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Gasdynamic effect of condensation at high pressure in Laval nozzle

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THE CLASSICAL picture of the flow behaviour when condensation occurs is as follows: in the subsonic region, flow is accelerated due to condensation whereas in the supersonic region flow is decelarated. By means of conservation equations for one-dimensional flow through the nozzle it is shown in the paper that this picture may not be adequate when the parameter $c_p T_s/h$ exceeds unity. It happens for water vapor at the pressure above 30 bar. In such a case decelaration or acceleration occurs inversely comparing to the classical picture. This of course influences the pressure disturbance. The qualitative analysis of the pressure behavior in the area of the beginning of condensation is presented in the paper.

Klasyczny obraz zachowania się przepływu przy występowaniu zjawiska kondensacji jest następujący: w obszarze poddźwiękowym przepływ wskutek kondensacji jest przyśpieszony, podczas gdy w obszarze naddźwiękowym jest opóźniony. Wykorzystując prawa zachowania dla jednowymiarowego przepływu przez dyszę wykazano, że obraz taki nie jest adekwatny, gdy parametr $c_p T_s/h$ przekracza jedność. Ma to miejsce w parze wodnej przy ciśnieniu przewyższającym 30 barów. W tym przypadku opóźnienie bądź przyśpieszenie występuje odwrotnie niż w przypadku klasycznym. Wpływa to oczywiście na zaburzenie ciśnienia. W pracy niniejszej przedstawiono jakościową analizę zachowania się ciśnienia w początkowej fazie kondensacji.

Классическая картина поведения течения при выступании явления конденсации следующая: в дозвуковой области течение вследствие конденсации ускорено, тогда как в сверхзвуковой области оно замедлено. Используя законы сохранения для одномерного течения через сопло показано, что такая картина не является адекватной, если параметр $c_p T_s/h$ превышает единицу. Это имеет место в водяном паре при давлении превышающем 30 бар. В этом случае замедление или ускорение выступают в обратных порядке, чем в классическом случае. Это конечно влияет на возмущение давления. В настоящей работе представлен качественный анализ поведения давления в начальной фазе конденсации.

1. Introduction

THE BEHAVIOUR of thermally perfect gas flow with heat addition or heat subtraction is the matter of classical analysis in fluid mechanics. The case of the flow with heat addition is a very adequate model of the flow with homogeneous condensation. This type of analysis has the phenomenological character and concerns the change of the flow parameters due to heat release without consideration of the physics of condensation. It clears up the gasdynamic effect of heat release for the thermally perfect gas in the acceleration of subsonic flow and deceleration of supersonic flow [1]. It has been proved experimentally that for atmospherical and low pressures condensing water vapor behaves in the classical manner, i.e., at supersonic domain the condensation pronounces by decelerating the flow and increasing the pressure [2].

Special attention has been recently paid to the investigation of water vapour expansion with condensation because of its technical application in steam turbines of great output. The development of atomic power plants with light water as cooling medium requires information about the flow with condensation at the pressures 60–70 bars. In these thermodynamic cycles the expansion in turbine starts from the saturation parameters at this range of pressures. The high pressure causes the deviation of gas behaviour from the model of thermally perfect gas and results in anomalous disturbances of parameters of condensing vapor.

Notation

$A(x)$	Cross-sectional area of channel as a given function of x,
c_p	specific heat at constant pressure,
$\bar{c}_p$	mean specific heat,
c	specific heat of liqpid,
$\bar{c}$	mean specific heat,
h	latent heat of condensation,
i	specific enthalpy of gas-liquid mixture,
i'''	enthalpy of vapor at saturation line,
i'	enthalpy of liquid at saturation line,
i_0	total enthalpy,
m_0	mass flow rate,
p	pressure,
T	temperature of gas,
T_w	temperature of liquid,
T_s	saturation temperature,
R	gas constant,
U	velocity,
v_2	relative volume of condensed liquid,
x	stream-wise distances,
ϱ	density of mixture,
ϱ_1	density of gas,
ϱ_2	density of liquid,
y	wetness fraction,
y_s	wetness fraction at thermodynamic equilibrium.

3. Governing equations

We restrict our consideration to the one-dimensional, stationary, ideal flow of condensing vapor with the assumption that there is no slip between condensing and condensed phase.

We shall apply the set of following conservation equations:

conservation of mass

$$\varrho U A = m_0, \tag{3.1}$$

conservation of momentum

(3.2) $$m_0\frac{dU}{dx} = -A\frac{dp}{dx},$$

conservation of energy

(3.3) $$\frac{U^2}{2}+i = i_0,$$

equation of state

(3.4) $$\frac{p}{T\varrho_1} = R$$

to the case of condensing vapor treated as the real gas.

The density of the mixture can be defined as:

(3.5) $$\varrho = (1-v_2)\varrho_1+v_2\varrho_2$$

and may be simplified because $v_2 \ll 1$. Then the Eq. (3.1) becomes:

(3.6) $$\frac{\varrho_1}{1-y}UA = m_0,$$

where the wetness fraction is defined as

(3.7) $$y = \frac{v_2\varrho_2}{\varrho_1+v_2\varrho_2}.$$

For the purpose of further analysis and also for the purposes of numerical calculation it is convenient to introduce the equation of momentum conservation (3.2) in the algebraic form:

(3.8) $$m_0U+pA = Z,$$

where Z is the "impulse function" of x and can be evaluated from the relation

(3.9) $$Z(x) = m_0U_0+p_0A_0+\int\limits_0^x p\frac{dA}{dx}dx.$$

The parameters U_0, A_0, p_0 should be defined at the starting point of calculation $x = 0$.

It is noteworthy that the function of $Z(x)$ has the minimum at the throat of Laval nozzle. In the procedure of calculation this function can be computed for extrapolated values of p.

Now, we introduce the enthalpy of the condensing vapor in the following form:

(3.10) $$i = \left[i''(p)+\int\limits_{T_s}^{T} c_p dT\right](1-y)+\left[i'(p)+\int\limits_{T_s}^{T_w} c dT\right]y.$$

It expresses the fact that $1-y$ mass fraction of gas is subcooled by the amount of T_s-T and y mass fraction of water is subcooled by the amount T_s-T_w. After introducing the mean value of $\bar{c}_p$ and $\bar{c}$ we obtain

$$i = [i''+\bar{c}_p(T-T_s)](1-y)+[i'+\bar{c}(T_w-T_s)]y.$$

Now, the equation of energy conservation can be rewritten in the form

(3.11) $$\frac{U^2}{2}+\bar{c}_pT(1-y)-\left(1-\frac{\bar{c}_pT_s}{h}-\frac{\bar{c}(T_w-T_s)}{h}\right)hy = i_0-i''+\bar{c}_pT_s$$

or taking into account that

$$i_0 = i'' + \int_{T_s}^{T_0} c_p dT = i'' + \bar{c}_p(T_0 - T_s) \tag{3.12}$$

and

$$\bar{c}_p T_0 \geqslant i''(p_0) - i''(p) - \bar{c}_p T_s(p_0) + \bar{c}_p T_s(p) \tag{3.13}$$

we can obtain

$$\frac{U^2}{2} + \bar{c}_p T(1-y) - \left(1 - \frac{\bar{c}_p T_s}{h} - \frac{\bar{c}(T_w - T_s)}{h}\right) hy = \bar{c}_p T_0. \tag{3.14}$$

Finally, we can get the set of governing equations for the vapor

$$\frac{\varrho_1}{1-y} UA = m_0, \tag{3.15}$$

$$m_0 U + pA = Z, \tag{3.16}$$

$$\frac{U^2}{2} + \bar{c}_p T(1-y) - \left[1 - \frac{\bar{c}_p T_s}{h} - \frac{\bar{c}(T_w - T_s)}{h}\right] hy = \bar{c}_p T_0, \tag{3.17}$$

$$\frac{p}{\varrho_1 T} = R(p, T) \tag{3.18}$$

with the six unknown functions

$$U(x),\ p(x),\ T(x),\ \varrho_1(x),\ y(x),\ T_w(x).$$

In order to close the system, two equations should be added describing the growth of mass fraction of condensing phase $y(x)$ and the temperature of condensing phase $T_w(x)$.

There are two limiting cases when the set (3.15)–(3.18) can be completed more easily. The first case is the one of purely subcooled expansion—frozen flow of vapor when $y = 0$. Then the closed set of equations (3.15)–(3.18) which resembles the case of thermally perfect gas flow is obtained.

The second case is the one of equilibrium expansion when

$$T = T_s = T_w.$$

In such a case we have to add the relation at the saturation line between p and T. The system of governing equations will then include five unknown functions

$$U(x),\ p(x),\ T_s(x),\ \varrho_1(x),\ y_s(x).$$

Let us consider the general case of expansion of the condensing vapor through Laval nozzle with the assumption that the condensed phase is at the saturation temperature $T_w = T_s(p)$. Then from the set (3.15)–(3.18) follows the quadratic form:

$$U^2 - \frac{2\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0}U + \frac{2R}{2\bar{c}_p - R}\left(1 - \frac{\bar{c}_p T_s}{h}\right) hy + \frac{2R}{2\bar{c}_p - R}\bar{c}_p T_0 = 0. \tag{3.19}$$

It is essential to the vapor that values R, $\bar{c}_p$, T_s, h are not constant. They only can be treated as being approximately constant when the small change of parameters takes place. The magnitude of "small change" is the matter of accuracy which one needs in calculation.

4. The pressure effect on condensing vapor

The condensation process in Laval nozzle may be divided into three characteristic regions. The first one is the region of subcooled vapor—frozen flow. It is characterized by $y = 0$. At the point of high nucleation, subcooling breaks down and we have the narrow (of order 10–30 mm) region, where

$$y > 0, \; \frac{dy}{dx} \gg \frac{dy_s}{dx}.$$

This region is called sometimes "condensation shock".

The third region is that of the expansion at nearly equilibrium, where

$$y \approx y_s, \quad 0 < \frac{dy_s}{dx} \ll \frac{dy}{dx}.$$

We shall restrict our consideration to the second region at which the condensation influences substantially gasdynamics parameters change due to the high value of dy/dx. Moreover, the pressure level at which this second region occurs plays also an important role in the character of parameter disturbances when condensation starts. The pressure effect in the Eq. (3.19) is expressed by the dependence of parameter $\bar{c}_p T_s/h$ on p. For the water vapor this parameter changes along the saturation line. Below the pressure about 30 bars $\bar{c}_p T_s/h$ is less than unity, at $\sim$ 30 bars it reaches unity and above 30 bars it exceedes unity as is shown in Fig. 1.

Let us differentiate the Eq. (3.19) with the assumption that all the material parameters can be treated as being locally constant. Thus we obtain[1]

$$\text{(4.1)} \qquad \frac{dU}{dx} = \frac{\dfrac{\bar{c}_p}{2\bar{c}_p - R}\dfrac{U}{m_0}p}{U - \dfrac{\bar{c}_p}{2\bar{c}_p - R}\dfrac{Z}{m_0}}\frac{dA}{dx} + \frac{\dfrac{R}{2\bar{c}_p - R}\left(\dfrac{\bar{c}_p T_s}{h} - 1\right)h}{U - \dfrac{\bar{c}_p}{2\bar{c}_p - R}\dfrac{Z}{m_0}}\frac{dy}{dx}.$$

(1) In (4.1) the term with $d\left(\dfrac{\bar{c}_p T_s}{h}\right)\Big/ dp$ has been neglected. This simplification is justified beyond the transonic conditions in the so called Wilson region where $y \cong 0.04$. The denominator in (4.1) can be expressed more exactly as follows

$$U\left(1 + \frac{R}{2\bar{c}_p - R} h\varrho y \frac{d[(\bar{c}_p T_s)/h]}{dp}\right) - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0}.$$

The second term in the brackets may be estimated according to the graph on Fig. 1 and steam tables. For the pressure region 50–0.05 bars

$$\frac{R}{2\bar{c}_p - R} h\varrho y \frac{d[(\bar{c}_p T_s)/h]}{dp} \approx 0.01$$

and this term is negligible far upstream and downstream of the throat.

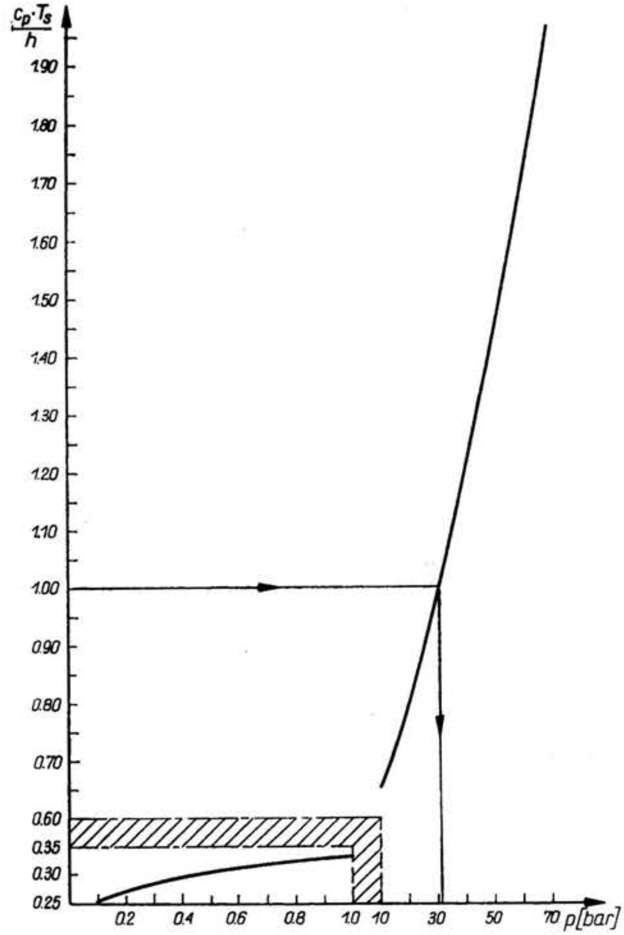

FIG. 1.

At the normal performance of Laval nozzle at the throat

(4.2)
$$U = \frac{\bar{c}_p}{2\bar{c}_p - R} \frac{Z_{min}}{m_0}$$

if the magnitude m_0 is chosen "exactly".

In the subsonic part of the nozzle

(4.3)
$$U < \frac{\bar{c}_p}{2\bar{c}_p - R} \frac{Z}{m_0},$$

in the supersonic part

(4.4)
$$U > \frac{\bar{c}_p}{2\bar{c}_p - R} \frac{Z}{m_0}.$$

The sign of the coefficient at dy/dx in the relation (4.1) depends not only on the character of the flow but also on the sign of $(\bar{c}_p T_s/h) - 1$.

The influence of these two factors on dU/dx is confronted in Table 1.

Table 1

	High pressure $\frac{\bar{c}_p T_s}{h} > 1$	Low pressure $\frac{\bar{c}_p T_s}{h} < 1$
Subsonic flow $U < \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0}$	$\frac{\frac{R}{2\bar{c}_p - R}\left(\frac{\bar{c}_p T_s}{h} - 1\right)h}{U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0}}\frac{dy}{dx} < 0$	$\frac{\frac{R}{2\bar{c}_p - R}\left(\frac{\bar{c}_p T_s}{h} - 1\right)h}{U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0}}\frac{dy}{dx} > 0$
Supersonic flow $U > \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0}$	$\frac{\frac{R}{2\bar{c}_p - R}\left(\frac{\bar{c}_p T_s}{h} - 1\right)}{U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0}}\frac{dy}{dx} > 0$	$\frac{\frac{R}{2\bar{c}_p - R}\left(\frac{\bar{c}_p T_s}{h} - 1\right)}{U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0}}\frac{dy}{dx} < 0$

At the low pressure (for water vapor below 30 bars) the influence of developing condensation on velocity change is the classical one. As we can see from the Table 1, condensation accelerates subsonic flow and decelerates supersonic one. But at the high pressure (for water vapor above 30 bars), gasdynamic effect of the developing condensation is the opposite one. Subsonic flow is decelerated and supersonic is accelerated. The physical reason of this anomaly results from the different behaviour of the mixture density at low and high pressure when the vapor is condensing.

The change of the mixture's density

$$\varrho = \frac{\varrho_1}{1-y} \tag{4.5}$$

can be written in the form:

$$\frac{d\varrho}{\varrho} = \frac{d\varrho_1}{\varrho_1} + \frac{dy}{1-y}. \tag{4.6}$$

The change of the gas density ϱ_1 consists in the change of the temperature and pressure (according to the equation of state)

$$\frac{d\varrho_1}{\varrho} = -\frac{dT}{T} + \frac{dp}{p}. \tag{4.7}$$

The change of the wetness fraction is due to the condensation and it is always coupled with the heat release and growth of the temperature. Taking into account the energy conservation equation for the extreme case of fully frozen flow with temperature T_f, we have

$$\frac{U_f}{2} + i''(p_f) + \bar{c}_p(T_f - T_s) = \frac{U^2}{2} + i''(p) + \bar{c}(T - T_s)(1-y) - yh. \tag{4.8}$$

We can find from (4.8) the approximate relation between y and T, assuming roughly that

$$\frac{U_f^2}{2}+i''(p_f) \approx \frac{U^2}{2}+i''(p), \tag{4.9}$$

then

$$y \approx \frac{\bar{c}_p(T-T_f)}{h-\bar{c}_p(T_s-T)}. \tag{4.10}$$

The relation (4.10) simply means that wetness fraction is defined by the deviation from frozen flow temperature. From this relation

$$\frac{dy}{1-y} \approx \frac{\bar{c}_p dT}{h-\bar{c}_p T_s+\bar{c}_p T}. \tag{4.11}$$

Substituting (4.11) and (4.7) into (4.6) yields

$$\frac{d\varrho}{\varrho} \approx \left(\frac{\bar{c}_p T_s}{h}-1\right)\frac{dT}{T}+\frac{dp}{p} \tag{4.12}$$

with the assumption that

$$\bar{c}_p(T_s-T) \ll h. \tag{4.13}$$

It is easy to notice that the change of the mixture density due to the growth of the temperature depends on the sign of the expression $(\bar{c}_p T_s/h)-1$. For the high pressures of water vapor, mixture gets denser due to the condensation, but at the low pressure density of the condensing mixture decreases. At the low pressures, condensing vapor behaves like thermally perfect gas flow with the heat addition. We can write the relation (4.5) in the form:

$$\varrho \approx \varrho_1(1+y). \tag{4.14}$$

In the process of liquidation of subcooling, due to the condensation, ϱ_1 always decreases and $1+y$ always increases. The influence on mixtures density change depends on prevalence of one of these factors. And this is determined by the dimensionless parameter $\bar{c}_p T_s/h$.

5. Pressure disturbance due to the condensation

The pressure distribution measurement along the channel is very wide-spread in the investigation of the flow with condensation. The condensation at the zone when it starts pronounces in the form of pressure deviation from the distribution corresponding to the expansion without condensation. From (4.1) and (3.2) follows the relation for the first derivative of the pressure along the nozzle:

$$\frac{dp}{dx} = -\frac{\dfrac{\bar{c}_p}{2\bar{c}_p-R}\dfrac{U}{A}p}{U-\dfrac{\bar{c}_p}{2\bar{c}_p-R}\dfrac{Z}{m_0}}\frac{dA}{dx}-\frac{\dfrac{R}{2\bar{c}_p-R}\left(\dfrac{\bar{c}_p T_s}{h}-1\right)\dfrac{hm_0}{A}}{U-\dfrac{\bar{c}_p}{2\bar{c}_p-R}\dfrac{Z}{m_0}}\frac{dy}{dx}. \tag{5.1}$$

The sign of dp/dx depends on the character of the flow (subsonic or supersonic) and the pressure, i.e., the sign of $(\bar{c}_p T_s/h)-1$. We shall consider the normal performance of de Laval nozzle when at the subsonic part we have

$$\frac{dA}{dx} < 0 \quad \text{and} \quad U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0} < 0$$

and at the supersonic part

$$\frac{dA}{dx} > 0 \quad \text{and} \quad U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0} > 0.$$

Then we can distinguish four characteristic cases.

I *Case*. Low pressure and subsonic flow. Due to the following conditions:

$$\frac{\bar{c}_p T_s}{h} - 1 < 0, \quad \frac{dA}{dx} < 0,$$

$$U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0} < 0, \quad \frac{dy}{dx} > 0,$$

we have

$$\frac{dp}{dx} < 0.$$

II *Case*. Low pressure and supersonic flow. The conditions:

$$\frac{\bar{c}_p T_s}{h} - 1 < 0, \quad \frac{dA}{dx} > 0,$$

$$U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0} > 0, \quad \frac{dy}{dx} > 0$$

lead from (5.1), either to

$$\text{a)} \quad \frac{dp}{dx} > 0 \quad \text{when } \frac{1}{A}\frac{dA}{dx} < \left(1 - \frac{\bar{c}_p T_s}{h}\right)\frac{h}{\bar{c}_p T}\frac{1}{1-y}\frac{dy}{dx},$$

or to

$$\text{b)} \quad \frac{dp}{dx} < 0 \quad \text{when } \frac{1}{A}\frac{dA}{dx} > \left(1 - \frac{\bar{c}_p T_s}{h}\right)\frac{h}{\bar{c}_p T}\frac{1}{1-y}\frac{dy}{dx}.$$

In subcase IIa we can expect the pressure "bump" because the positive sign of dp/dx may appear due to the condensation, at the part of nozzle where pressure normally drops.

III *Case*. High pressure and subsonic flow. Because of

$$\frac{\bar{c}_p T_s}{h} - 1 > 0, \quad \frac{dA}{dx} < 0,$$

$$U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0} < 0, \quad \frac{dy}{dx} > 0,$$

we may expect from (5.1) either

$$\text{a)} \quad \frac{dp}{dx} > 0 \quad \text{when } \frac{1}{A}\frac{dA}{dx} > \left(1 - \frac{\bar{c}_p T_s}{h}\right)\frac{h}{\bar{c}_p T}\frac{1}{1-y}\frac{dy}{dx},$$

or

$$\text{b)} \quad \frac{dp}{dx} < 0 \quad \text{when} \quad \frac{1}{A}\frac{dA}{dx} < \left(1 - \frac{\bar{c}_p T_s}{h}\right)\frac{h}{\bar{c}_p T}\frac{1}{1-y}\frac{dy}{dx}.$$

The case IIIa corresponds to pressure "bump".

IV *Case*. High pressure and supersonic flow. From the conditions

$$\frac{\bar{c}_p T_s}{h} - 1 > 0, \quad \frac{dA}{dx} > 0,$$

$$U - \frac{\bar{c}_p}{2\bar{c}_p - R}\frac{Z}{m_0} > 0, \quad \frac{dy}{dx} > 0$$

follows

$$\frac{dp}{dx} < 0.$$

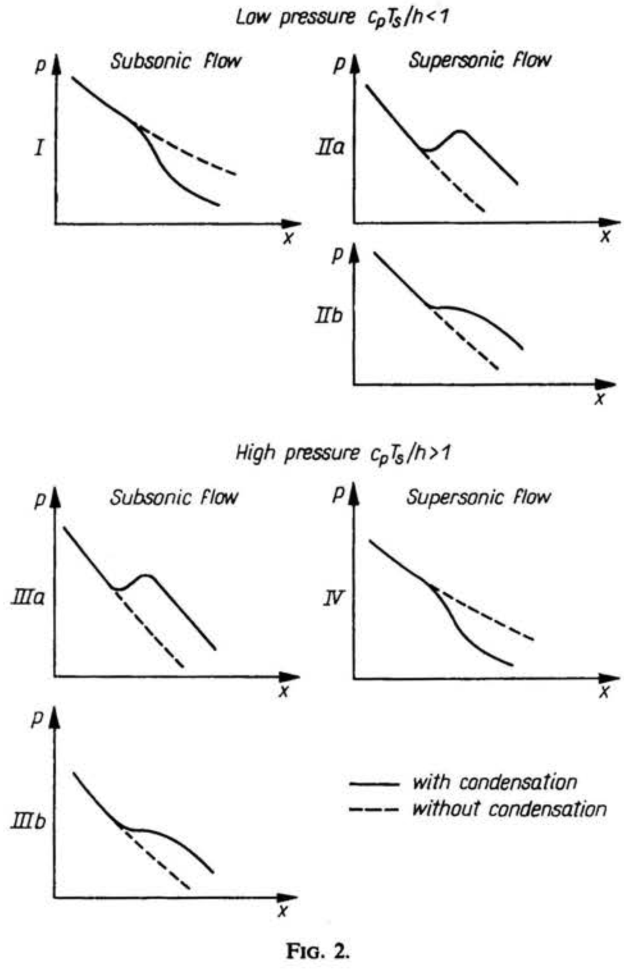

FIG. 2.

In the Fig. 2 a graphical presentation of the four discussed situations is shown. In addition to these cases unstable conditions due to condensation which have been reported in [3] may happen in transonic region.

6. References

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