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Shape optimization problem for coupling of elasticity and Navier-Stokes equations

I. Lasiecka, K. Szulc, A. Żochowski

Instytut Badań Systemowych
Polska Akademia Nauk

Systems Research Institute
Polish Academy of Sciences

POLSKA AKADEMIA NAUK

Instytut Badań Systemowych

ul. Newelska 6

01-447 Warszawa

tel.: (+48) (22) 3810100

fax: (+48) (22) 3810105

Kierownik Zakładu zgłaszający pracę:
Prof. dr hab. inż. Antoni Żochowski

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SYSTEMS RESEARCH INSTITUTE
POLISH ACADEMY OF SCIENCES

Irena Lasiecka, Katarzyna Szulc, Antoni Żochowski

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Contents

Chapter 1

Problem formulation

Let $D \subset \mathbb{R}^2$ be a bounded domain with a piecewise regular boundary ∂D consisting of two sub-domains Ω_1 and Ω_2, as shown in Fig.1.1. The boundary of the interior part of the domain $\partial\Omega_1$ is denoted by $\Gamma_{\text{int}} \cup \Gamma_1$ and the exterior boundary $\partial\Omega_2$ is denoted by $\Gamma_{\text{ext}} = \Gamma_{\text{in}} \cup \Gamma_{\text{out}} \cup \Gamma_{\text{wall}}$. In the interior subdomain Ω_1 we consider a problem of linear elasticity for elastic body, and in the exterior subdomain Ω_2 we consider a problem of Navier-Stokes for motion of fluid.

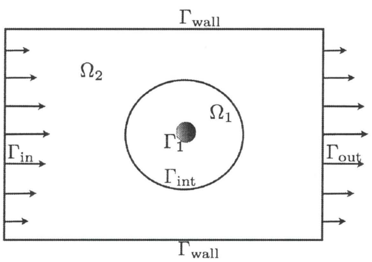

Figure 1.1: Domaine $D = \Omega_1 \cup \Omega_2$ with its boundary $\Gamma_{\text{in}} \cup \Gamma_{\text{out}} \cup \Gamma_{\text{wall}}$.

Linear elasticity. The equilibrium equations for a linear elastic body occupying Ω_1 are given as follows.

$$-\mathrm{div}\sigma(\mathbf{u}) = 0 \quad \text{in} \quad \Omega_1, \tag{1.1}$$

$$\sigma(\mathbf{u}) = A\varepsilon(\mathbf{u}) \quad \text{in} \quad \Omega_1, \tag{1.2}$$

$$\mathbf{u} = 0 \quad \text{on} \quad \Gamma_1, \tag{1.3}$$

$$\sigma(\mathbf{u})\cdot \mathbf{n}_{\Omega_1} = \mathbf{t}(\mathbf{u},\mathrm{p}) \quad \text{on} \quad \Gamma_{\text{int}}, \tag{1.4}$$

where $\mathbf{u} = (u_1, u_2)$ is the displacement field, $\sigma = \{\sigma_{ij}\}, i,j = 1,2$ are the stress tensor components.Elasticity tensor $A = \{a_{ijkl}\}, i,j,k,l = 1,2$ is given and satisfies the usual properties of symmetry and positive definiteness

$$\begin{array}{ll} a_{ijkl}\xi_{kl}\xi_{ij} \geq c_0|\xi|^2, \quad \forall\xi_{ij}, \xi_{ij} = \xi_{ji}, \quad c_0 = const, \\ a_{ijkl} = a_{klij} = a_{jikl}, \quad a_{ijkl} \in L^\infty(\Omega_1). \end{array} \tag{1.5}$$

Relation (1.1) are equilibrium equations, and (1.2) is the Hooke's law, $u_{ij} = \dfrac{\partial u_i}{\partial x_j}$, $(x_1, x_2) \in \Omega_1$. All functions with two lower indeces are symmetric in these indeces, i.e. $\sigma_{ij} = \sigma_{ji}$ etc. Summation convention is assumed over repeated indices throughout the paper. Here $\mathbf{t}(\mathbf{u},\mathrm{p})$ is the traction force depending on the pressure in the fluid and displacement on the surface Γ_{int}.

Transformation of the domain. Suppose that an incompressible viscous flow occupies Ω_2. One of the difficulties in the paper is modification of the interior boundary Γ_{int}. We propose the following procedure for the boundary displacement. Let the interior boundary be the set defined as follows:

$$\Gamma_{\text{int}}(\mathbf{u}) = \{\mathbf{x} : \mathbf{x} = \mathbf{x}^p + \mathbf{u}(\mathbf{x}^p),\ \mathbf{x}^p \in \Gamma_{\text{int}}\}. \tag{1.6}$$

where $\mathbf{u} = (u_1, u_2)^\top$. We define the transformation of the domain $\Omega_2(0)$ by

$$\begin{array}{rcll} \Delta\phi_1 & = & 0 & \text{in} \ \ \Omega_2(\mathbf{u}), \\ \phi_1 & = & u_1 & \text{on} \ \ \Gamma_{\text{int}}(\mathbf{u}), \\ \phi_1 & = & 0 & \text{on} \ \ \Gamma_1, \end{array} \tag{1.7}$$

and

$$\begin{array}{rcll} \Delta\phi_2 & = & 0 & \text{in} \ \ \Omega_2(\mathbf{u}), \\ \phi_2 & = & u_2 & \text{on} \ \ \Gamma_{\text{int}}(\mathbf{u}), \\ \phi_2 & = & 0 & \text{on} \ \ \Gamma_1, \end{array} \tag{1.8}$$

and $\Phi(\mathbf{x}) = \mathbf{x} + \phi(\mathbf{x}) = \mathbf{x} + \begin{bmatrix} \phi_1(\mathbf{x}) \\ \phi_2(\mathbf{x}) \end{bmatrix}$.

Then $\Omega_2(u) = \Phi(\Omega_2(0))$. Observe that if derivatives $\mathbf{u}_{i/j}$ are small, so are the derivatives of φ_1, φ_2. Such defined Φ is the smoothest possible transformation of the domain $\Omega_2(0)$ and $\Omega_2(0) = \Phi^{-1}(\Omega_2(\mathbf{u}))$. Denote the coordinates in $\Omega_2(\mathbf{u})$ as $\mathbf{y}$, i.e.

$$\mathbf{y} = \Phi(\mathbf{x}), \quad \mathbf{x} \in \Omega_2(0). \tag{1.9}$$

Navier-Stokes equation. The state equation for the flow is given in the above coordinates by the following system of stationary Navier-Stokes equations:

$$-\nu\Delta_{\mathrm{y}}\mathbf{w} + (\mathbf{w}\cdot\nabla_{\mathrm{y}})\mathbf{w} + \nabla_{\mathrm{y}}\mathrm{p} = 0 \quad \text{in} \quad \Omega_2(\mathbf{u}), \tag{1.10}$$

$$\mathrm{div}_{\mathrm{y}}\mathbf{w} = 0 \quad \text{in} \quad \Omega_2(\mathbf{u}), \tag{1.11}$$

$$\mathbf{w} = 0 \quad \text{on} \quad \Gamma_{\mathrm{int}}(\mathbf{u}), \tag{1.12}$$

$$\mathbf{w} = 0 \quad \text{on} \quad \Gamma_{\mathrm{wall}}, \tag{1.13}$$

$$\partial_{\mathbf{n}}\mathbf{w} + \mathrm{p}\cdot\mathbf{n} = 0 \quad \text{on} \quad \Gamma_{\mathrm{out}}. \tag{1.14}$$

Here $\mathbf{w} = (w_1, w_2)^T$ is a velocity field, p the pressure, ν the kinematic viscosity of the fluid ($\nu = \frac{1}{Re} > 0$, where Re is the Reynolds number). The non-linear term $(\mathbf{w}\cdot\nabla)\mathbf{w}$ in (1.10) is a symbolic notation for the vector

$$(w_1\frac{\partial w_1}{\partial y_1} + w_2\frac{\partial w_2}{\partial y_2}, w_1\frac{\partial w_2}{\partial y_1} + w_2\frac{\partial w_1}{\partial y_2})^T.$$

A parallel flow in a channel is considered.

After transformation $\mathbf{y} = \Phi(\mathbf{x})$, this system is defined in $\Omega_2(0)$, but has variable coefficients:

$$-\nu\nabla_x(A(\mathbf{u})\nabla_x\mathbf{w}) + \mathbf{w}(K(\mathbf{u})\nabla_x)\mathbf{w} + H(\mathbf{u})\nabla_x\mathrm{p} = 0 \text{ in } \Omega_2(0). \tag{1.15}$$

Here $A(\mathbf{u}) = A(\mathbf{x})$, $K(\mathbf{u}) = K(\mathbf{x})$ and $H(\mathbf{u}) = H(\mathbf{x})$ are complicated expressions depending on $\phi_{i/j}$. Our idea is to linearise them, leaving only first powers of $\phi_{i/j}$. This facilitates both numerical computations and theoretical analysis of the whole coupled system.

Coupling of N-S equations and elasticity. The coupling of velocity and displacement fields acts through the expression

$$\mathrm{t}(\mathbf{u}, \mathrm{p}) = \mathrm{p}\cdot\tilde{B}(\phi)\cdot\mathbf{n}_{\Omega_1}.$$

For illustration, the linearised version is as follows

$$\tilde{B} = (1 - \frac{1}{2}\mathbf{n}^\top B\mathbf{n})I + C = (1 - \mathbf{n}^\top C\mathbf{n})I + C, \tag{1.16}$$

where

$$B(\phi) = \begin{bmatrix} 2\phi_{2/2} & -(\phi_{1/2} + \phi_{2/1}) \\ -(\phi_{1/2} + \phi_{2/1}) & 2\phi_{1/1} \end{bmatrix}, \tag{1.17}$$

and

$$C = C(\phi) = \begin{bmatrix} \phi_{2/2} & -\phi_{2/1} \\ -\phi_{1/2} & \phi_{1/1} \end{bmatrix},$$

so that $B = C + C^{\top}$. The solution of a coupled system is done using the fixed point iteration.

The ultimate goal of the research is to study the effect of small holes inside the elastic body on the hydrodynamic drag. The shape functional that we consider here is the integral functional describing the aerodynamic resistance and written in the following form

$$\mathcal{I} = \int_{\Gamma_{\text{int}}(\mathbf{u})} \mathrm{p} \cdot \mathbf{n}_{\Omega_2(\mathbf{u})} \cdot \mathrm{e}_1 \mathrm{d}s, \tag{1.18}$$

where $\mathbf{e_1}$ is a unit vector directed to the right.

1.1 Wellposedness of nonlinear problem

The first step toward optimization is a good understanding of wellposedness of the system with respect to existence, uniqueness and continuous dependence on the data in the respective topologies. This will amount showing that with given boundary data $(g_1, g_2) = (w_1|_{\Gamma_{in}}, w_2|_{\Gamma_{in}})$ which are "small" with respect to suitable topology on the boundary, one obtains existence of the solutions in a suitable state space. The choice of topology is critical-as in all quasilinear problems. In the present case we shall consider $W^{s,p}$ spaces for suitable values of p, s .

Theorem 1.1.1 *Assume that* $\mathbf{g} = (g_1, g_2) \in W^{1-\frac{1}{p},p}(\Gamma_{in})$ *with suitably small norm. For dimension* Ω *equal 2 we take* $p > 2$ *and for dimension* Ω *equal 3, we take* $p > 3$. *Then, there exists unique solution* $\mathbf{u} \in W^{2,p}(\Omega_1)$, $(\mathbf{w}, p) \in W^{2,p}(\Omega_2) \times W^{1,p}(\Omega_2)$. *which depends continuously on the data in the topologies listed above.*

Proof. We shall carry the proof for $n = 2$. In the case of $n = 3$ the numerology can be easily adjusted. In order to carry out the proof we shall rewrite the original system as follows;

$$\mathbf{u} = N\mathrm{t}(\mathbf{u}, \mathrm{p}), \tag{1.19}$$

Where the map $N : W^{s,p}(\Gamma_{\text{int}}) \to W^{s+1+\frac{1}{p},p}(\Omega_1)$ is Neuman solver for the system of elasticity. The flow map transforming variable domain into static domain is given by:

$$\Phi(\mathbf{u}) = I + D(\mathbf{u}|_{\Gamma_{\text{int}}}), \text{ in } \Omega_2(\mathbf{u}) \tag{1.20}$$

where D is a standard Dirichlet harmonic extension. Thus, $\Omega_2(\mathbf{u}) = \Phi(\Omega_2(0))$ The traction force $\mathrm{t}(\mathbf{u}, \mathrm{p})$ is determined by $\mathrm{p}\mathbf{n}$ in the reference domain given by

$$\mathrm{t}(\mathbf{u}, \mathrm{p}) = \mathrm{p}\tilde{B}(\Phi(\mathbf{u}))\mathbf{n}|_{\Omega_1} \tag{1.21}$$

where $\tilde{B}$ is obtained via change of variables

$$\tilde{B}(\cdot) = (I - \mathbf{n}^\top C\mathbf{n})I + C$$

and $C(\phi)$ is given above. The elastic system $\mathbf{u}$ is fed by the force t, hence the pressure p obtained from quasilinear Stokes equation defined on a reference domain $\Omega_2(0)$.

$$\begin{aligned} \nu\nabla_x(A(\mathbf{u})\nabla_x\mathbf{w}) + \mathbf{w}(K(\mathbf{u})\nabla_x)\mathbf{w} + H(\mathbf{u})\nabla_x\mathrm{p} &= 0 \\ div_{A(\mathbf{u})}\mathbf{w} &= 0 \\ \mathbf{w} &= g, \Gamma_{in} \\ \frac{\partial_{A(\mathbf{u})}\mathbf{w}}{\nu} + \mathrm{p}\mathbf{n} &= 0, \Gamma_{int} \end{aligned} \tag{1.22}$$

The above formulation leads to a fixed point determined from the chain of implications

$$\mathbf{u} \to \Phi(\mathbf{u}) \to (\mathbf{w}(\mathbf{u}, \mathbf{g}), \mathrm{p}(\mathbf{u}, \mathbf{g})) \to \mathrm{t}(\mathbf{u}, \mathrm{p}) \to N\mathrm{t}(\mathbf{u}, \mathrm{p}) = \mathbf{u}$$

The equation for fluid is quasilinear and will be treated as a perturbation of the linear part. This leads to a map $T(\bar{\mathbf{u}}, \bar{\mathbf{w}}, \bar{\mathrm{p}}) \to (\mathbf{u}, \mathbf{w}, \mathrm{p})$

$$(\bar{\mathbf{u}}, \bar{\mathbf{w}}, \bar{\mathrm{p}}) \to (\mathbf{u}, \mathbf{w}, \mathrm{p})$$

where for a given $\bar{\mathbf{u}}, \bar{\mathbf{w}}, \bar{\mathrm{p}}$ one solves the linear problem for $(\mathbf{u}, \mathbf{w}, \mathrm{p})$.

$$\begin{aligned} \nu\nabla_x(\nabla_x\mathbf{w}) + \nabla\mathrm{p} &= \nu\nabla_x((-A(\bar{\mathbf{u}}) + I)\nabla_x\mathbf{w}) \\ &+\mathbf{w}((-K(\bar{\mathbf{u}}))\nabla_x)\mathbf{w} - (H(\bar{\mathbf{u}}) - I)\nabla_x\bar{\mathrm{p}} \\ \text{div } \mathbf{w} &= \text{div}_{(I-A(\bar{\mathbf{u}}))}\bar{\mathbf{w}} \\ \mathbf{w} &= g, \text{on } \Gamma_{\text{in}} \\ \frac{\partial\mathbf{w}}{\partial\nu} + \mathrm{p}\mathbf{n} &= -\frac{\partial_{A(\bar{\mathbf{u}})-I}\bar{\mathbf{w}}}{\partial\nu}, \text{on } \Gamma_{\text{int}} \end{aligned} \tag{1.23}$$

where $(\bar{\mathbf{u}}, \bar{\mathbf{w}}, \bar{\mathrm{p}})$ are taken from $B_r(X)$ where

$$X \equiv \{(\bar{\mathbf{u}}, \bar{\mathbf{w}}, \bar{\mathrm{p}}) \in W^{2,p}(\Omega_1) \times W^{2,p}(\Omega_2) \times W^{1,p}(\Omega_2)\}$$

$B_r(X)$ denotes a ball in X with a radius equal to $r > 0$.

Step 1. We shall show that the map T takes a ball into a ball for sufficiently small r.

The above choice of X leads to the estimates

$$\begin{aligned}
||A(\bar{\mathbf{u}}) - I||_{L_\infty} \leq ||\bar{\mathbf{u}}||_{1,\infty,\Omega_1} \leq ||\bar{\mathbf{u}}||_{2,p,\Omega_1} \leq C_X r \\
||K(\bar{\mathbf{u}})||_{L_\infty} \leq C(||\bar{\mathbf{u}}||_{2,p,\Omega_1} + 1) \leq C_X \\
||H(\bar{\mathbf{u}}) - I||_{L_\infty} \leq C||\bar{\mathbf{u}}||_{2,p,\Omega_1} \leq C_X r \\
||\nabla_x(A(\bar{\mathbf{u}})\nabla_x \bar{\mathbf{w}}||_{0,p,\Omega_2} \leq ||\bar{\mathbf{w}}||_{1,\infty}||\bar{\mathbf{u}}||_{2,p,\Omega_1} \leq ||\bar{\mathbf{w}}||_{2,p,\Omega_2}||\bar{\mathbf{u}}||_{2,p,\Omega_1} \leq C_X r||\bar{\mathbf{w}}||_{2,p,\Omega_2} \\
||\bar{w}(K(\bar{\mathbf{u}})\nabla_x \bar{\mathbf{w}})||_{0,p,\Omega_2} \leq ||K(\bar{u})||_{L_\infty}||\bar{\mathbf{w}}\nabla_x||_{0,p,\Omega_2} \leq ||K(\bar{\mathbf{u}})||_{L_\infty}||\bar{\mathbf{w}}||_{L_\infty}||\nabla_x \bar{\mathbf{w}}||_{0,p,\Omega_2} \leq \\
||\bar{\mathbf{u}}||_{2,p,\Omega_1}||\bar{\mathbf{w}}||_{1,p,\Omega_2}||\bar{\mathbf{w}}||_{2,p,\Omega_2}
\end{aligned} \tag{1.24}$$

By maximal regularity corresponding to the refernce Stokes problem one obtains the estimate

$$\begin{aligned}
||\mathbf{w}||_{2,p,\Omega_2} + ||\mathrm{p}||_{1,0,\Omega_2} \leq C|g|_{1+1/p,p,\Gamma_{in}} + C_X r||\bar{\mathbf{w}}||_{2,p,\Omega_2} + \\
||\frac{\partial_{A(\bar{\mathbf{u}})-I}\bar{\mathbf{w}}}{\partial \nu}||_{1/p,p,\Gamma_{int}} + C_X r||\bar{\mathrm{p}}||_{1,p,\Omega_2} + ||div_{I-A(\bar{\mathbf{u}})}\bar{\mathbf{w}}||_{1,p,\Omega_2} + \\
||\bar{\mathbf{u}}||_{2,p,\Omega_1}||\bar{\mathbf{w}}||_{1,p,\Omega_2}||\bar{\mathbf{w}}||_{2,p,\Omega_2}
\end{aligned} \tag{1.25}$$

The above estimate along with (1.24) leads to

$$\begin{aligned}
||\mathbf{w}||_{2,p,\Omega_2} + ||\mathrm{p}||_{1,p,\Omega_2} \leq C|g|_{1+\frac{1}{p},p,\Gamma_{in}} + ||\bar{\mathbf{u}}||_{W^{1,\infty}(\Omega)}||\bar{\mathbf{w}}||_{2,p,\Omega} + ||\bar{\mathbf{w}}||_{W^{1,\infty}(\Omega)} \\
||\bar{\mathbf{u}}||_{2,p,\Omega} + ||\bar{\mathbf{u}}||_{2,p,\Omega_1}||\bar{\mathrm{p}}||_{1,p,\Omega_2} + ||\bar{\mathbf{u}}||_{2,p,\Omega_1}||\bar{\mathbf{w}}||_{1,p,\Omega_2}||\bar{\mathbf{w}}||_{2,p,\Omega_2} \\
\leq C|g|_{1+1/p,p,\Gamma_{in}} + C_X r + C_X r^2
\end{aligned} \tag{1.26}$$

The force $\mathbf{t}$ has the estimate

$$|t(\mathbf{u}, \mathrm{p})|_{1-1/p,p,\Gamma} \leq ||\mathrm{p}||_{1,\Omega} + ||\Phi(\mathbf{u})||_{W^{1,\infty}(\Omega)}$$

which gives back

$$||\mathbf{u}||_{2,p\Omega_1} \leq C|g|_{1+1/p,p,\Gamma} + C_X r$$

where C_X and C are generic constants depending only on Ω_1, $\Omega_2(0)$. Taking the boundary data $|g|_{1+1/p,p,\Gamma_{in}}$ sufficiently small (with respect to $1/2r$) one shows that the map T for small r takes B_r into itself.

Step 2. Showing that the map T is contractive. We show that for $\mathfrak{w}$ and $\mathfrak{v}$ we have

$$||T\mathfrak{w} - T\mathfrak{v}||_X \leq \kappa||\mathfrak{w} - \mathfrak{v}||_X$$

Let us denote $\tilde{\mathbf{u}} = \mathbf{u}_1 - \mathbf{u}_2$, $\tilde{\mathbf{w}} = \mathbf{w}_1 - \mathbf{w}_2$, and $\tilde{\mathrm{p}} = \mathrm{p}_1 - \mathrm{p}_2$. Then, according to (1.24) we get:

$$\begin{aligned}
||A(\tilde{\mathbf{u}}) - I||_{L_\infty} \leq ||\tilde{\mathbf{u}}||_{1,\infty,\Omega_1} \leq ||\tilde{\mathbf{u}}||_{2,p,\Omega_1} \leq C_X r, \\
||K(\tilde{\mathbf{u}})||_{L_\infty} \leq C(||\tilde{\mathbf{u}}||_{2,p,\Omega_1} + 1) \leq C_X, \\
||H(\tilde{\mathbf{u}}) - I||_{L_\infty} \leq C||\tilde{\mathbf{u}}||_{2,p,\Omega_1} \leq C_X r, \\
||\nabla_x(A(\mathbf{u}_1)\nabla_x\mathbf{w}_1 - \nabla_x(A(\mathbf{u}_2)\nabla_x\mathbf{w}_2||_{L_\infty} \\
\leq ||\nabla_x(A(\tilde{\mathbf{u}})\nabla_x\mathbf{w}_1||_{0,p,\Omega_2} + ||\nabla_x(A(\mathbf{u})\nabla_x\tilde{\mathbf{w}}||_{0,p,\Omega_2} \\
\leq ||\mathbf{w}_1||_{1,\infty,\Omega_2}||\tilde{\mathbf{u}}||_{2,p,\Omega_1} + ||\tilde{\mathbf{w}}||_{1,\infty,\Omega_2}||\mathbf{u}_2||_{2,p,\Omega_1} \\
\leq C_X r||\mathbf{w}_1||_{2,p,\Omega_2} + C_X r||\tilde{\mathbf{w}}||_{2,p,\Omega_2}, \\
||\mathbf{w}_1(K(\mathbf{u}_1)\nabla_x(\mathbf{w}_1)) - \mathbf{w}_2(K(\mathbf{u}_2)\nabla_x(\mathbf{w}_2))||_{L_\infty} \\
\leq ||\tilde{\mathbf{w}}K(\mathbf{u}_1)\nabla_x\mathbf{w}_1 + \mathbf{w}_2K(\tilde{\mathbf{u}})\nabla_x\mathbf{w}_1 + \mathbf{w}_2K(\mathbf{u}_2)\nabla_x\tilde{\mathbf{w}}|| \\
\leq ||\tilde{\mathbf{w}}||_{L_\infty}||\mathbf{u}_1||_{2,p,\Omega_2}||\mathbf{w}_1||_{1,\infty,\Omega_2} \\
+||\mathbf{w}_2||_{L_\infty}||\tilde{\mathbf{u}}||_{2,p,\Omega_1}||\mathbf{w}_1||_{1,\infty,\Omega_2} \\
+||\mathbf{w}_2||_{L_\infty}||\mathbf{u}_2||_{2,p,\Omega_1}||\tilde{\mathbf{w}}||_{1,\infty,\Omega_2} \\
\leq Cr^2||\mathbf{u}_1||_{2,p,\Omega_2} + Cr^2||\mathbf{w}_1||_{2,p,\Omega_2} + Cr^2||\tilde{\mathbf{w}}||_{2,p,\Omega_2}
\end{aligned} \tag{1.27}$$

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