_0/m)^{\frac{1}{2}}$) and the force of interaction, connected with motion (the term in the brackett {...} proportional to the velocity q).

The interaction terms are inversely proportional to the square of the distance d and those which are connected with motion are proportional to the velocity of the system. It is characteristic that the interaction connected with motion decreases the drag.

By suitable choosing of the temperatures of the spheres, every force or the total force acting on the system can be made equal to 0. In such a case, the spheres of the system could move without drag. We may also receive the total force directed according to the velocity of the system — in such a case, the system would be accelerated; we would then have something of a kind of the free-molecular engine.

3. *The system moving along the axis connecting the centres with hypersonic velocity in great distances approximation*

The first term in $\mathbf{F}_1$ represents the force descending from the particles impinging directly from the medium (the screening of the sphere K_1 by the sphere K_1 is taken into account); the second term in $\mathbf{F}_1$ represents the force, descending from the particles coming from the sphere K_2 and from all particles reflected from the sphere K_1. The first term in $\mathbf{F}_2$ represents the force evoked by particles coming directly from the medium (impinging on the sphere K_2 and reflected from K_2); the second term represents the force descending from the particles coming from the sphere K_1.

The obvious drag is proportional to the square of the velocity of the system, the interaction drag (the interaction force connected with the motion) — to the product of the velocity q of the system and thermal velocity C_T of the particles reflected from the first or second sphere. In the force $\mathbf{F}_1$ the interaction increases the drag, in $\mathbf{F}_2$ decreases the drag.

It is characteristic that the effects of the interaction — in the force $\mathbf{F}_1$ — are inversely proportional to the fourth power of the distance, however — in $\mathbf{F}_2$ — to the square of the distance. The effects of the screening are proportional to $\cos^2\Theta^+$ or $\cos^3\Theta^+$ for the terms representing the particles impinging or reflected, respectively.

4. *The heat exchange for the system moving with hypersonic velocity in the arbitrary direction in the case when $T_1 = T_2$*

The analysis of the screening (the shade thrown in uniform stream on the sphere K_1 by K_2) requires the partition of the problem into 13 cases (with reference to the proportion of radii R_1/R_2 and magnitude of the angle of attack Θ_q). For all these cases has been found the general expression for the heat exchange for the screened sphere K_1. The effect of screening is characterized by geometrical factor I_g. The value I_g we obtain by substituting the proper parameters, characterizing the case of screening. The factor $\pi/2$ in the bracket [...] corresponds to the situation when the sphere K_1 is not screened.

For example, from the general expression for the exchange E_1 has been calculated I_g for 5 values of the angle of attack (the details in the paper [1, 2]).

The term in E_1, representing the impinging molecules, is proportional to the third power of the system velocity q; the term representing the reflected molecules is proportional to the product of the velocity of the system and the temperature of the spheres.

The most important fact is that the interaction heat exchange has geometrical character only and results completely from the screening of one sphere by another.

The problems 1, 2, 3, 4 are presented in detail in the papers [1, 2, 3].

Possibilities of the utilizing of the results obtained

On the basis of the results obtained we may set up many interesting cognitive experiments in earthly conditions or in the cosmic space around the Earth, where the conditions of free-molecular medium and hypersonic velocity can be realized. The list of the experiments proposed we present in Table 2.

In the earthly conditions can be proposed, for example, the following experiment.

In the reservoir of the gas with free-molecular conditions and constant temperature of the walls we may hang up two small spheres, next heat them to different temperatures and observe their deviations from the balance point. For the receiving of the conditions mentioned above it suffices to take for example the reservoir with radius 1 m and to fill it with the gas, whose pressure is $4-40\times10^{-6}$ atm in the temperature $T^0 - 300^0$K. In such conditions two spherical envelopes, with density $\varrho = 3$ g/cm^3, heated to the temperatures 600–700^{0}K, should deviate at small distances by the angle 1–9^0.

Changing the materials of the spheres we may investigate the phenomena of interaction in dependence on the kind of surface. This experiment reminds the experiment from electrostatics.

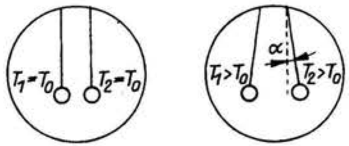
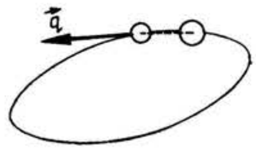
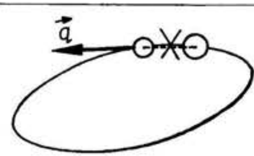
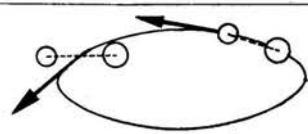
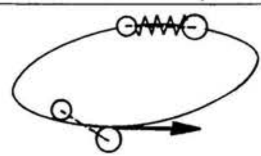
In the cosmic space of Earth we may set up such experiments as:

1) the determining of the drag of the system by observation of its trajectories,

2) the determining of the interaction forces by the measurement of the change of the distance of the spheres or their relative velocity (to this end it suffices to cut the chains connecting the spheres).

On the basis of results obtained the total acceleration and the interaction acceleration

Table 2. Proposed experiments

Subject of experimet	Scheme	Measurements	Remarks
Interaction of two bodies		Deviation out of balance point	Earthly conditions $\lambda \gg l \gg R$ (Example — envelopes $\varrho = 3g/cm^3$ $\lambda = 1$m, $T_0 = 300$ K, $p_0 = 4-40\times10^{-7}$ atm. $(R_1+R_2)/d \approx 1$, $T_1, T_2 \approx 600-700$ K, $\alpha \approx 1-9°$)
Drag of system		Trajectory	Cosmic conditions acceleration for the spheres $\Big($$\varrho = 3g/cm^3$, $R_2/R_1 = \frac{1}{2}$, $k_1^2 = 1/10$, $\sqrt{\frac{2kT}{m}}\Big/q = \frac{1}{10}$, $q = 7{,}8$ km/sek$\Big)$ $H = 130$ km, $R_1 = 10^2$cm, $1{,}23\times10^{-2}$ cm/s², $H = 50$, $R_1 = 10^{-4}$, $1{,}7\times10^9$ cm/s²
Interaction forces		Distance or relative velocity of the spheres	Cosmic conditions Interaction acceleration, envelopes $H = 130-200$ km, $\sqrt{2kT/m}/q = \frac{1}{10}\varrho = 3g/cm^3$ $(R_1+R_2)/d = 1$, $10^{-2}-10^{-1}$cm/s.
Verification of diffusion model		Heat exchange for different geometrical configurations	Cosmic conditions $\left.\begin{matrix} Q_I/Q_{II} = f(G) \\ G \equiv (R_1, R_2, d, \Theta_q)\end{matrix}\right\}$ only for diffusion model
Parameters of cosmic atmophere		Forces and heat exchange	Cosmic conditions $F = f(n_0, T_0)$, $Q = Q(n_0, T_0)$ $F_I/F_{II} = f(T_0)$, $Q_I/Q_{II} = f(T_0)$ Parameters in calibration of diffusion model

for the system of the spheres moving in cosmic conditions (the velocity of the system $q = 7.8$ km/s at different heights were calculated.

For example, the acceleration for the spheres with parameters:

$$\varrho = 3\ \mathrm{g/cm^3},\ R_2/R_1 = \frac{1}{2},\ k_1^2 = \frac{1}{10},\ T_1 = T_2 = mq^2/8k$$

is in the case $R_1 = 10^2$ cm$-1{,}23\times 10^{-2}$ cm/s^2 at the height $H = 130$ km and in the case $R_1 = 10^{-4}$cm$^2 - 1.7\times 10^9$ cm/s^2 at the height $H = 50$ km. The interaction acceleration corresponding to these conditions and parameters for the spherical envelopes at small distances $\big((R_1+R_2)/d \approx 1\big)$ and at the heights H 130–200 km achieve the values 10^{-2} cm/s^2. Although the accelerations and interaction accelerations for the bodies with dimensions $\approx 10^2$ cm are small, their effects are easily observable because such quantities as the way and the velocity of the system, the relative distance and relative velocity of the bodies are cumulating in the time. The solution of the problem of the heat exchange we may utilize to set up the experiment for verification of the assumed diffusion model of gas-wall interaction. In the case considered in our work, when the temperatures of the spheres are equal, the proportion of the magnitudes of the heat exchange for two geometrical configurations (that is: different distances, different angles of attack, different radii) depends on the geometry only. Because such situation takes place for the diffusion model only, the experiment gives the possibility of precise verification of diffusion model.

The above experiments we may utilize in a quite different way. Namely, we may state the problem inversely. If we consider the parameters of the cosmic atmosphere as unknown,

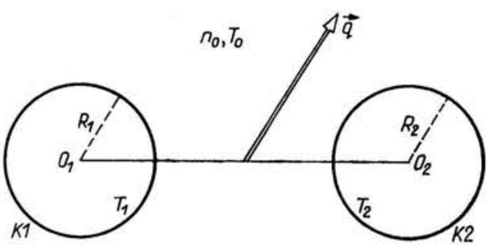


Fig. 1

we may determine them from the measurements of the forces and heat exchange. But the parameters obtained in this way will be the parameters in calibration of diffusion model only.

At last, the results of the work we may use for creating the theory of the multicomponent gases, in which one component creates for the other component the free-molecular conditions (for example: highly dispersed aerosols, the neutral gas and thermal radiation, the atoms and electrons or protons). The particles of one component of such gas, namely this one, for which the second component creates the free-molecular conditions, will interact. The state equation of such gases should be improved.

References

1. S. Kosowski, *Stationary interaction of the system of two spheres moving in free-molecular medium*, IFTR Reports (Polish Academy of Sciences), 36/1973.

2. S. KOSOWSKI, *Stationary interaction of the system of two spheres, resting or moving in free-molecular medium along the axis connecting the centres*, Archives of Mechanics, **26**, 2, 1974.
3. S. KOSOWSKI, *The steady heat exchange for the system of two spheres with equal temperatures moving in free-molecular medium with arbitrary velocity*, Archives of Mechanics, **26**, 4, 1974.
4. S. KOSOWSKI, *Stationary interaction of the system of two spheres moving in free-molecular medium with hypersonic velocity along the axis connecting the centres*, Acta Geophysica Polonica, **22**, 2, 1974.

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