916.

[NOTE ON A THEOREM IN MATRICES.]

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PROF. CAYLEY remarks that a "simple instance [of the theorem] is that, if the real symmetric matrix

$$\left(\begin{array}{ccc} a, & h, & g \\ h, & b, & f \\ g, & f, & c \end{array}\right)$$

has two latent roots each $=0$, and therefore a vacuity $=2$, then it has also a nullity $=2$ [which may be shown as follows], viz. the conditions for a vacuity $=2$ are

$$\left|\begin{array}{ccc} a, & h, & g \\ h, & b, & f \\ g, & f, & c \end{array}\right| = 0, \quad bc+ca+ab-f^2-g^2-h^2=0,$$

or, if as usual the determinant is called K, and if

$$(A,\ B,\ C,\ F,\ G,\ H) = (bc-f^2,\ ac-g^2,\ \ldots),$$

then, if

$$K=0,\quad A+B+C=0,$$

these equations give

$$BC-F^2=Ka=0,\quad AC-G^2=Kb=0,\quad AB-H^2=Kc=0,$$

i.e.

$$BC=F^2,\quad AC=G^2,\quad AB=H^2;$$

and therefore

$$A\,(A+B+C)=A^2+H^2+G^2,$$
$$B\,(A+B+C)=H^2+B^2+F^2,$$
$$C\,(A+B+C)=G^2+F^2+C^2;$$

or, if $A+B+C=0$, then for real values

$$A=B=C=F=G=H=0,$$

i.e. nullity $=2$."