714.

VARIOUS NOTES.

[From the *Messenger of Mathematics*, vol. VII. (1878), pp. 69, 115, 124, 125.]

An Identity.

THE following remarkable identity is given under a slightly different form by Gauss, *Werke*, t. III., p. 424,

$$1+\left(\frac{\frac{1}{2}}{1}\right)^3 x+\left(\frac{\frac{1}{2}\cdot\frac{3}{2}}{1\,.\,2}\right)^3 x^2+\left(\frac{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}}{1\,.\,2\,.\,3}\right)^3 x^3+\text{\&c.}$$

$$=\left\{1+\left(\frac{\frac{1}{4}}{1}\right)^2 x+\left(\frac{\frac{1}{4}\cdot\frac{5}{4}}{1\,.\,2}\right)^2 x^2+\left(\frac{\frac{1}{4}\cdot\frac{5}{4}\cdot\frac{9}{4}}{1\,.\,2\,.\,3}\right)^2 x^3+\text{\&c.}\right\}^2.$$

On two related quadric functions.

Assume

$$\phi x=a^2(c-x)-x(c^2-b^2-cx),$$

$$\psi x=b^2(c-x)-x(c^2-a^2-cx):$$

then

$$\phi\left(\frac{a^2}{c-x}\right)=\frac{a^2}{(c-x)^2}\psi x,$$

$$\psi\left(\frac{b^2}{c-x}\right)=\frac{b^2}{(c-x)^2}\phi x.$$

In the first of these for x write $\dfrac{b^2}{c-x}$; then

$$\phi\left\{\frac{a^2(c-x)}{c^2-b^2-cx}\right\}=\frac{a^2(c-x)^2}{(c^2-b^2-cx)^2}\,\frac{b^2}{(c-x)^2}\,\phi x=\frac{a^2b}{(c^2-b^2-cx)^2}\,\phi\,(x).$$

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A Trigonometrical Identity.

$$
\begin{aligned}
&\cos(b-c)\cos(b+c+d)+\cos a\cos(a+d)\\
&=\cos(c-a)\cos(c+a+d)+\cos b\cos(b+d)\\
&=\cos(a-b)\cos(a+b+d)+\cos c\cos(c+d)\\
&=\cos a\cos(a+d)+\cos b\cos(b+d)+\cos c\cos(c+d)-\cos d.
\end{aligned}
$$

Extract from a Letter.

"I wish to construct a correspondence such as

$$(x+iy)^3+(x+iy)=X+iY,$$

or, say, for greater convenience

$$4(x+iy)^3-3(x+iy)=X+iY;$$

viz. if

$$x+iy=\cos u,$$

then

$$X+iY=\cos 3u.$$

Suppose $3u_0$ is a value of $3u$ corresponding to a given value of $X+iY$, then the three values of $x+iy$ are of course $\cos u_0$, $\cos\left(u_0\pm\frac{2\pi}{3}\right)$; but I am afraid that the calculation of u_0, even with cosh and sinh tables, would be very laborious. Writing

$$X+iY=R(\cos\Theta+i\sin\Theta),$$

the intervals for Θ might be 5°, 10° or even 15°, those of R, say 0·1 from 0 to 2, and then 0·5 up to 4 or 5; and 2 places of decimals would be quite sufficient; but even this would probably involve a great mass of calculation.

It has occurred to me that perhaps a geometrical solution might be found for the equation $X+iY=\cos 3u$."

October 31, 1877.

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