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A Decision Support System on the Forex Market Using the Fuzzy Multicriteria Approach

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Abstract

The paper deals with the problem of multiple criteria decision making on the Forex market. Existing fully automated systems mostly fail to achieve profits in a wide time horizon, and simple manual decision support systems are ineffective due to the large number of variants that are available to the decision maker.

We propose a new decision support system based on concepts of fuzzy sets theory and multicriteria analysis. The system includes an original dominance-based algorithm. Algorithm sequentially analyzes subsets of variants generated by the trading system, what visibly decreases the algorithm's computational complexity. As opposed to classical crisp approaches, where a binary activation function is used, here membership functions are proposed to extend the viability of trading systems. Thus every instrument on the market is considered as a single multicriteria variant.

The system assures the decision maker's sovereignty as he/she can make decisions according to his/her different levels of willingness to take a risk and to use a different set of market indicators.

The effectiveness of the proposed approach is confirmed with the use of numerical experiments on real data from the Forex market for two and three criteria examples with successive readings and different parameter settings.
Keywords: Multicriteria decision making, foreign exchange, fuzzy sets, decision support

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1. Introduction

Currently we can observe tendency, that the number of instruments available to the trader, i.e. the decision maker on the Forex market, easily exceeds over 100 instruments and continues to increase. Introducing new instruments not only related to currency pairs but also to stock market indices as well as other derivatives entails the need for the introduction of new concepts that were not available to the decision maker several years ago. Manual trading systems are overrun by partial or fully automated trading systems capable of controlling every single aspect of the transaction related to starting the position as well as maintaining and closing the position. A fully automated system is understood as an application in which predefined rules are used to completely maintain decision maker's account. Examples of fully automated trading systems can be found in (Chourmouziadis & Chatzoglou, 2016; Cervelló-Royoa, Guijarroa, & Michniuk, 2015; Ozturk & Fidan, 2016).

Vast majority of such automated systems are based on the set of rules defined on the basis of different market indicators and will be further called rule-based trading systems. The efficiency of such an approach was confirmed in multiple articles related to the subject, and results related to the practical application of technical analysis indicators were presented in (Taylor & Allen, 1992, 2006). On the other hand, fundamental analysis as a tool for generating the rules is used seldom and is mostly related to text mining concepts (Nassirtoussi & Ngo, 2015). A summary related to the subject of technical analysis on the Forex market can be found in (Cheol-Ho Park, 2007). Author stated, that in early studies (until early 1990s) technical trading strategies were capable to generate consistent profits on the Forex market and futures market. While at the same time later studies point out some inconsistencies and mixed results. The mentioned problems involved for example data snooping, difficulties in risk estimation, skipping the transaction costs or even ex post selection of trading rules.

Two problems related to generation of the buy and sell signals in the existing systems can be observed. The first one arises when the system takes into account a large number of instruments and the signal is generated independently for each instrument satisfying any of the assumed conditions. A potential number of signals generated by these systems often exceeds 10-20 signals in every single time step. It is far beyond the possibilities of decision makers. In general, decision makers have limited information about the efficiency of the signals especially that some of them can be dominated by others.

The second problem relates to a specific market situation when the signal is generated only when all indicators considered in the system satisfy assumed conditions at the same time. It is obvious that increasing the number of market indicators included in the system improves the quality of the signals;

however, the number of signals is very small and very often reaches zero in a given reading, even when the number of considered instruments is very large.

The above problems create gaps which we propose to fill up with the use of a specially constructed decision support system that indicates currency pairs that are potentially interesting to the decision maker. We focus on the concept where one set of indicators is used for a large set of currency pairs in order to emphasize the advantages of our solution. The proposed approach includes the application of fuzzy sets concepts to generate signals for currency pairs by the trading system and a comparison of the currency pair variants by the domination relations used in multicriteria analysis. An algorithm generating the set of non-dominated variants according to the decision maker's preferences is proposed. The efficiency of the algorithm is proved by the respective theorem and shown in a series of calculation experiments on real data from the Forex market.

In the classical existing crisp-based approach the time in which the decision maker can open a transaction is limited. Such an approach excludes the opportunity to open the position before the binary signal is generated. Moreover, the two-fold signal mechanism based on "signal / no signal" function do not allow to identify the dominated variants. Such mechanism seems be be inefficient in the case of large number of instruments.

We propose a fuzzy concept which will allow to estimate the efficiency of a given buy / sell signal for a currency pair value in the range of $\langle 0, 1 \rangle$ for every used indicator. Every currency pair in a given market situation at a time t is represented in the analysis as a variant with a vector of criteria related to particular indicators. Each criterion is valuated by a membership function. The shape of membership function is indicated by the function defining the given market indicator. The possibility to freely extend or reduce the membership function is limited to the crisp rule definition. All membership functions included in the proposed system are directly related to the simple rule definitions that are commonly used in the trading systems. In comparison to the crisp approach the information generated by our system is extended and quantified.

In our approach the original crisp signal is included in the membership function and the time interval in which it is possible to open the position is extended; however, a greater time range to take the position also means a greater risk related with this position. The willingness to take a risk is set by the decision maker. A more conservative approach could include narrow membership functions, while a decision maker with a more aggressive trading approach could extend the range of the membership functions. Every transaction in the crisp approach has some probability of being effective; however, introducing the fuzzy membership function decreases this probability proportionally to the distance of the original crisp signal. Thus, increasing the willingness to take a risk is understood as an extension of the membership range. On the other hand, such an extension positively affects the number of signals generated by the system. So, in general, risk in this concept is the ratio between the number of possible signals and the safety of transactions.

Fully automated trading systems in general exclude the actions of the decision maker. Knowledge about the details related to criteria defined by the decision maker as well as his/her variability of preferences is not taken into account in the systems. The approach proposed in this paper was developed on the basis of multicriteria decision-making concepts. The system should generate outcomes according to preferences of decision makers. Our motivation is only to support the decision maker. The final decision is not made automatically. In fact we try to deliver to the decision maker as many beneficial options as possible. Moreover, this approach can be freely extended in such way, that the different indicators can be considered jointly. Membership functions can be used for a single indicator as well as for groups of indicators and also in the case of fundamental analysis.

The algorithm proposed in this article allows to deliver example solutions even when the simple crisp approach returns an empty set of acceptable variants. The key factor introduced in the algorithm is the mechanism of expanding the set of non-dominated variants in every step of the algorithm. The mechanism allows to shorten the calculation time by narrowing the set of variants that have to be evaluated if a new variant suspected of not being dominated appears. The set of non-dominated variants is built in every single step of the algorithm. Eventually, the final solution is equal to the sum of all sets of non-dominated variants calculated for every step of the algorithm. Thus novelties of this article include:

- a new template for the rule-based trading system based on the fuzzy sets theory and multicriteria analysis is proposed;
- a new low computation complexity algorithm capable to deliver the full set of non-dominated variants is presented;
- a mechanism related to the willingness to take a risk by the decision maker is proposed;

- efficiency of the proposed solution is compared with two different rule-based trading systems commonly found in literature.

The article outline is as follows. In Section 2 we present some works related to the subject of trading systems and basic concepts used in the article. Section 3 introduces the details of the crisp approach and the proposed fuzzy approach along with the trading rules that are used in the framework. Section 4 introduces the dominance concept and the dominance-based algorithm with a simple illustrative example. The last two sections of the article include detailed examples based on real world problems and some conclusions together with a description of future work.

2. Background and related works

In a decision support system (DSS) we take into account 3 equally important market indicators; however, the proposed approach can include any number of indicators. Fig. 1 presents a schematic diagram of a simple trading system in the context of decision making process.

In our proposed approach we concentrate especially on the set of technical indicators. However fundamental indicators could be used as well. All of the considered indicators are presented below. The moving average equation is given as follows:

$$MA_p(t) = \frac{\sum_{i=t}^{t-p} price_i}{p}, \tag{1}$$

where $MA_p(t)$ is the value of the moving average for period p in time t, $price_i$ is a currency pair value for a given time i, and p is the number of included values. An example concept based on moving averages may be found in Holt (2004).

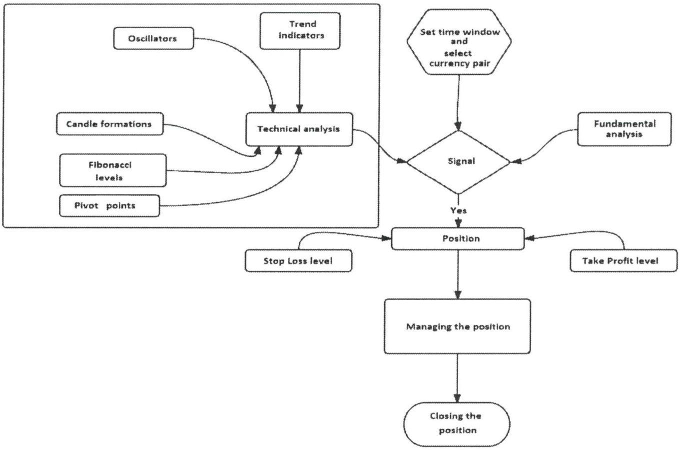

Figure 1: A simple rule based trading system in a decision making process

Two oscillators were also included in the trading system. The first is the Relative Strength Index (RSI):

$$RSI_p(t) = 100 - \frac{100}{1 - \frac{avg_{gain}}{avg_{loss}}}, \tag{2}$$

where $RSI_p(t)$ is the value of the RSI indicator calculated on the basis of the last p periods in time t, avg_{gain} is the sum of gains over the past p periods and avg_{loss} is the sum of losses over the past p periods. The second oscillator to be used is the Commodity Channel Index (CCI):

$$CCI_p(t) = \frac{1}{\alpha} \cdot \frac{price_{typical} - MA_p(t)}{\sigma(price_{typical})}, \tag{3}$$

where $CCI_p(t)$ is the value of the CCI indicator calculated on the basis of p periods in time t, $price_{typical}$ is the typical price calculated as the average value of the Close, Low and High price from a given period, σ is the mean absolute deviation and α is the constant value used for scaling the mean absolute deviation value.

Rule-based trading systems are not the only methods used on the Forex market. A separate group of methods represent trading systems based on neural networks. Publications related to this subject emerged already in 1995, when the neural network was used as a preliminary signal (Chan & Teong, 1995). To the best of the authors' knowledge, one of the first articles dedicated to using neural networks on the market was described in Mizuno & Komoda (1998). Newer approaches concerned the use of neural networks for data prediction (Jingtao & Lim, 2000) or the application of technical indicators as neural network entry points (Thawornwong & Dagli, 2003). A very interesting connection was presented between technical analysis indicators and fundamental indicators (Eng & Lee, 2008). In Ciskowski & Zaton (2010), a self-organizing map was used as a mechanism of detecting correlations between Japanese candlesticks. A similar concept was proposed in Chmielewski & Kaleta (2015), where the k-means algorithm was used to detect some Japanese candlestick patterns.

The second group of articles is related to systems based on new trading rules generated on the basis of existing technical analysis indicators. These approaches often rely on approximate mechanisms such as metaheuristics. Examples of such papers include genetic programming (Lee & Loh, 2002), grammar evolution (Brabazon & O'Neill, 2004), and evolutionary algorithms (Bodas-Sagi, Soltero & Risco-Martin, 2009). Such approaches are based on the concept of setting optimal values of the technical indicators. More recent papers in this field rely on the concept of evolutionary multicriteria algorithms (Bodas-Sagi, Fernandez-Blanco & Soltero-Domingo, 2013) and particle swarm optimization (Bagheria & Akbaric, 2014). Approximate approaches in this field are presented in Ozturk & Fidan (2016) and Moscinski & Zakrzewska (2016).

The last group of articles is related to using new technical indicators as support for decision makers. These approaches are called hybrid indicators and are often built on the basis of classical technical analysis indicators. It is worth noting that new technical analysis indicators are only a small group of articles in this field. An example of such work may be found in Fliess & Join (2009), in which a new indicator that was correlated with the risk factor was presented. This concept was based on the mathematical evidence of trends in financial data that was presented in Fliess & Join (2009). Another example of a new indicator is the moving min-max described in Silagadze (2011). This indicator was used as a chart smoothing tool allowing to ignore small price corrections. It is worth noting that there are very few approaches in which decision support is preferred over automated trading, as in most of the approaches the complete trading system is treated like a black-box, where the set of input data is transformed into the output data signal; however, in article Kissell & Malamut (2006) the authors proposed an approach in which it is possible to set such parameters as risk aversion.

We make use of the system based on fuzzy sets as were originally introduced by Zadeh (1999). There are multiple publications related to the use of fuzzy concepts with stock data (rarely with Forex data) such as (Wang, 2002, 2003; Hadavandi & Ghanbari, 2010). One of the crucial concepts from the point of view of the current article are the so-called aspiration levels as introduced by Wierzbicki (1982). Wierzbicki provided a mathematical background to satisfy the decision making. A newer work including a definition of the reference point is Kahneman (1992). Examples of articles related to the subject of decision making supporting the multicriteria analysis and fuzzy sets can be found, for example, in (Carlsson & Fuller, 1996; Ribeiro, 1996), and in the more recent Lazim (2013).

3. Traditional crisp and the proposed fuzzy approach

In the classical crisp approach the rule-based trading system includes a predefined set of transaction rules related with the technical indicators. Every indicator can be described by the set of rules, which can be transformed into the binary activation function. A signal is generated only in the case, when the function value is equal to 1. In the fully automated-trading system positive function value is equal to opening the transaction, while in the crisp decision support system information about the signal is derived in the system and presented to the decision maker.

We propose a fuzzy concept, in which information about a market situation is transformed into a value of the membership function for each considered indicator. Such value is calculated for every considered currency pair. So each currency pair in a given market situation in a time t is represented in the analysis as a variant with the vector of criteria related to particular indicators. To estimate the efficiency of such approach we compare it with the classical crisp approach, where criteria for all indicators are built on the basis of the binary activation function. To describe accurately the proposed concept, we consider buy signals, however the same concept can be used in the case of sell signals.

3.1. Binary activation functions in the crisp approach

For each considered indicator there is a condition for which the binary activation function should be set to "true", which is denoted as the signal to open the position. The value 1 is assigned to every "true" value. We denote these conditions as $cond_{MA}$, $cond_{RSI}$ and $cond_{CCI}$; thus the potential BUY signal may be considered as follows:

$$f_{buy} = true \text{ if } (cond_{MA_{Buy}} = true \vee cond_{RSI_{Buy}} = true \vee cond_{CCI_{Buy}} = true), \quad (4)$$

where $cond_{MA_{Buy}}$ is the binary activation function for the moving averages. In other case the considered variant is rejected. Motivations to open the position rise when the number of indicators with the "true" value increases. In the proposed approach we adapted the basic rules used with the technical indicators. For the moving averages the condition is given as follows:

$$cond_{MA_{Buy}} = true \text{ if } (MA_{fast}(t) > MA_{slow}(t)) \wedge (MA_{fast}(t-1) < MA_{slow}(t-1)), \quad (5)$$

$MA_{fast}(t-1)$ is a value of the moving average with a lower period in time $t-1$, $MA_{slow}(t-1)$ is a value of the moving average with a higher period in time $t-1$. An example signal is generated if two moving averages crosses each other. In the case of oscillators RSI and CCI, the binary activation functions are built on the basis of crossing the indicator with some predefined levels. For the RSI this will be 30. In the case of CCI the value is -100:

$$cond_{RSI_{Buy}} = true \text{ if } (RSI_p(t-1) < 30) \wedge (RSI_p(t) > 30), \quad (6)$$

$RSI_n(t-1)$ is a RSI value in time $t-1$. The binary activation function for the second oscillator is defined as follows:

$$cond_{CCI_{Buy}} = true \text{ if } (CCI_p(t-1) < -100) \wedge (CCI_p(t) > -100). \quad (7)$$

Both indicators are based on overbought and oversold levels. Overbought is a level in which there is a high probability of a price fall, whereas the oversold level indicates the high possibility of a price rise.

3.2. Membership functions in the fuzzy approach

We propose a fuzzy concept, in which a market situation is transformed into a value of the membership function for each considered indicator. Such value is calculated for every considered currency pair. So each currency pair in a given market situation in a time t is represented in the analysis as a variant with the vector of criteria related to particular indicators. To estimate the efficiency of such approach we compare it with the classical crisp approach, where criteria for all indicators are built on the basis of the binary activation function. A basic assumption is that every criterion value is in the range of $\langle 0 : 1 \rangle$, where 0 is no signal, while 1 is a strong BUY. On the basis of external factors the decision maker may approach the presented signal differently and ignore all BUY/SELL signals when the currency pair is in the middle of consolidation. These factors imply the eventual possibility of changing the range of the criterion which is considered as satisfactory for the decision maker. Due to limited space we introduce only membership functions related to the BUY signals, whereas similar membership functions can be defined for the SELL signals:

$$\mu_{MA-BUY}(c) = \begin{cases} \frac{max-f_{low}}{max} \text{ if } (MA_{fast}(t) > MA_{slow}(t)) \wedge (f_{low} < max) \\ \wedge(MA_{fast}(t-1) > MA_{slow}(t-1)) \\ 1 \text{ if } (MA_{fast}(t) > MA_{slow}(t)) \\ \wedge(MA_{fast}(t-1) < MA_{slow}(t-1)) \\ \frac{f_{high}}{max} \text{ if } (MA_{fast}(t) < MA_{slow}(t)) \wedge (f_{high} < max) \\ \wedge(MA_{fast}(t-1) < MA_{slow}(t-1)) \\ 0 \text{ in other case} \end{cases} \tag{8}$$

where c denotes the currency pair for which the conditions on the right side of the equation are checked, max is the maximal number of readings used in the calculations, f_{high} is a function used to count readings above the moving average with a higher period and f_{low} is a function used to count readings below the moving average with a higher period. The transaction system collects information from the market and calculates the values of the indicator at a given time t.

The membership function defined for the RSI indicator is given as follows

$$\mu_{RSI-BUY}(c) = \begin{cases} \frac{RSI_p(t)}{30} \text{ if } (RSI_p(t) < 30) \\ 1 \text{ if } ((RSI_p(t-1) < 30) \wedge (RSI_p(t) > 30)) \vee (RSI_p(t) = 31) \\ \frac{0.9}{RSI_p(t)-30} \cdot \alpha \text{ if } (RSI_p(t) > 31) \\ \wedge(RSI_p(t) < 50) \wedge (RSI_p(t-1) \leq 30) \\ 0 \text{ if } (RSI_p(t) > 50) \end{cases} \tag{9}$$

In the case of the CCI indicator the membership function is defined as follows:

$$\mu_{CCI-BUY}(c) = \begin{cases} 0 \text{ if } (CCI_p(t) < CCI_{min}) \\ \frac{CCI_p(t)-CCI_{min}}{-CCI_{min}-100} \text{ if } (CCI_p(t) > CCI_{min}) \wedge (CCI_p(t) < -100) \\ 1 \text{ if } (CCI_p(t-1) < -100) \wedge (CCI_p(t) > -100) \\ \frac{CCI_p(t)+50}{-50} \text{ if } (CCI_p(t) > -100) \wedge (CCI_p(t) < -50) \\ \wedge (CCI_p(t-1) > -100) \\ 0 \text{ if } (CCI_p(t) > -50) \end{cases} \tag{10}$$

where $CCI_p(t)$ is a value of the CCI indicator in the present time, CCI_{max} is the maximal considered CCI value and CCI_{min} is the minimal considered CCI value. A vector of scalar values in the range of $\langle 0; 1 \rangle$ is generated in a given time t for all of the given indicators.

In the fuzzy approach, the original signal taken from the crisp approach is still included; however, the neighboring values of the indicator can also be included by calculating the membership function. Example comparison of such a situation can be seen in Fig. 2, where the crisp approach (upper indicator chart) and the fuzzy approach (lower indicator chart) are visible. The gray color marks the window in which the signal value is greater than zero. In the case of the crisp approach, the non-zero value is observable only in the specific time tick; however, for the fuzzy approach the signal can be observed with the specific time window. In the crisp approach the decision maker has a limited time to open the transaction when the signal was generated.

4. Preference relations and Domination cones

Let c be a currency pair valuated by a vector y of n criteria $y = (y_1, \ldots, y_n)$. Variants of the vectors are analyzed in the criteria space $\mathbb{R}^n$. The criteria

Figure 2: Comparison of the length of the time window in which a signal generated in the crisp approach and fuzzy approach is active

refer to the particular indicators with the values of membership functions. In the case of MA indicator considered as the first criterion $y_1 = \mu_{MA}(c)$ for a given currency pair c, and following $y_2 = \mu_{RSI}(c)$, $y_3 = \mu_{CCI}(c)$, etc. for n considered indicators. The trading system generates some number of such variants in a given time window. By the time window we understand a time, that is needed to generate a new value on the price chart.

In the crisp approach each criterion can take only 1 or 0 value. In the proposed fuzzy approach the set of analyzed variants is extended due to the definitions of the membership functions.

The preferences of the decision maker have to be defined in the set of analyzed variants. The decision maker, i.e. trader, tries to find a variant with possibly maximal values of all the criteria; therefore, we can define the following relations between variants in $\mathbb{R}^n$ space:

Definition 1. *Variant y is at least as preferred as variant z if each criterion* *of y is not worse than the respective criterion of z.*

$$y \succeq z \Leftrightarrow (y_1 \geq z_1) \wedge (y_2 \geq z_2) \wedge \ldots \wedge (y_n \geq z_n). \tag{11}$$

Definition 2. *Variant y is more preferred (better) than variant z according to the logical formulae:*

$$y \succ z \Leftrightarrow (y \succeq z) \wedge \neg(z \succeq y). \tag{12}$$

Definition 3. *Variant y is incomparable with variant z if*

$$\neg(y \succeq z) \wedge \neg(z \succeq y). \tag{13}$$

The strict domination relation $\succ$ defines a preorder in the n dimensional space of criteria. This domination relation can be formulated and presented with the use of domination cones. The cones are formulated in the n dimensional criteria space $\mathbb{R}^n$ defined by the membership functions for all n indicators. The maximal attainable value of each criterion in the space is equal to 1 according to the definitions of the membership functions. A point $u = \{u_1, u_2, ..., u_n\}$ with $u_i = 1$ for all $i = 1, \ldots, n$ is called the aspiration point. It relates to the best theoretical ideal variant. In general, such a variant may not exist. The definition differs from the definition typically used in the multicriteria analysis where the ideal point is formulated by the maximal values of the criteria of existing variants.

Definition 4. *We say that an element y* ***dominates*** *an element z where $y, z \in \mathbb{R}^n$ and write $y \succ z$ if $y \in (z + D_+)$ where D_+ is the domination cone defined as $D_+ = \{y : y_1 \geq 0, \ldots, y_n \geq 0,$ and $y \neq 0\}$.*

Definition 5. *The* ***set of points dominating a given point*** *y is defined by $(y + D_+)$, i.e. by cone D_+ moved to point y.*

Definition 6. *The **set of points dominated by a given point** y is defined by* $(y + D_-)$ *where* $D_- = \{y : y_1 \leq 0, \ldots, y_n \leq 0,$ *and* $y \neq 0\}$.

Definition 7. *An element* $y \in Y \subset \mathbb{R}^n$ *is **non-dominated** in set* Y *if there does not exist any element* $z \in Y$ *such that* $z \in (y + D_+)$.

The domination relation formulated with the use of cones in $\mathbb{R}^2$ is illustrated in Fig. 3. The figure presents variants y^1, y^2, y^3, y^4 in relations to a given variant y. Variant y^1 dominates y as it belongs to the set $(y + D_+)$ defined by the positive cone D_+ moved to the point y in $\mathbb{R}^2$. Variant y^2 belongs to the set $(y + D_-)$, so it is dominated by y. Variant y^3 as well as y^4 is non-comparable with y as it does not dominate y nor is it dominated by y.

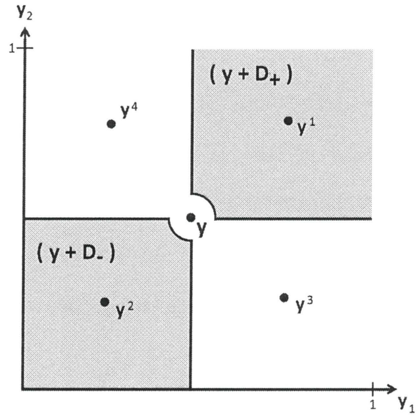

Figure 3: Domination concepts with the use of cones

We assume that the decision maker first proposes a point $x \in \mathbb{R}^n$, called the reservation point, defining non-accepted variants which should be removed from further analysis:

$$x = \{x_1, x_2, ..., x_n\}, \forall_i 0 < x_i < u_i, \tag{14}$$

where n is the number of criteria, x_i is the value of criterion i.

Point x is of course dominated by the aspiration point, i.e. $x \in (u + D_-)$. All variants dominated by point x are removed, i.e. each variant y such that $y \in (x + D_-)$ is removed from further analysis. Let Y denote the set of all variants generated by the trading system. Having point x, we can define the set of non-accepted variants $Y_- = (x + D_-)$ and set $Y_+ = Y \setminus Y_-$ of variants accepted for further analysis, in which non-dominated variants are looked for. Point x relates to the risk aversion of the decision maker with respect to particular criteria. It defines an extension of the set of accepted variants in the fuzzy approach in comparison to the crisp case.

In the set of accepted variants we construct a vector $\tau \in \mathbb{R}^n$ defining a direction of concessions from the aspiration point u to the reservation point x, such that $x = u - \tau$. Particular points s^j on the concession line linking u and x can be calculated by $s^j = u - t^j \cdot \tau$, where t^j are real numbers $0 \leq t^j \leq 1$.

In the proposed approach the set of accepted variants is divided into several subsets which are sequentially analyzed to uncover all non-dominated variants. The subsets are defined by domination cones with vertices s^j lying on the concession line:

$$s^j = \{s_1^j, s_2^j, ..., s_n^j\}, \forall_k \, x_k \leq s_k^j \leq u_k, k = 1, 2, ..., n. \tag{15}$$

This scheme is illustrated in Fig. 4. Initially, s^0 relates to the aspiration point u. Then it is moved according to the direction of concession, with an increasing value of σ until point x is reached. A respective domination cone and a subset of analyzed variants is constructed for each s^j.

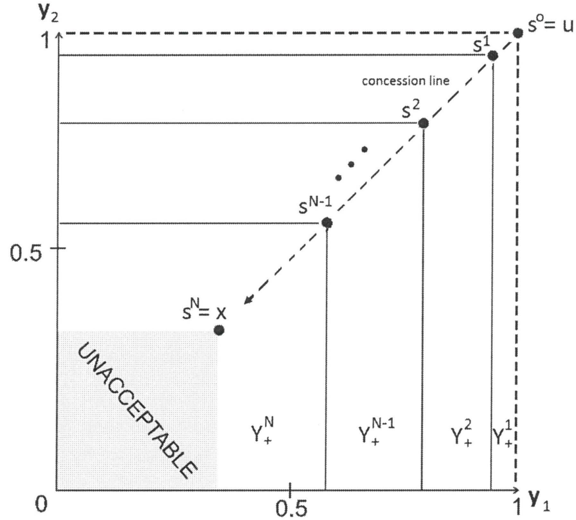

Figure 4: Set of accepted variants in the proposed fuzzy approach

4.1. Dominance-based algorithm

Below, algorithm 1 is proposed which enables the generation of all non-dominated variants in the set of accepted variants Y_+.:

Points u and x are fixed in the initial part of the algorithm (lines $1-5$). Then the sets used in the algorithm are defined.

The reservation point x is assumed by the decision maker. The point defines the set Y_- of the non-accepted variants. All variants belonging to the set are removed from further analysis. A position of the point in the criteria space is related to a measure of the risk undertaken by the decision maker

```
begin
    Fix the aspiration point $u$
    Create the set $Y$ and set $ND = \emptyset$
    Decision maker sets the point $x$ defining the non-accepted variants
    Decision maker sets the number of steps $N$ in the algorithm
    Derive sets $Y_-$ and $Y_+$
    if there exists $y \in Y$ such, that $y = u$ then
        $ND = \{y\}$ End of the algorithm
    end
    Fix $s^0 = u$ and derive $\sigma = \frac{s^0 - x}{N}$
    Derive $Y_-^0 = Y_-$
    for $j = 1, \ldots, N$ do
        Derive $s^j = s^{j-1} - \sigma$ and $Y_-^j = Y_-^{j-1}$
        Create $ND^j = \emptyset$
        Derive $Y_+^j$
        for each variant $y$ in $Y_+^j$ do
            if $y \in Y_-^j$ then
                Delete $y$ from further analysis, i.e. $Y_+ = Y_+ \setminus \{y\}$
            end
            else if $y \notin Y_-^j \wedge ND^j = \emptyset$ then
                Add $y$ to the set $ND^j$ and Update the set $Y_-^j$ and $Y_+ = Y_+ \setminus \{y\}$.
            end
            else
                for $z \in ND^j$ do
                    if $y \succ z$ then
                        Delete $z$ from $ND^j$
                    end
                    else if $z \succ y$ then
                        Mark $y$ as dominated. Delete it from $Y_+$, and BREAK
                    end
                end
                if $y$ is non-dominated then
                    Add $y$ to $ND^j$
                    Update the set $Y_- = Y_- \cup (y + \mathbb{R}_-^n \setminus \{0\})$
                end
                Update the set $Y_-^j$
            end
        end
        if $Y_+ = \emptyset$ then
            $ND = ND^1 \cup ND^2 \cup \cdots \cup ND^N$ and end the algorithm
        end
    end
end
```

Algorithm 1: Dominance-based algorithm who extends the set of analyzed variants in the fuzzy approach in comparison to the crisp case. The set $Y_+ = Y \setminus Y_-$ is derived. The set ND is created. It is initially empty. All non-dominated variants generated in the algorithm will be collected in the set. The decision maker defines a number of steps N of the algorithm. In lines 6 – 7, a variant referring to the aspiration point u is looked for. If such a variant exists, then it dominates all of the other variants. It is added to the set ND as the solution, i.e. $ND = \{u\}$. This also means the end of the algorithm.

If such a variant does not exist then other non-dominated variants are looked for in the following steps of the algorithm. The set of accepted variants Y_+ is divided into N subsets using domination cones. Each domination cone is defined by vertex s^j where $j = 1, 2, \ldots, N$ denotes a step in the algorithm. For each $j = 1, 2, \ldots, N$ $s^j = s^{j-1} - \sigma$ where $s^0 = u$ and

$$\sigma = \frac{s^0 - x}{N}. \tag{16}$$

Lines 10 – 29 include the main part of the algorithm. Vertex s^j defines the cone of the dominated variants $(s^j + D_-)$. In step j, variants belonging to set $Y_+^j = (s^j + D_+) \setminus (s^{j+1} + D_-)$ are analyzed. A set ND^j is created, which is initially empty, in which sequentially all non-dominated variants found in set Y_+^j are collected. For each vertex s^j, a set of non-accepted variants $Y_-^j = Y_-^{j-1}$ is derived which is initially equal to the set from the previous step.

A given analyzed variant is removed if it belongs to set Y_-^j of non-accepted variants; if not, it is treated as a potentially non-dominated variant. It is added to set ND^j if the set is empty. Otherwise, it is compared to each variant from set ND^j.

There are three possible situations:

- first – an analyzed variant can be dominated by a variant that is in set ND^j, then it is removed from further analysis;
- second – it can dominate a variant that is in the set, then it replaces the dominated variant in ND^j and the dominated variant is removed;
- third – if it is not dominated or does not dominate any of the compared variants, it is added to ND^j.

For any variant y added to set ND^j at line 26, set Y_-^j is enlarged by the domination cone moved to this point y, i.e. $Y_-^j = Y_-^j + (y + D_-)$.
Each analyzed variant is removed from Y_+.

Set Y_+ is checked at the end of each step. If it is empty, the mail loop of the algorithm is finished. The decision maker obtains a set of non-dominated variants ND:

$$ND = ND^1 \cup ND^2 \cup \cdots \cup ND^N. \tag{17}$$

Each variant in this set is not worse than the reservation point x.

Theorem 1. *For a given nonempty set of accepted variants $Y_+ = Y \setminus (X + D_-)$, the algorithm generates all non-dominated variants in the set.*

Proof of the Theorem We will prove by induction that all non-dominated variants in set $Y_+ = Y \setminus (X + D_-)$ of the accepted variants are generated by Algorithm 1. Set Y_+ of accepted variants is divided into N subsets $Y_+^j = (s^j + D_+) \setminus (s^{j+1} + D_-)$,where $j = 1, 2, \ldots, N$ is iteration number, $s^j = (s^{j-1} - \sigma)$ with $s^0 = u$ and $\sigma = \frac{s^0 - x}{N}$ in the algorithm.

Base case. The first iteration $j = 1$.

All variants in set Y_+^1 are checked. All non-dominated variants are collected in set ND^1 according to lines 14 – 26 of the algorithm. Let us denote the variants by $y^{1,k}$, where $k = 1, \ldots, p$ is the number of the non-dominated variant in set ND^1, including p of such variants. Set Y_-^1 is enlarged for each non-dominated variant, so $Y_-^1 = Y_- \cup [\bigcup_{k=1,\ldots,p}(y^{1,k} + D_-)]$; therefore, all variants dominated by points $y^{1,k}, k = 1, \ldots, p$ are excluded from further analysis.

Induction step. The iteration $j > 1$.

In the iteration we have a set of non-dominated variants ND^{j-1} that were found in the previous iteration and set Y_-^{j-1} of elements dominated by the points. According to lines 14 – 26 of the algorithm, the variants collected in set ND^j satisfy the property that no variant dominates over the others in the set and that no variant is dominated by any other variant from the set. All of the variants are dominated by the s^j point. Therefore, according to the transitivity of the domination relation defined in Definition 1, variants in set ND^j cannot dominate any of the variants from the ND^{j-1} set. On the other hand, the variants in ND^i do not belong to the Y_-^{j-1} set, therefore none of these variants can be dominated by any of the variants from the ND^{j-1} set.

Via the principle of induction, it is true that for all $j = 1, 2, \ldots, N$, each set ND^j includes all variants that are non-dominated in Y_+ found in Y_+^j.

The sum of sets Y_+^j, for $j = 1, \ldots, N$ is equal to set Y_+, therefore ND in line (29) includes all variants that are non-dominated in Y_+.

4.2. Illustrative example

This example illustrates the initial phase and the three following steps of the algorithm leading to the selection of the final set of non-dominated variants ND. Let the set of variants Y include 10 variants enumerated 1 to 10. The positions of the variants, set Y_- of the non-accepted variants and the aspiration – ideal point u, are presented in Fig. 5a in the space of two criteria y_1, y_2.

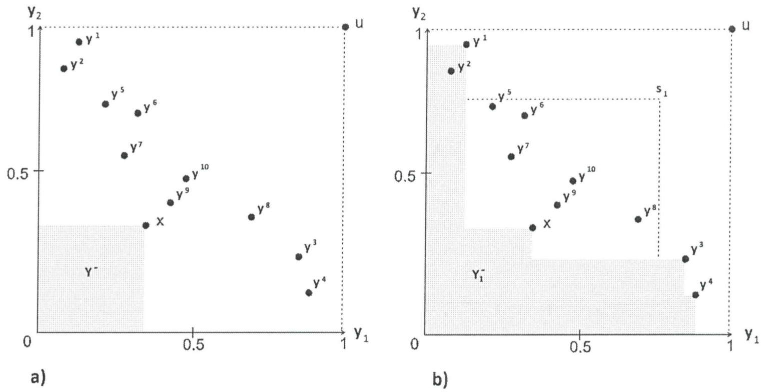

Figure 5: a) Sets of accepted and non-accepted variants: Y_+ and Y_-; b) step 1 of the algorithm

Neither of the existing variants relates to the aspiration - ideal point u. Therefore, this example is not trivial and further analysis is required in the algorithm.

Variants y^1, y^2, y^3 and y^4 are analyzed in the first phase of the algorithm. Variant y^1 does not belong to the set of non-accepted variants, therefore it is added to the empty set ND^1. Set Y_-^1 is enlarged by the set of variants dominated by y^1, i.e. by variants belonging to the moved cone $(y^1 + D_-)$. Variant y^2 is removed from further analysis because it belongs to the updated set Y_-^1. Variant y^3 is not dominated nor dominates variant y^1 and is added to set ND^1. Set Y_-^1 is updated again. It is enlarged by the variants dominated by y^3. Variant y^4 is treated in the same way, as it is not worse and not better than y^1 or y^3. Set Y_-^1 is updated again. Step s^1 is finished. The updated set of non-accepted variants and the discovered non-dominated variants are presented in Fig. 5b.

There are four variants y^5, y^6, y^7 and y^8 in set Y_+^2 analyzed in the second step of the algorithm. Variant y^5 does not belong to set Y_-^2, therefore it is added to set ND^2, and set Y_-^2 is enlarged. Variant y^6 also does not belong to the set of non-accepted variants. It is not better than variant y^5. It is added to set ND^2, and set Y_-^2 is updated again. Similar moves are taken for variant y^8, which is also added to set ND^2. The results of moves made in the second step are presented in Fig. 6a.

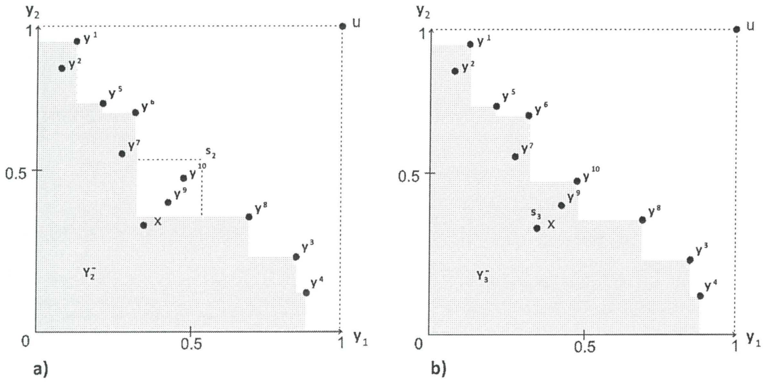

Figure 6: a) Step s^2 of the algorithm; b) all non-dominated variants at the end of the algorithm

The two last variants, y^9 and y^{10}, are in set Y_+^3 in the third step. Variant y^9 does not belong to Y_-^3; it is added to ND^3 and set Y_-^3 is updated. Variant y^{10} dominates variant y^9. Therefore it replaces y^9 in set ND^3 and again set Y_-^3 is enlarged. Finally, in this step set ND^3 includes only one variant, namely y^{10}. This situation is presented in Fig. 6b.

The set of variants remaining in Y_+ is empty. All non-dominated variants are included in the set $ND = ND^1 \cup ND^2 \cup ND^3$. A total of 7 variants in set ND are presented to the decision maker. This is the end of the algorithm.

5. Numerical experiments

In this section we present the results of experiments obtained with the use of the dominance-based algorithm. Two separate systems, i.e. with two and three criteria, were included. In the first part of the experiments we focused on a visualization of the solution for the two different criteria.

In the second part of the experiments we tested a given number of variants that would be potentially interesting to the decision maker and achievable with the use of the dominance-based algorithm. We tested 30 successive readings, in which the algorithm was run for every reading and the obtained set of non-dominated variants was presented to the decision maker (every new situation on the price chart corresponds to the new reading). We selected three different time frames corresponding to the scalping system with aggressive trading (the length of a single time frame was equal to 5 minutes), the intraday system (the length of a single time frame was equal to 1 hour) and finally long-term trading with the length of a single time frame equal to 1 day. Thus the overall length of the experiments in the case of the scalping system was equal to $30 \cdot 5 = 150$ minutes, for the intraday system the overall length of the experiments was equal to 30 hours, and 30 days for long-term trading.

We analyzed the fuzzy approach, which included two criteria based on the CCI and RSI indicators, while in the case of the three criteria we used the CCI, RSI and Moving Averages indicators. The number of variants (currency pairs) available in every reading was always equal to 68. A crucial aspect from the point of view of the decision maker was to derive the information about the size of the non-dominated set ND for every considered reading and for every considered time frame. In the experiments, the position of the reservation point x and its impact on the results and information about the computation time were also included. Four different values of the reservation point were included $x = (x_1, x_2, ...x_n)$, $\forall_i\ x_i = 0.7,\ x_i = 0.8,\ x_i = 0.9,\ x_i = 0.95$. In order to verify the efficiency of the domination-based algorithm, we assumed that the case with a variant equal to the aspiration point u among the set of available variants would be omitted, thus it was necessary to calculate the set ND for all of the conducted experiments.

Two different concepts of Crisp systems commonly used on the Forex market were selected as a comparison with the fuzzy approach proposed in this article: namely $Crisp^*$ and $Crisp^{**}$, where in the Crisp* approach each variant is analyzed in the decision making process only when at least one of the conditions defined by the binary activation function is satisfied, while in the second considered approach Crisp** at least $n-1$ conditions must be fulfilled. Both concepts were adapted to the problem with three criteria, thus in the first case we assumed that the signal is generated if any of the considered criteria are equal to the value of 1, while in the second case at least two different criteria are equal to 1. Obviously, the case with all criteria equal to 1 corresponds to the case in which the aspiration point u is achievable.

As was mentioned before, we analyzed three different time frames with 30 readings each. This gives a total count of 90 readings with 68 variants analyzed in each reading. A detailed summary of the time frames that were analyzed in the further part of the experiments can be found in Table 1. We considered examples of two and three indicators.

Table 1: Data sets summary

	Starting Date	Starting Hour	Ending Date	Ending Hour
5 Minutes	2017 IV 03	8.00	2017 IV 03	10.25
1 Hour	2017 I 02	7.00	2017 I 03	12.00
1 Day	2017 II 03	00.00	2017 III 15	00.00

5.1. Data example with two indicators

We selected an approach with two criteria based on the CCI and RSI indicators. We selected the most popular 1-hour time frame and the date of January 3, 2017, which was connected with the opening of the New York trading session. All generated variants are presented in the two-dimensional criteria space shown in Fig. 7a.

A characteristic arrangement of variants in the criteria space can be observed in the case of the system based only on two criteria. A large number of variants has a very high value only in the case of a single criterion, while the other criterion is often below the acceptable value. The results for the fuzzy approach are presented in Fig. 7b. The dot lines in Fig. 7b and Fig. 7c denote the position of the reservation point $x = (0.75, 0.75)$. First, all variants dominated by the reservation point x are excluded from further analysis. This situation can be observed in Fig. 7b. After obtaining a set of variants that are potentially acceptable by the decision maker, the proposed

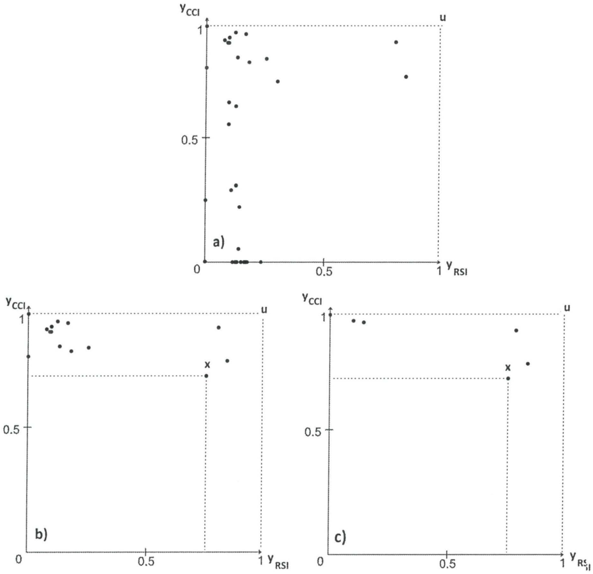

Figure 7: Two-criteria example. a) all 68 variants generated; b) set of considered variants along with the reservation point x. c) set of non-dominated variants derived for the decision maker

algorithm is used to generate a set of non-dominated solutions ND. These variants can be seen in Fig. 7c. Finally, 5 non-dominated, different variants were derived for the decision maker.

These observations confirm that for two criteria the number of variants derived for the decision maker is relatively small in the case of the Crisp* approach, while the fuzzy approach can be used to extend the set of non-dominated variants derived for the decision maker.

The trading systems with only two criteria leads to the situation, in which many variants are located near the maximal possible criterion value, while the second criterion value is very small (situation observed in Fig. 7). The proposed fuzzy system allows to consider such variants in the final set derived to the decision maker. At the same time it is possible to graduate variants for which only one criterion is equal to 1. Such action is not possible in the case of crisp approach.

5.2. Data example with three indicators

In the next part of the experiments we focus on a problem that occurs when three or more criteria (indicators) are involved in the decision process. For a large number of criteria (by large we mean three or more), Crisp* overproduces the number of variants available to the decision maker, thus the selection of a single variant from the subset of variants derived for the decision maker on the basis of Crisp* is still extremely difficult. Crisp** produces a decreasing number of variants while the number of criteria is increasing. Even in the case of three criteria this often leads to an empty set of variants derived for the decision maker. The proposed fuzzy approach fills this gap between the Crisp* and Crisp** approach, and will be shown in details in the further parts of this section.

In our experiments we calculate the number of non-dominated variants accessible to the decision maker in successive readings. In Table 2 we can see the results for the 5-minute time frame. The first four columns of the table are related to the proposed fuzzy approach with different positions of the reservation point x ($x = a$ can be read as $x = (x_1, x_2, ...x_n), \forall_i x_i = a$), while the two remaining columns are related to the results achieved by the crisp methods. Thus *Crisp** is the version in which at least one criterion must be equal to 1, while *Crisp*** corresponds to the method in which at least two criteria must be equal to 1. The last two columns present the results for systems that are commonly used by the decision makers. By comparing the *Crisp** and *Crisp*** results we can observe visible disadvantages that can be limited in the case of the fuzzy approach. First, the number of variants that are potentially interesting (in the case of *Crisp**) to the decision maker increases while the number of indicators used in the system increases. This makes the whole decision process increasingly difficult. At the same time the *Crisp*** approach, in the case of the three indicators, significantly decreases the number of variants available to the decision maker. In Table 2 we can see that in most cases this approach cannot guarantee deriving even a single variant for the decision maker. The fuzzy approach generates relatively small sets of non-dominated variants which are far easier to analyze by the decision maker than the classical crisp approaches that are used today, although the problem with a large set of variants is still observed and can be seen in reading 6 (with 10 variants available to the decision maker even for high x values) or in readings 21 and 25 (respectively variants 8 and 7), but this does not take place as often as in the case of the crisp approach.

We used bold font to indicate the desirable situations that occurred in readings 5, 11, 22, 24, 28, where a relatively small number of variants was eventually derived for the decision maker. We assumed that a desirable number of variants is three, and such situations were marked in the table; however, 2 and 4 are also acceptable. It is worth noting reading number 24, in which for all situations with different values of the reservation point it was possible to generate a relatively small number of variants available to the decision maker. At the same time, we used italic font to indicate situations which are connected with the empty set of variants derived for the decision

Table 2: Number of variants available to the decision maker for the 5-minute time frame

	x = 0.7	x = 0.8	x = 0.9	x = 0.95	Crisp*	Crisp**
Reading 1	7	6	5	5	8	1
Reading 2	6	6	6	6	*16*	1
Reading 3	10	10	6	6	*11*	**2**
Reading 4	9	8	7	5	9	1
Reading 5	6	**4**	**3**	**3**	*12*	1
Reading 6	*12*	*12*	*11*	10	9	*0*
Reading 7	7	7	6	5	**3**	*0*
Reading 8	8	8	8	7	*15*	5
Reading 9	7	7	7	**4**	10	*0*
Reading 10	9	8	8	8	5	*0*
Reading 11	**4**	**4**	**3**	**2**	7	1
Reading 12	6	5	**4**	**3**	*13*	*0*
Reading 13	8	7	6	5	7	*0*
Reading 14	6	5	5	5	*12*	**2**
Reading 15	9	9	5	**4**	*11*	*0*
Reading 16	10	8	6	5	9	**2**
Reading 17	*11*	8	7	6	**4**	*0*
Reading 18	6	6	**4**	**4**	10	**3**
Reading 19	9	8	5	**4**	6	*0*
Reading 20	8	7	6	6	5	*0*
Reading 21	*12*	*11*	10	8	8	*0*
Reading 22	**4**	**4**	**3**	**3**	*13*	1
Reading 23	8	8	6	**4**	10	*0*
Reading 24	**3**	**3**	**3**	**3**	5	**2**
Reading 25	7	7	7	7	5	*0*
Reading 26	5	**4**	**4**	**4**	7	1
Reading 27	5	5	5	5	*11*	**3**
Reading 28	**4**	**3**	**3**	**3**	8	1
Reading 29	9	9	9	8	10	*0*
Reading 30	8	7	7	5	9	*0*

maker and can be observed, e.g. in readings 6, 7, 9, 10, 12, 13, etc. In the case of the Crisp** and fuzzy approach an undesirable situation takes place when a large number of variants is eventually derived for the decision maker. Again, these situations are marked with italic font.

The results are also presented in graphical form for the fuzzy approach, where $x = 0.95$ (Fig. 8). This allows to easily visualize the disproportions in the number of variants available to the decision maker in both of the crisp methods, while the fuzzy approach allows to balance the number of variants available to the decision maker. It is crucial to understand that all variants derived for the decision maker in the case of the fuzzy approach are non-dominated, while in the case of the Crisp* approach (due to the binary activation function) many variants will be equally important due to one of the considered criteria. This leads to an important observation that in the case of both crisp approaches a single variant is treated as acceptable if any criterion is equal to 1. Thus, in the case of two variants, $y^1 = (0, 0.05, 1)$ and $y^2 = (0, 0.95, 1)$, both of them are treated as $(0, 0, 1)$, while in the fuzzy approach it is possible to distinguish these two variants in favor of the latter which strictly dominated the first variant. Situations such as the one presented above are observed increasingly often when the number of criteria is large.

Similar experiments were conducted for the 1-hour time frame and the results are presented in Table 3. In the case of a large time frame the problem related with the crisp approach seems to be even more visible. The *Crisp** column in the table shows that the number of variants with at least one criterion equal to 1 is surprisingly high, while at the same time in most cases *Crisp*** was not capable of deriving even a single variant. It is worth noting

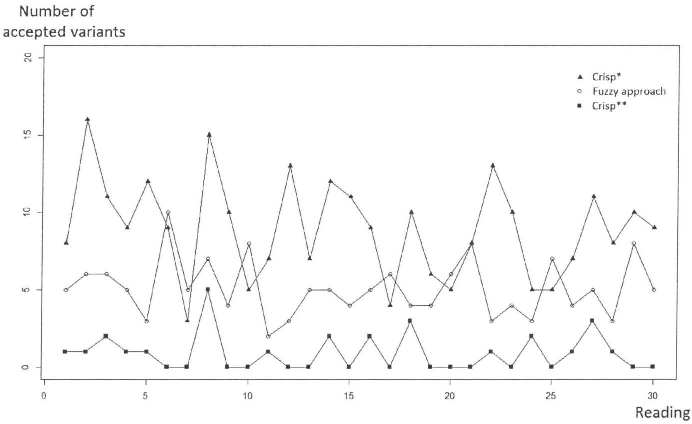

Figure 8: 5-minute time frame linear chart for the fuzzy approach with $x = 0.95$, $Crisp^*$ and $Crisp^{**}$

that in the case of reading 15 both of the crisp methods failed to deliver variants, while the fuzzy approach derived some number of non-dominated variants (depending on the position of x). Once again, we used the bold font to denote the readings in which the given method was able to produce a set of variants that was relatively small in order to be easily analyzed by the decision maker. While the small value of the reservation point ($x = 0.7$) generated sets of variants often exceeding the assumed limits, the position of the reservation point closer to the aspiration point u was able to produce many promising solutions. Such examples can be observed in readings 11, 15, 16, 17, 20, 21, 22. At the same time, in most cases $Crisp^{**}$ was not able to derive a nonempty set of variants for the decision maker, while the size of the solution set derived by $Crisp^*$ was too large (these exceeding 10 variants per reading are shown in italic font).

Table 3: Number of variants available to the decision maker for the 1-hour time frame

	x = 0.7	x = 0.8	x = 0.9	x = 0.95	Crisp*	Crisp**
Reading 1	10	8	7	6	**4**	*0*
Reading 2	9	8	5	**4**	*15*	**2**
Reading 3	*11*	9	6	5	5	*0*
Reading 4	8	6	5	5	*13*	*0*
Reading 5	6	6	5	5	*15*	*0*
Reading 6	7	6	**4**	**4**	10	*0*
Reading 7	10	9	9	8	9	1
Reading 8	7	7	5	**4**	*15*	*0*
Reading 9	9	8	7	7	9	2
Reading 10	7	**4**	**4**	**4**	5	*0*
Reading 11	**4**	**3**	**3**	**3**	7	*0*
Reading 12	6	6	5	**4**	*19*	**3**
Reading 13	10	9	5	5	*12*	2
Reading 14	6	5	5	5	**2**	1
Reading 15	7	5	**3**	**2**	*0*	*0*
Reading 16	**3**	**3**	**3**	**3**	10	*0*
Reading 17	**4**	**4**	**3**	**3**	7	*0*
Reading 18	6	**4**	**4**	**4**	**4**	*0*
Reading 19	6	6	5	**4**	**2**	*0*
Reading 20	**4**	**4**	**3**	**2**	9	*0*
Reading 21	5	**4**	**3**	**3**	9	*0*
Reading 22	7	5	**4**	**3**	6	*0*
Reading 23	**4**	**3**	**3**	**3**	7	*0*
Reading 24	**4**	**4**	**4**	**3**	*13*	2
Reading 25	**4**	**4**	**4**	**3**	7	1
Reading 26	6	6	5	**3**	10	*0*
Reading 27	6	6	**4**	**4**	*11*	1
Reading 28	5	5	**3**	**3**	7	*0*
Reading 29	5	**4**	**4**	**3**	6	*0*
Reading 30	**3**	**3**	**3**	**3**	5	*0*

Again, the same results in graphical form for both of the crisp methods and the fuzzy approach with $x = 0.95$ are presented in Fig. 9. The tendency in which the number of variants derived for the decision maker on the basis of the fuzzy method visibly fills the gap between the relatively small number of variants derived for the decision maker on the basis of Crisp** and the too large number of variants derived with the use of Crisp*. This problem will become even more important when the number of considered variants is smaller and, eventually, the Crisp** approach will not be able to produce any variants. Under these circumstances, extending the Crisp** approach with additional criteria will lead to a situation in which in the most readings it will not be possible to derive even a single variant for the decision maker.

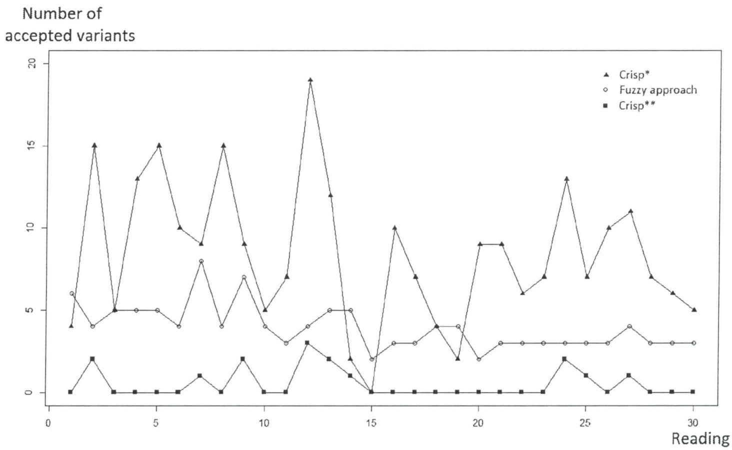

Figure 9: 1-hour time frame linear chart for the fuzzy approach with $x = 0.95$, *Crisp** and *Crisp***

Finally, the results for the largest 1-day time frame are presented in Table 4. The results in the case of the crisp methods are similar as were observed in the 1-hour time frame, where $Crisp^{*}$ derived a set of variants that could not effectively be analyzed by the decision maker, while $Crisp^{**}$ allowed to generate only 13 variants that were potentially interesting to the decision maker during the whole 30-day analyzed period, which the case of 68 variants in a single reading was far beyond the expected numbers. The fuzzy approach, in turn, once again allowed to obtain a reasonable number of non-dominated variants in successive readings. In the largest considered time frame an interesting situation could be observed in readings 4, 18 and 20. This is an extreme case in which the $Crisp^{**}$ approach could not deliver even a single variant while $Crisp^{**}$ generated a number of variants that greatly exceeded the analytical capabilities of the decision maker. At the same time, the fuzzy approach generated an optimal number of non-dominated variants.

It is worth noting that in our experiments we assumed that the position of the reservation point x remained unchanged during the new readings; however, there were no contra-indications to set a new position of x in successive readings. Moreover, such action is supported by the framework, as the overall computation time is less than 3 seconds, which includes the process of calculating the values of the criteria and generating the set of non-dominated variants. Thus a recalculation of the new ND set can be done even within the same reading; lower values of x support a more aggressive trading option, where the initially non-acceptable variants with lower x values will eventually be included in the ND set. However, the fuzzy approach still allows to easily distinguish variants where one of the criteria is equal to 1 while the others have lower values; this cannot be done in the crisp approach.

Finally, the graphical representation for the fuzzy approach with $x = 0.95$

Table 4: Number of variants available to the decision maker for the 1-day time frame

	x = 0.7	x = 0.8	x = 0.9	x = 0.95	Crisp*	Crisp**
Reading 1	9	9	7	**4**	**4**	*0*
Reading 2	**3**	**3**	**3**	**2**	8	1
Reading 3	6	6	5	**4**	7	*0*
Reading 4	**4**	**3**	**3**	**3**	12	*0*
Reading 5	7	7	7	7	5	*0*
Reading 6	7	6	**4**	**4**	9	*0*
Reading 7	8	6	6	5	8	*0*
Reading 8	6	5	**4**	**4**	9	1
Reading 9	6	6	**2**	**2**	8	*0*
Reading 10	9	7	6	6	14	1
Reading 11	8	5	**3**	**2**	6	1
Reading 12	5	**4**	**4**	**4**	10	1
Reading 13	9	8	8	8	*17*	*0*
Reading 14	8	7	6	5	*11*	2
Reading 15	8	6	6	6	**4**	*0*
Reading 16	5	5	5	5	7	1
Reading 17	**4**	**4**	**4**	**4**	5	*0*
Reading 18	6	**4**	**4**	**3**	*12*	*0*
Reading 19	6	5	5	5	8	*0*
Reading 20	**3**	**3**	**3**	**3**	*17*	2
Reading 21	8	6	6	**4**	*12*	*0*
Reading 22	7	7	6	6	9	*0*
Reading 23	8	8	7	7	*11*	1
Reading 24	7	5	**3**	**3**	7	*0*
Reading 25	*11*	9	8	6	9	*0*
Reading 26	7	7	5	5	6	*0*
Reading 27	7	7	6	5	*18*	1
Reading 28	8	8	7	7	*14*	2
Reading 29	*11*	10	7	6	**4**	*0*
Reading 30	9	9	7	6	10	*0*

and the two crisp approaches for the 1-day time frame are presented in Fig. 10. Just as in the two previous experiments with a lower time frames, once again we can observe a situation in which the proposed approach allows to balance a very small (or empty) set of variants derived for the decision maker in the case of $Crisp^{**}$ and a large set of variants when at least one criterion is equal to 1. It is worth noting that by adding more criteria $Crisp^{*}$ will eventually allow to obtain an even larger number of variants, which then cannot be effectively analyzed by the decision maker. The proposed fuzzy approach seems to be resistant to such problems.

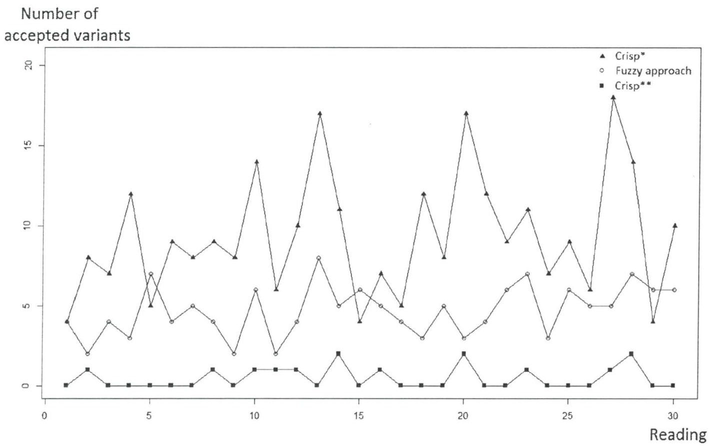

Figure 10: 1-day time frame linear chart for the fuzzy approach with $x = 0.95$, $Crisp^{*}$ and $Crisp^{**}$

To sum up, we prepared a boxplot chart for all of the 18 data sets and the results achieved for different time frames; the results are presented in Fig. 11. The first six boxplots are related to the smallest 5-minute time frame (the first four boxes are connected with increasing values of x and the last two boxes are, respectively, $Crisp^*$ and $Crisp^{**}$); boxes in the middle of the chart are connected with the medium time frame equal to 1 hour and, finally, the last 6 boxes are connected with the highest 1-day time frame. Every single box gives information about both the minimal and maximal values of variants derived for the decision maker as well as the first and third quartile. The line in the middle of the box is the median value. The outlier values in the charts are presented by empty circles. For higher values of the reservation point x and closer distance to aspiration point u), most of the observed values fall directly in the gap between results obtained on the basis of both crisp approaches. This confirms that the proposed solution allows to generate a number of variants that are interesting from the point of view of the decision maker in situations in which both crisp approaches fail. Similar conclusions are true for four and a larger number of criteria used in the system.

The number of solutions generated in the case of Crisp* fairly exceeds analytical capabilities of the decision maker, while Crisp** often generates no solutions at all. The proposed fuzzy approach gives the possibility to control the number of generated variants on the basis of the risk aversion adjusted with the use of the reservation point. At the same time it may be easily extended on the trading systems with four and more criteria.

6. Conclusions and future work

In this article we presented a decision support framework based on the fuzzy sets concept and the dominance-based algorithm. The proposed method was adapted to effectively support the decision maker in the field of financial

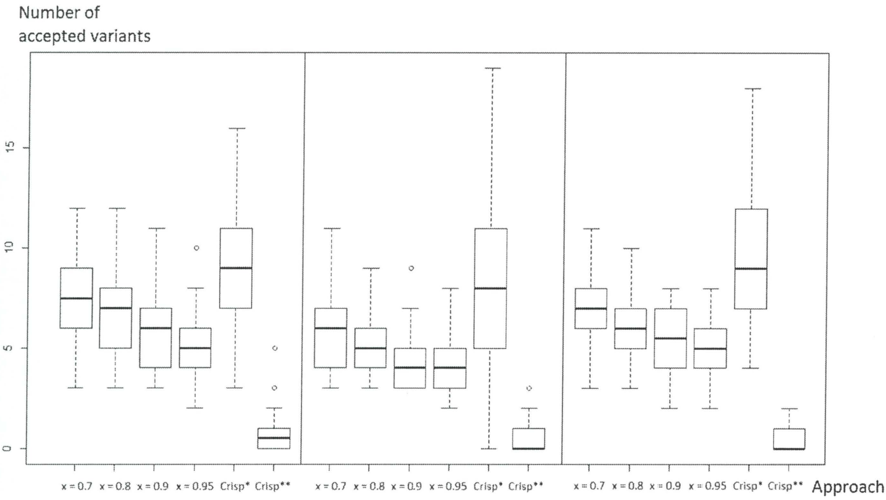

Figure 11: Boxplots for all considered data sets.

data. We focused on currency pairs, i.e. data considered to be the most dynamic and difficult to analyze. Concepts related with the fuzzy sets theory were adapted to transform values of indicators into values of membership functions which were further used in the decision process. The crisp approach was included to compare and estimate the efficiency of the proposed approach. The main contribution of this article is a new dominance-based algorithm which is capable of delivering a set of non-dominated variants that are potentially interesting for the decision maker. All sets of variants are divided into smaller subsets (depending on the number of steps of the algorithm) and eventually a subset of non-dominated variants is derived for each of them. This approach guarantees that the decision maker will obtain a full set of non-dominated variants available to him/her at the time t. The proposed approach may be considered as an extension of the crisp approach, in which all variants considered by the decision maker can be found only on the edge of the set of acceptable solutions. It should be noted that the proposed approach allows for an analysis of all variants which are described as potentially interesting to the decision maker. The concept of reservation point relates to risk aversion of the decision maker. Lower values of the reservation point (further distance from the aspiration point u) are related to a higher risk taken by the decision maker. A short illustrative example was also described.

To confirm the efficiency of the proposed algorithm, a decision support framework which included the proposed fuzzy approach and the typical crisp approaches was developed. Numerical tests including systems with two and three criteria were also presented. The system with two criteria was used to visualize the problem that takes place when the crisp approach delivers a large set of variants but any of them equals to the aspiration point u.

The numerical tests with three criteria allowed to show gaps in the crisp approaches. We used two different systems based on the crisp approach, namely Crisp* and Crisp**. An interesting problem occurred when two not correlated criteria (two indicators) were included in the system. The situation when both indicators can generate an effective crisp signal is rare, thus the extension of the crisp approach with the use of fuzzy concepts allows to visibly increase the number of reasonable variants derived for the decision maker.

The methodology introduced in the article allows for fast and effective calculation of the non-dominated set of variants (currency pairs) which are derived for the decision maker. The proposed system assures sovereignty of the decision maker. The reservation point concept may be identified as a tool allowing to maintain the risk at an level acceptable on the part of the decision maker.

The obtained results confirm that the dominance-based algorithm proposed in the article can be effectively used by decision makers not only on the Forex market, but also on other markets. Flexibility allows to freely modify the set of criteria included in the decision process. Our future work will focus on deriving a dominance rank mechanism which will not only deliver a set of non-dominated variants but also a rank of variants generated on the basis of the trade-off approach.

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