933.

TABLES OF PURE RECIPROCANTS TO THE WEIGHT 8.

[From the *American Journal of Mathematics*, t. XV. (1893), pp. 75—77.]

IN the tabulation of Pure Reciprocants it is convenient to write $a=1$; we thus have for all the reciprocants of a given weight a single column of literal terms which (as in the Seminvariant Tables) I arrange in alphabetical order AO, and the several reciprocants have then each of them its own column of numerical coefficients: the form of the table is thus similar to that of the seminvariant table, the only difference being that for reciprocants the final terms are not in general power-enders: as in the seminvariant table, the columns of the table are arranged *inter se* with their final terms in AO. As remarked in my paper, "Corrected Seminvariant Tables for the Weights 11 and 12," *Amer. Math. Journ.*, t. XIV. (1892), pp. 195—200, [926], it is not in every case the top term of a column which should be regarded as the initial term; but to the extent 8, to which the reciprocant tables are here carried, this remark has no application.

I recall that the notation is the modified one employed by Halphen, and by Sylvester* in his 12th and subsequent lectures, viz. $a, b, c, d, \ldots$ denote

$$\tfrac{1}{2}\frac{d^2y}{dx^2},\ \tfrac{1}{6}\frac{d^3y}{dx^3},\ \tfrac{1}{24}\frac{d^4y}{dx^4},\ \tfrac{1}{120}\frac{d^5y}{dx^5},\ldots$$

respectively. As already noticed, a is put $=1$, but it is to be in the several terms restored in the proper powers so as to obtain for the reciprocant a homogeneous expression of a degree equal to the original degree of the final term; thus $d-3bc+2b^3$ is to be read as standing for $a^2d-3abc+2b^3$.

The ultimate verification of the expression for a pure reciprocant consists (as is known) in its annihilation by the operator

$$V=2a^2\partial_b+5ab\partial_c+(6ac+3b^2)\,\partial_d+(7ad+7bc)\,\partial_e+(8ae+8bd+4c^2)\,\partial_f+\&c.,$$

or, say

$$V=2\partial_b\ \ +5b\partial_c\ \ +(6c\ \ +3b^2)\,\partial_d+(7d\ \ +7bc)\,\partial_e+(8e\ \ +8bd+4c^2)\,\partial_f+\&c.;$$

[* *American Journal of Mathematics*, t. IX. (1887), p. 7.]

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thus for the reciprocant $50e - 175bd + 28c^2 + 105b^2c$, the result obtained is

$$2(-175d + 210bc) + 5b(56c + 105b^2) + (6c + 3b^2)(-175b) + (7d + 7bc)(50),$$

or, collecting, this is

$$\begin{array}{c|l|l} d & -350 \qquad\qquad\qquad +350 & \pm\ 350 \\ bc & +420 + 280 - 1050 + 350 & \pm\ 1050 \\ b^3 & \qquad\ \ +525 - \ \ 525 & \pm\ 525, \end{array}$$

$= 0$, as it should be.

The tables are

c	$+4$
b^2	-5
	$+4$
	-5

d	$+1$
bc	-3
b^3	$+2$
	± 3

e	$+50$	
bd	-175	
c^2	$+28$	$+16$
b^2c	$+105$	-40
b^4		$+25$
	$+183$	$+41$
	-175	-40

f	$+10$	
be	-40	
cd	-12	$+4$
b^2d	$+65$	-5
bc^2	$+16$	-12
b^3c	-39	$+23$
b^5		-10
	± 91	± 27

g	$+14$			
bf	-63			
ce	-1350	$+800$		
d^2	$+1470$	-875	$+125$	
b^2e	$+1782$	-1000		
bcd	-4158	$+2450$	-750	
c^3	$+2130$	-1344	$+256$	$+64$
b^3d			$+500$	
b^2c^2		$+35$	$+165$	-240
b^4c			-300	$+300$
b^6				-125
	$+5576$	$+3250$	± 1018	$+364$
	-5508	-3254		-365

h	$+7$			
bg	-35			
cf	-539	$+560$		
de	$+605$	-650	$+50$	
b^2f	$+735$	-700		
bce	$+306$	-290	-150	
bd^2	-2135	$+2275$	-175	
c^2d	$+1001$	-1036	$+28$	$+16$
b^3e	-1485	$+1500$	$+100$	
b^2cd	$+3465$	-3710	$+630$	-40
bc^3	-1295	$+1988$	-84	-48
b^4d			-350	$+25$
b^3c^2		$+63$	-259	$+152$
b^5c			$+210$	-155
b^7				$+50$
	$+6119$	± 6386	± 1018	± 243
	-5489			

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i	+ 420						
bh	− 2310						
cg	− 32704	+ 1176					
df	+ 57750	− 8085	+ 20433				
e^2	− 20460	+ 7040	− 21542	+ 625			
b^2g	+ 45500	− 1470					
bcf	− 28392	+ 18963	− 61299				
bde	− 90900	− 16940	+ 69062	− 4375			
c^2e	+ 103740	− 27160	+ 80248	+ 49700	+ 3200		
cd^2	− 38320	+ 26460	− 85554	+ 55125	− 3500	+ 500	
b^3f	− 69615	− 9555	+ 40866				
b^2ce	+ 83538	+ 28098	− 106218	+ 128625	− 8000		
b^2d^2	+ 92820	+ 12740	− 54782	− 61250	+ 4375	− 625	
bc^2d	− 102102	− 52822	+ 191590	− 156800	+ 9800	− 3000	
c^4		+ 21560	− 73304	+ 84868	− 5376	+ 1024	+ 256
b^4e			− 378	− 78750	+ 5000		
b^3cd			+ 1176	+ 183750	− 12250	+ 5750	
b^2c^3				− 102165	+ 6580	− 620	− 1280
b^5d						− 2500	
b^4c^2					+ 175	− 2025	+ 2400
b^6c						+ 1500	− 2000
b^8							+ 625
	+ 383768	+ 116037	+ 403375	+ 452993	+ 29130	+ 8774	+ 3281
	− 384803	− 116032	− 403077	− 453040	− 29126	− 8750	− 3280

I remark that in the last of these tables the first column, say $i \infty bc^2d$, which ends in bc^2d, is a more simple form than Sylvester's $P_8, = i \infty c^4$, (*Amer. Math. Journ.*, t. IX. p. 35), which ends in c^4; P_8 is in fact a linear combination, first col. + 6 second col. of the first and second columns of the table: the second column, say $cg \infty c^4$ is Sylvester's (a^2cg), t. IX. p. 124.

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