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A study of history for control engineers

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***Abstract*—The evolution of civilizations according to A.J. Toynbee can be explained by a simple feedback model that may be used as a stimulating "tool for thought". Even if the forward path simply consists of an integrator and the feedback path of a pure delay element, the model gives rise to a variety of responses, thus demonstrating clearly the counterintuitive behavior of closed–loop systems. The relation of the model with those used in control system design is pointed out.**

Keywords: Feedback, Time–delay systems, Step response, History of civilizations.

I. Introduction

Although the title might seem to allude maliciously to the need of introducing a new topic in the control engineering curriculum, it is simply intended to specify the nature of a potential nontechnical application of classic control theory. At the same time, it is plainly allusive to the famous 12–volume *opus magnum* by Arnold J. Toynbee entitled "A Study of History" [18], finished in 1961, in which the author traces the development and decay of all of the major world civilizations in the historical record. The first 10 volumes of Toynbee's monumental work have been abridged by David C. Somervell in 2 volumes which are themselves a great historical achievement [16] and were kept as *livre de chevet* by Antonio M. Lepschy (1931–2005), a founding father of the Italian control community, whose studies on the dynamics of historical events date back to the mid–70s [9]. Indeed, the interest of the present authors in this kind of problems was aroused by their long–lasting collaboration with Lepschy [20].

The intent of this paper is to show how the evolution of ancient civilizations, as described by Toynbee and his followers, can be explained by means of classic control theory tools. According to the authors' experience, this uncommon application is particularly appealing to engineers who realize that the properties of feedback systems can be exploited far beyond industrial processes. Of course, these authors are well aware that a thorough understanding of the laws underlying complex historical phenomena would require competences that cannot be expected from an average engineer, and in fact the study of the evolution of civilizations has been the lifelong concern of highly knowledgeable scholars (see, e.g., [13] [5] [2] [17]). However, with due awareness of its cultural limitations, the suggested application lends itself well to demonstrate some essential properties of feedback, with particular regard to the emergence of counterintuitive behaviors (in the sense of [6]) from the interconnection of simple component parts such as integrators and static gains. The value of the following exercise rests mainly on its use as a "tool for thought", borrowing an expression of Rheingold [14].

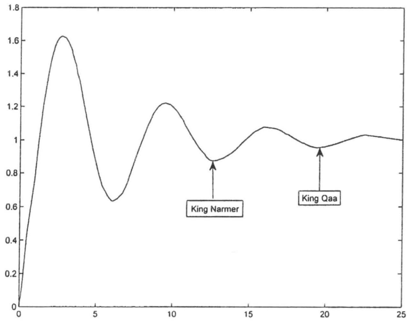

Fig. 1. The "normal evolution" exhibited by the Nile River civilization from about 3650 B.C. to 2650 B.C. (adapted from [3]). One time unit corresponds to 40 years. The ordinate represents a suitable "societal index" [2] (normalized to steady state). In particular, the second rally corresponds to the unification of Upper and Lower Egypt under King Narmer and the third rally to King Qàa's upheaval.

Even if neither Toynbee [18] nor Somervell [16] offered a pictorial representation of the evolution of civilizations, they did provide a precise description of the course of all known historical civilizations. Essentially, they distinguish three kinds of civilizations:
- the "normal civilizations" whose evolution is characterized by a first period of rapid growth followed by four "routs" separated by three "rallies" as in Fig. 1 (precisely, Toynbee's words are: "the normal rhythm seems to be rout–rally–rout–rally–rout–rally–rout: three and a half beats"),
- the "arrested civilizations" characterized by a monotonic growth ending rather rapidly (to this category belong the Polynesian, Eskimo, Nomadic, Ottoman, and Spartan civilizations),

and
- the "abortive civilizations" which after a short initial growth return to the condition before the growth (to this category belong the Far Western Christian, Far Eastern Christian, Scandinavian, and Syriac civilizations).

No explicit measure of civilization level is provided by Toynbee. Notable attempts in this direction have been made by Gray [7] and Blaha [2]. In particular, the latter defines a *societal index* based on a variety of social and technological aspects. Since this paper focuses on evolution patterns and explanatory models [15] rather than accurate descriptions, units of maesure will be neglected.

Previous work on the modeling of historical phenomena has concentrated mainly on data fitting methods. The approximations based on the solutions of linear differential equations can be included in this category since they try to account for the observed patterns by means of combinations of exponential modes that depend, in a rather cryptic way, on the coefficients of a characteristic equation [9]. Instead, the adopted feedback model is meant to convey insight into the "mechanism" that gives rise to the observations and allows us to evaluate the effects of changes of individual parameters on the response patterns. As observed in [10], a similar approach was followed by A.W. Phillips in the modeling of a closed economy for stabilization purposes [12] [19]. Indeed, there is a close relationship between socio–economic phenomena and the evolution of civilizations.

By pursuing previous ideas [4], Section II presents a simple time–delay feedback model and shows how its response depends on the forward–path gain and the extent of the delay in the feedback path. The model differs from that proposed in [4] in the number and location of its component parts: it turns out that only two ingredients are required to reproduce all of the behaviors described by Toynbee, i.e., an integrator and a delay element. Section III shows how to obtain different responses and discusses the choice of the input and the location of the delay inside the loop.

II. Feedback Model

A widely adopted approach to the design of standard controllers in feedback control systems assumes that the plant can adequately be represented by a first–order rational transfer function in series with a time delay. Indeed, such a model approximates well the behaviour of typical complex industrial processes characterized by many left half–pale (LHP) real poles and a large pole/zero excess, and allows us to evaluate easily system robustness, e.g., in terms of modulus margin [8], which would be difficult by referring to more elaborate models. On the other hand, time lags are actually present in many applications due to transportation, measurement or communication delays (cf., e.g., [21]).

It seems reasonable to adopt a similar model inside a loop for complex historical processes, too. However, an essential difference with respect to the control of industrial plants is related to the fact that the feedback action in these kind of non–technical systems may not be associated with a subsystem that is physically distinct from the rest of the system, i.e., the feedback is *intrinsic* or *endogenous*, as is often the case in systems biology [1]. Of course, time scales are much different from those encountered in technological applications: historical time is usually measured in decades or even centuries.

Based on the previous considerations, the adopted explanatory model is structured as in Fig. 2. It consists of a closed–loop system whose forward path contains only an integrator with gain k and the negative feedback path only a delay element of duration t_d. Despite its disarming simplicity, this two–parameter model can generate a variety of oscillatory and aperiodic behaviors depending on the values of the t_d and k. The latter can be viewed as the proportional gain of a standard controller.

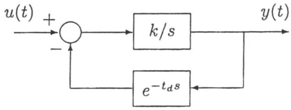

Fig. 2. Simple negative feedback model of a civilization: the output of an integrator with transfer function k/s is fed back to the input through a pure delay element with transfer function is $e^{-t_d s}$.

The transfer function from $u(t)$ to $y(t)$ is clearly

$$W(s) = \frac{k}{s + k e^{-t_d s}}. \tag{1}$$

According to the Nyquist criterion, stability is ensured if, and only if, the Nyquist diagram of the loop function $L(j\omega) = k e^{-j t_d \omega}/j\omega$ does not encircle the critical point $-1+j0$. Simple calculations show that this condition is satisfied for

$$k < \frac{\pi}{2t_d}. \tag{2}$$

Due to the presence of the delay term, the system may well exhibit oscillations even if the forward–path transfer function has just one pole in the origin, which makes the system a type–1 system with zero steady–state error to a step input. Fig. 3 shows the step responses of system (1) for $t_d = 1$ and a number of values of k.

Given the value of t_d, oscillations are present only when k exceeds a certain value, under which the system response is overdamped. For example, when $t_d = 1$, as in Fig. 3, this discriminating value is $k_c = 0.36$ which leads to a critically damped response. Clearly, the overshoot of an underdamped response increases with k since the system approaches instability. The settling time, too, increases with k; correspondingly, the rise time decreases and the system becomes more reactive.

The effects of the delay duration t_d on the system response are shown in Fig. 4. An increase of t_d reduces the stability margins and, thus, increases the tendency to oscillate. As is expected, also the pseudo–period T_p of the oscillations, i.e., the distance between two consecutive maxima, increases with t_d and tends to $4t_d$.

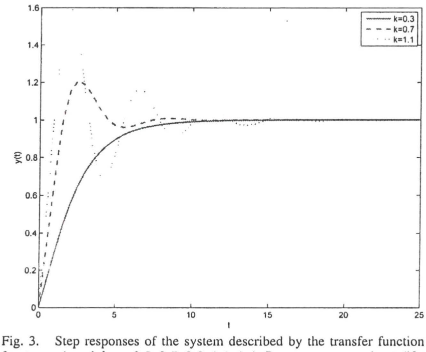

Fig. 3. Step responses of the system described by the transfer function (1) for $t_d = 1$ and $k = 0.2, 0.5, 0.8, 1.1, 1.4$. Promptness, overshoot (if any) and settling time increase with k.

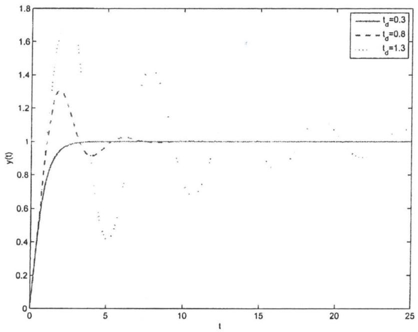

Fig. 4. Step responses of the system described by the transfer function (1) for $k = 1$ and $t_d = 0.1, 0.4, 0.7, 1.0, 1, .3$. Oscillations become more persistent for larger values of t_d, and their pseudo–periods longer.

Without pretense of expert analysis, the feedback model of Fig. 2 lends itself to a natural interpretation. Precisely, the integrator in the forward path accounts for an accumulation process (e.g., of knowledge and experience) leading to an increased exploitation of resources. After a certain time, however, this augmented consumption deprives the system of resources that are necessary for further development. For such a depriving action accounts the *negative* feedback channel. The gain k can be thought of as a measure of system responsiveness, whereas t_d is somehow related to the availability of resources (either natural or manmade). The overall process may be considered efficient if it reaches rapidly a steady state compatible with the input and then maintains it without excessive oscillations, which depends on both parameters.

III. Simulation of Different Behaviors

As shown in the previous section, the behavior of a normal civilization can be simulated well by the step response of the feedback model in Fig. 2. For example, the evolution of the Nile river civilization depicted in Fig. 1 can be obtained by setting $t_d = 1.5$ and $k = 0.75$. In this way, the distance between two consecutive intersections of the oscillatory response with the steady–state value is a little more than $2t_d = 3$ and the pseudo–period a little more than $T_p = 6$, i.e., almost a quarter of a millennium by assuming a time unit of 40 years.

The evolution of an arrested civilization, too, can be reproduced by the step response of such a model. For instance, by assuming $t_d = 1.5$, a behavior of this kind is obtained for $k \leq 0.25$. The upper curve in Fig. 5 shows the critically–damped behavior corresponding to $k = k_c = 0.25$.

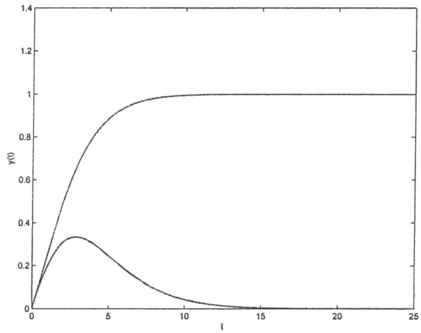

Fig. 5. The upper curve shows the evolution of an arrested civilization. It is obtained as the step response of the feedback model in Fig. 2 for $t_d = 1.5$ and $k = 0.25$ (critically–damped behavior). The lower curve shows the evolution of an abortive civilization. It is obtained as the response of the same model to the input $e^{-t/2}$, $t > 0$.

The behavior of an abortive civilization cannot be obtained from the system of Fig. 2 using a step input because the step response of a type–1 system tends asymptotically to reproduce the input. To allow for an eventual decay without adding further elements to the block diagram of the closed–loop system, an asymptotically decreasing input must be adopted. Such a choice corresponds to the relaxation and eventual disappearance of the thrust exerted on the system, which seems to be a reasonable assumption in many cases. The response of system (1) with $t_d = 1.5$ and $k = 0.25$ to the decreasing exponential input $u(t) = e^{-t/T}$, $t > 0$ with time constant $T = 2$ is also shown in Fig. 5 (lower curve).

Clearly, the same result can be achieved using a step input and inserting a unit–gain smoothed–derivative filter characterized by the transfer function

$$F(s) = \frac{Ts}{1 + Ts}, \qquad (3)$$

with $T = 2$, in cascade with the closed–loop system of Fig.

2. This additional component could account for a progressive deterioration or obsolescence of resources (Toynbee uses the term "disintegration" to describe this process).

The same procedure can be followed to generate evolutions that fall off to the final value in an oscillatory fashion. A behavior of this kind is exhibited by the Greek scientific civilization according to Napolitani [11]. Fig. 6 reproduces the curve shown in [11]. It has been obtained by filtering the oscillatory step response of (1) with $t_d = 1$ and $k = 1$ by means of (3) with $T = 10$.

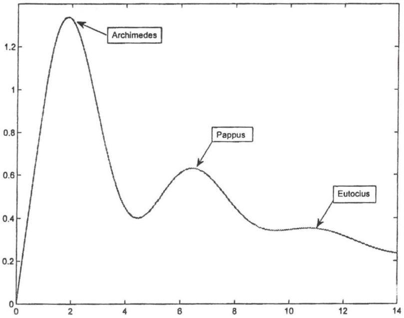

Fig. 6. Evolution of the Greek scientific civilization according to [11]. The curve starts increasing at the middle of the fifth century B.C. (Hyppocrates of Chios), reaches a first maximum at the middle of the third century B.C. (Archimedes of Syracuse) and then declines slowly until the end of the fifth century A.D. (Eutocius of Ascalon) presenting, however, a second rather flat maximum around the end of the third century B.C. (Pappus of Alexandria).This diagram has been obtained by filtering the step response of (1) with $t_d = 1$ and $k = 1.03$ by means of (3) with $T = 10$.

A final remark concerns the location of the delay. If the delay is placed entirely in the feedback path, as in Fig. 2, the model response $y(t)$ starts immediately after the application of the input. Instead, if the delay is placed entirely in the forward path of the loop, the output is equal to zero for $t < t_d$. An intermediate situation occurs when one part of the delay is placed in the forward path and the remaining part in the feedback path. However, since the loop function does not change whatever the position of the delay inside the loop is, the shape of the response remains the same. Fig. 7 shows the step responses for $k = 1$ and $t_d = 1$ when the delay element is located entirely in the forward path (no output delay) and when it is located entirely in the feedback path (output delayed by t_d).

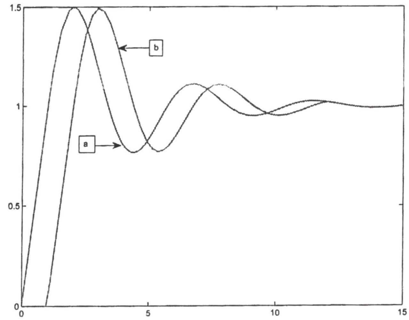

Fig. 7. Step responses of the feedback system for $k = 1$ and $t_d = 1$ when the delay t_d is placed entirely in the feedback path (curve a) and when it is placed entirely in the forward path (curve b). Curve b is simply a shifted version of curve a.

IV. Conclusions

Despite its remarkable simplicity, the model of Fig. 2 can explain well the behaviour of all the civilizations studied by Toynbee and his successors. Its feedback structure and the nature of its two component parts lend themselves to suggestive interpretations, making the model a useful tool for thought even in a nontechnical context.

However, the main purpose of this contribution has been to show how the feedback connection can generate a variety of system behaviors depending on the values of very few parameters.

Based on the authors' academic experience, the presentation of these concepts in a historical guise is appealing to control engineers, as well as to experts in the social sciences, and helps them appreciate the "mysterious" properties of feedback.

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