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Mathematical modeling, planning and analysis of sewage sanitary networks

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Abstract. In the paper the basic questions connected with modeling of communal sewage networks are presented and the formulas of modelling the basic network parameters are analyzed. The problem described concerns gravitational sewage networks divided by nodes into branches and sectors. Hydraulic calculation of sewage networks are commonly carried out by means of nomograms being in form of charts in which relations between network parameters like canal diameters, flow rates, hydraulic slopes and flow velocities are designed. In traditional planning the hydraulic values of sewage networks are simply read from the nomogram chart tables. Another way of networks calculation is use of professional software like SWMM that models the sewage flows in the canals by means of differential liquid equations. In both approaches the user is a mechanical operator of fixed procedures that mostly does not know the meaning of his actions. In the paper the another way of executing hydraulic calculations of sewage networks is presented. The numerical solutions of nonlinear equations describing the physical phenomena of sewage flows are used and explained. The presented algorithms developed for static and dynamic sewage network modelling enable a quick analysis of net parameters and open the possibility of fast, simple and comprehensible network modeling and planning.

1. INTRODUCTION

Modelling and planning of communal sewage networks is a difficult problem because of the complexity of mathematical equations that are to use if one wants to describe exactly the wastewater flows in network canals and because of the variety of networks types. In case of drink water networks which are pressure systems their main parameters are water flows and pressures whose values are dependent on pipe diameters and on the water pressures produced by the pump stations located on the networks. Contrary to this the wastewater networks are gravitational ones whose main hydraulic parameters are sewage flows and filling heights in the canals and the factors deciding about their values are canal diameters, slopes and profiles. The classic approach of planning the sewage systems consists in using the co called nomograms which are diagrams visualizing relations between the canal diameters, slopes and filling heights as well as the sewage flow intensities and velocities. The wanted values of these parameters are picked off from the nomograms which are results of former calculations made with the formulas commonly used for computing the sewage networks (formulas of Chezy, Colebrooke-White and of Manning) [1, 2, 3, 10]. More advanced approach for modelling the communal sewage systems means the use of hydraulic models of wastewater networks like SWMM software developed by EPA (*Environmental Protection Agency*) what requires however some knowledge of and skills in informatics [7]. The classical approach of sewage systems modelling is very mechanical and the second one is very complicated.

In the paper an indirect approach to calculate the hydraulic parameters of wastewater networks is proposed and using it some algorithms for a simple numerical solution of nonlinear equations resulted from the main hydraulic formulas and rules describing the networks are presented. This approach makes possible fast analysis of the main network parameters, i.e. of canal filling heights and sewage flow velocities, and it enables fast and simple simulation of the investigated sewage system. One of the algorithms presented has been used for static simulation and planning of an exemplary wastewater network of sanitary type. Changing the values of intensities of sewage inflows into the network in some chosen network nodes one can simply calculate new values of canal filling heights and flow velocities sewage passing the canals. Other algorithms presented are stated for the case of networks dynamic simulation. The approach proposed enables to understand clearly the relations combining different hydraulic parameters of sewage canals.

2. BASIC ASSUMPTIONS

In general the following kinds of sewage can be distinguished: housekeeping (sanitary) sewage, industrial sewage, rain wastewater, drainage sewage and ground water. The following sewage networks can be marked out depending on the kind of the wastewater transported:

a) rain water network

b) housekeeping network

c) combined network.

In a combined network all kinds of the wastewater are led through the common canals. At the present time the separated sewage systems are mostly used in which the rain water network and the housekeeping one are divided from one another.

In this paper the following basic assumptions are made:

- Only housekeeping or combined sewage nets are considered, divided by nodes into branches and segments.
- The nodes are the points of connection of several network segments or branches or the points of changing the network parameters as well as of location of sewage inflows into the network (sink basins, rain inlets, connecting basins). In the connecting nodes the flow balance equations and the condition of levels consistence are satisfied.
- The nets considered are of gravitational type.

Modelling and planning the sewage networks means the solution of the following tasks:

- Hydraulic calculation of the network for known section crosses and for known canal slopes. Then the calculation of canal filling heights as well as of flow velocities depending on the sewage flow rates must be done. This calculation is done for the respective net segments using the earlier received flow values.
- Designing new segments of the network. It concerns the case when some new segments of the network are to add to the existing ones. In this situation diameters and slopes for new canals must be chosen. It is assumed that the sewage inflows are known.

3. BASIC HYDRAULIC DEPENDENCES IN SEWAGE NETWORK FOR STATIC SIMULATION

It is assumed that all segment parameters such as shape, canal dimension, bottom slope or roughness are constant. Thanks to these assumptions all following relations concern the steady state problem.

According to Manning formula [1] the flow velocity of sewage depends on hydraulic radius R and radius R depends on the filling height H. The Manning formula for velocity v has the form:

$$v = \frac{1}{n} \cdot R^{\frac{2}{3}} \cdot J^{\frac{1}{2}} \qquad (1)$$

where: R – hydraulic radius, J – canal slope, n – roughness coefficient, v – flow velocity.

The relations presented in the following concern the canals with circular section. From Manning formula and taking into account canal geometry the following relations result:

for $H <= 0.5d$:

$$A = \frac{d^2}{8} \cdot (\varphi - \sin\varphi) \qquad (2a)$$

$$\varphi = 2 \cdot \arccos\left(1 - 2 \cdot \frac{H}{d}\right) \qquad (2b)$$

$$R = \frac{1}{4} d \left(1 - \frac{\sin\varphi}{\varphi}\right) \qquad (2c)$$

for $H > 0.5d$:

$$A = \frac{\pi d^2}{4} - \frac{d^2}{8} \cdot (\varphi - \sin\varphi) \qquad (3a)$$

$$\varphi = 2 \cdot \arccos\left(2 \cdot \frac{H}{d} - 1\right) \qquad (3b)$$

$$R = \frac{d}{4} + \frac{d}{8} \cdot \frac{\sin\varphi}{\pi - 0.5\varphi} \qquad (3c)$$

where: A - cross-section area, H – filling height, r – radius of circular canal, φ - central angle, d - canal inside diameter.

From the above expressions one can see that for circular canals the cross-section area A and hydraulic radius R depend on the canal filling height H and as a result the sewage flow velocity v depends on canal filling height H when canal slope J and diameter d are given.

We define in the following the canal filling degree in form H/d. In Fig.1 the relations between A and H/d and between R and H/d for different values d are shown. The figure shows that section area A increases monotonically with growing canal filling H/d. For greater diameter values the increase of section area is faster and its values are greater. The greatest value of A is in case of total canal filling and it equals $\pi d^2/4$. Hydraulic radius R increases from zero and achieves its maximum for the filling ratio of 81,3% and then it decreases to the value equal to half of the canal height. For the total filling and for the half canal filling the value of radius is $d/4$. For greater diameters d also the hydraulic radius grows but the shape of the curves does not depend on d.

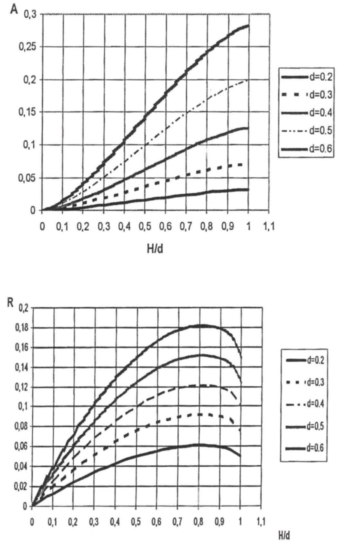


Fig. 1. Dependences between cross-section area A and canal filling degree H/d (top), and between hydraulic radius R and canal filling degree H/d for different diameters d (down).

The sewage velocity depends on the canal parameters like diameter, canal slope and roughness coefficient and on the canal filling degree (see Fig. 2). The sections of the surface from Fig. 2*on-the-top*, received by using planes J=const., are presented in Fig. 2*down.* They show that the function describing velocity v depending on filling degree H/d have the shapes similar to the functions describing hydraulic radius R. The sewage velocity increases from zero and achieves its maximum for the filling degree of 81,3% and then it decreases to the value equal to half of the canal height. Greater diameters *d* increase only velocities *v* but the shape of the curves presented does not depend on *d*.

An exemplary section of the surface from Fig. 2*on-the-top* got by the use of plane *H/d*=const. is shown in Fig. 3*(top)*. It shows that the flow velocity increases monotonically with the growing canal slope for the given filling degree. Fig. 3*(down)* shows the relation between flow velocity *v* and canal filling *H/d* for different slope values *J*. One can see from this Figure that greater values of canal slope increase only velocity values and they do not influence the shape of the curves designed.

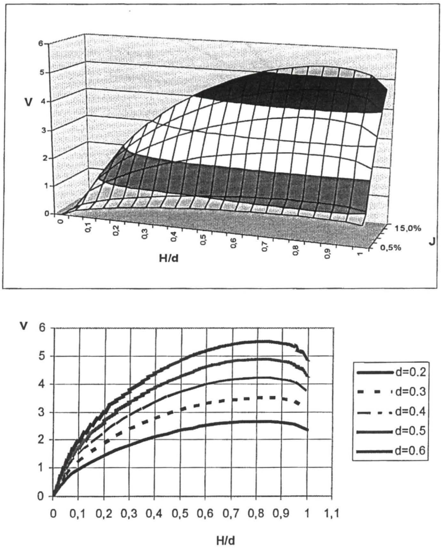
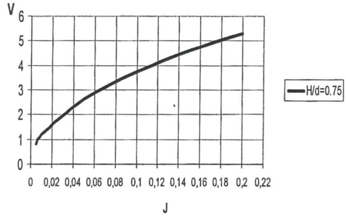


Fig. 2. Dependences between flow velocity v, canal filling degree H/d and canal slope J for roughness coefficient n=0,013 and for canal diameter d=0.6 (on the top), and between v and H/d for n=0,013 and J=5% for different values d (down).



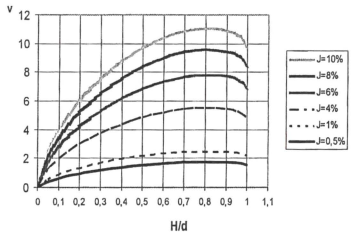


Fig. 3. Dependences between flow velocity v and canal slope J for given canal filling H/d (top), and between v and H/d for roughness coefficient n= 0,013 and for diameter d=0.6 for different J (down).

4. ALGORITHMS FOR THE CALCULATION OF WASTE-WATER NETWORKS IN STEADY STATE

4.1. The algorithm for modelling canal filling heights and flow velocities

The algorithm presented requires the following data for its calculation:

- type of the network– housekeeping sewage net or combined sewage net
- structure of the network – numbers of segments and of nodes and the type of nodes

- maximal sewage inflow into the network and the corresponding input node number
- slopes of canal and the canal dimensions.

The task of the algorithm is to determine the following values for given values of rate inflows Q_i:

- filling heights in each wastewater network segment
- flow velocity for each network segment.

The calculation scheme presented below is for the canals with circular section. The algorithm consists of the following steps (see Fig. 6):

Step 1. Entering the network structure and input data, i.e. number of nodes *NW*, number of segments *N*, set of nodes $W=\{j=1,\ldots, NW\}$, set of segments $U=\{i=1,\ldots, N\}$, set of diameters $\{d_i\}$, set of slopes for segments J_i, $i=1,\ldots, N$, roughness coefficients n_i.

Step 2. Calculating the inflow rates for network input nodes; they are calculated depending on the kind of sewage. For the housekeeping and industrial sewages the maximal hour inflow Q for given network segment can be calculated according to the relation [1, 3, 4, 5]:

$$Q_{h\,max} = \frac{N_{h\,max}\, M \cdot q_{śr}}{24} \tag{4}$$

where: M – number of residents for the given segment of the net, $q_{śr}$ – average wastewater amount for average housekeeping unit, N_{hmax} – rate of irregularity for 24 hours.

For the rain the wastewater inflow can be expressed as follows [1, 3, 5]:

$$Q = q_d \cdot \psi \cdot F \cdot \varphi \tag{5}$$

where: Q – rain wastewater inflow caused by infiltration [dm^3l/s], F – area of drainage basin for the canal segment considered [ha], ψ –ratio between the rain wastewater amount passing into canals and the rain wastewater amount coming from the whole area defined for the modelling, φ – rate of delay between the rain time and the time of infiltration result, q_d – rain intensity.

Step 3. For given rate inflows Q_i in segments i=1,...., N one can determine the following values: filling heights H_i, hydraulic radius values R_i and flow velocities v_i.

1. From the Manning formula and taking into account the canal geometry one can lead out the following relations for:

$$x = \frac{H}{d}$$

For $x \leq 0,5$:

$$\beta \cdot F_1(x) - Q = 0 \tag{6a}$$

$$F_1(x) = \frac{(\varphi_1(x) - \sin(\varphi_1(x)))^{\frac{5}{3}}}{\varphi_1(x)^{\frac{2}{3}}} \tag{6b}$$

$$\varphi_1(x) = 2 \cdot \arccos(1 - 2 \cdot x) \tag{6c}$$

For $x > 0,5$:

$$\beta \cdot F_2(x) - Q = 0 \tag{7a}$$

$$F_2(x) = 2 \cdot \frac{(\pi - 0,5 \cdot \varphi_2(x) + 0,5 \cdot \sin(\varphi_2(x)))^{\frac{5}{3}}}{(\pi - 0,5 \cdot \varphi_2(x))^{\frac{2}{3}}} \tag{7b}$$

$$\varphi_2(x) = 2 \cdot \arccos(2 \cdot x - 1) \tag{7c}$$

$$\beta = 0,5 \cdot \frac{1}{n} \cdot (d)^{\frac{8}{3}} \cdot \left(\frac{1}{4}\right)^{\frac{5}{3}} \cdot J^{\frac{1}{2}} \tag{8}$$

where: H – filling height, φ – central angle, d – inside canal diameter, J – canal slope, n – roughness coefficient, Q – rate inflow, H/d – canal filling degree.

Parameter β in (8) depends on canal diameter d and on canal slope J and for the fixed diameter values and canal slopes it is constant. Solving equations (6a–7b) one can obtain canal filling degree H/d as a function of flow rate Q. In Fig. 4 the relation between parameter β, canal diameter d and canal slope J is shown.

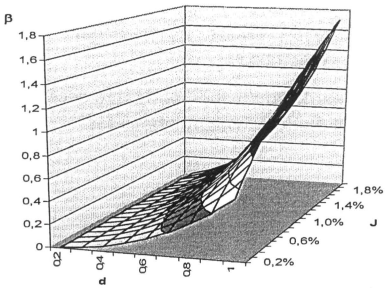


Fig. 4. Relation between parameter β, d and J.

2. For canal filling *H/d* calculated above the hydraulic radius R_i can be determined according to the formula:

For $x <= 0.5$:

$$R=\frac{1}{4}d\left(1-\frac{\sin\varphi}{\varphi}\right) \tag{9a}$$

$$\varphi = 2 \cdot arccos\left(1 - 2 \cdot \frac{H}{d}\right) \tag{9b}$$

For $x > 0.5$:

$$R = \frac{d}{4}\left(\frac{\pi - 0.5\varphi + 0.5\,sin(\varphi)}{\pi - 0.5\varphi}\right) \tag{10a}$$

$$\varphi = 2 \cdot arccos\left(2 \cdot \frac{H}{d} - 1\right) \tag{10b}$$

3. The flow velocity is to calculate from the formula:

$$v = \frac{1}{n} R^{\frac{2}{3}} \cdot J^{\frac{1}{2}} \tag{11}$$

Knowing the network geometry, i.e. slopes, shapes and diameters of the canals as well as the wastewater inflows Q_i, one can calculate filling heights and flow velocities for each network canal. The calculation is carried out for each network segment beginning from the farthest one and going step by step to the nearest segment regarding the wastewater treatment plant.

In Fig. 5 relations between canal filling degree x and canal diameter d for the different flow values Q are designed.

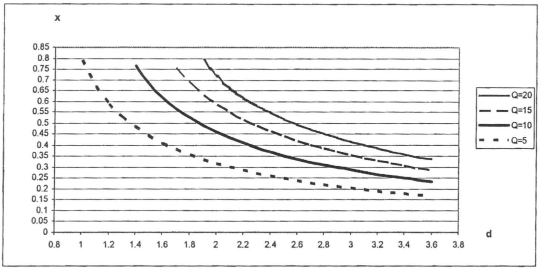


Fig. 5. Relations between canal filling degree x and canal diameter d for the different values of Q.

Step 4. The equations of flow balances $\sum_{j \neq i} Q_j = 0$ and the conditions of surface levels equality are calculated in each network node.

Step 5. The whole network is to calculate one after another with the wastewater inflows changed. Under assumption of constant sewage flows into the network segments the sewage system simulation can be executed for a sequence of time steps, for a couple of hours or days; by such the calculation the change of the wastewater inflows occurring with the time must be considered.

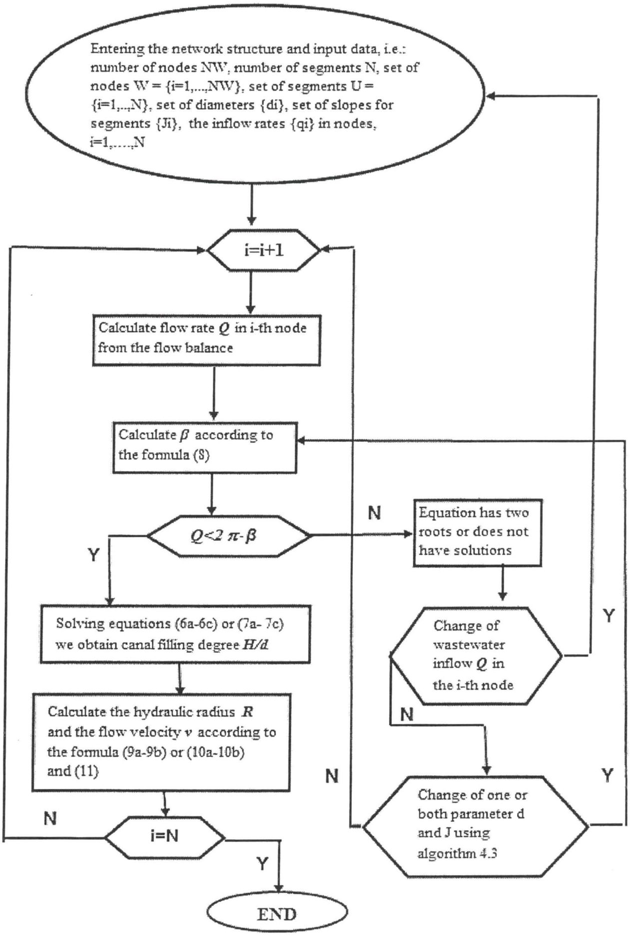
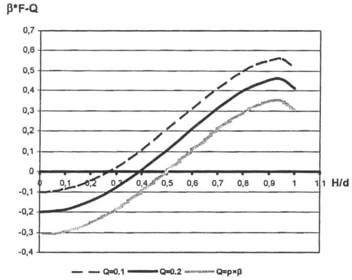


Fig. 6. Scheme of the algorithm for calculating canal filling heights and flow velocity.

4.2 Analysis of equations (6a-6c) and (7a-7c)

Equations (6a–7b) for calculating the canal filling degree are nonlinear and to solve them the standard numerical methods for solving nonlinear algebraic equations can be applied. In order to determine the equation roots some conditions for parameter β and sewage flow Q must be fulfilled that will be discussed in the following.



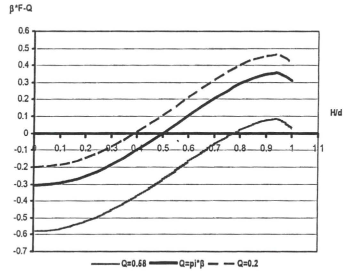


Fig. 7. Diagrams of function β·F(x)–Q for different values of Q in values range (0; π·β> (top) and in values range (0; 2π·β> (down).

Function $F(x)=F_1(x)+F_2(x)$ is continuous in values range (0; 1>. For $x = 1$, i.e. for the full canal filling, there is $F = 2\pi$ and for $x = 0.5$ we get $F = \pi$. In values range (0; 0.8> function $F(x)$ is growing monotone. In values range (0.8; 1> the function reaches its maximum $F_{max} = 6.7588$ for

$x = 0.9381$. It is diminishing in values range (0.9381; 1>. This analysis has been done for $d = 0.6$, $J = 1\%$ and $n = 0.013$. For fixed network parameters like canal diameter d and canal slope J, equation $\beta \cdot F(x) - Q = 0$ has the solutions depending on sewage flow Q (see Fig. 7).

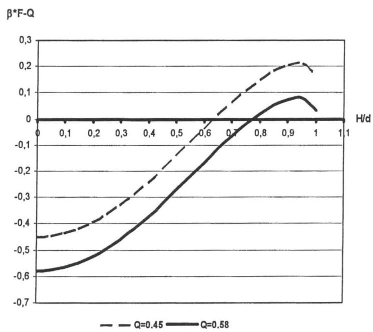
Equation $\beta \cdot F(x) - Q=0$ has the following roots:

1. For $x \in (0; 0.5>$ there is only one root and the following inequality must be fulfilled: $0 < Q \leq \pi \cdot \beta$. This inequality defines a values range for sewage flows Q for fixed canal diameters d and canal slopes J.

2. For $x \in (0.5; 1>$ equation $\beta \cdot F(x) - Q=0$ has the following roots:

- one root for $x \in (0.5; 1)$ and $\pi \cdot \beta < Q < 2\pi \cdot \beta$
- two roots for $x \in (0.5; 1>$ and $2\pi \cdot \beta \leq Q < \beta \cdot 6.7586936$ whereas for $Q = 2\pi \cdot \beta$ there are $x_1 = 1$ and $x_2 = 0.81963$.

The results of the discussion are shown in Fig. 8 in which the case with two roots of equation $\beta \cdot F(x) - Q = 0$ is presented for $Q = 2\pi \cdot \beta$ and $Q = 0.63$ ($Q < \beta$ 6,7588) (Fig. 8 *top).*



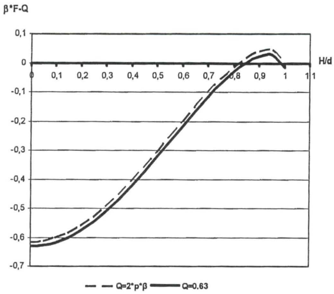


Fig. 8. Diagrams of function β·F(x) – Q for π·β < Q < 2π·β (top) and of function β·F(x) – Q for 2π·β ≤ Q < β*F_{max} (down).

For the fixed network parameters such as a canal diameter *d* and canal slope *J* the above relations let to decide what are the solutions for the given flow *Q* and whether the value of *Q* is not greater than the upper limit *β*·6.7586936, what means the lack of solutions. In such the case a change of one or of both of the fixed network parameters *d* and *J* must be considered.

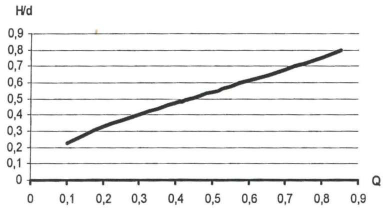


Fig. 9. Relation between the solution of β·F(x)– Q = 0 and flow Q for d = 0.6.

The result of the above relations says that the flow value *Q* depends on parameter *β*. On the another side parameter *β* depends on the canal diameter *d* and on the canal slope *J*. The equation describing the dependence between canal filling and flow in range (0; $2\pi{\cdot}\beta$) has only one solution and that is why this range is highly relevant.

In Fig. 9 the relation between the solution of equation $\beta{\cdot}F(x) - Q = 0$ and flow Q for $d = 0.6$, $J = 2\%$, $n = 0.013$ and $0 < Q < 2\pi{\cdot}\beta$ and in Fig. 10 the same relation for different d and $J = 1\%$ where $J = 1/d$ are designed.

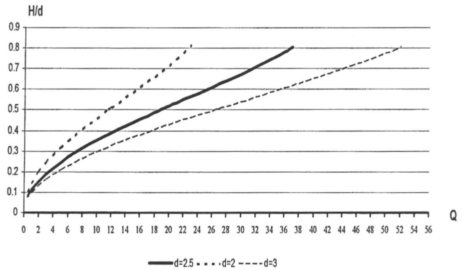


Fig. 10. Relation between the solution of $\beta{\cdot}F(x) - Q = 0$ and flow Q for diferent d.

The result of the above relations says that the flow value Q depends on parameter β. On the another side parameter β depends on the canal diameter d and on the canal slope J. The equation describing the dependence between canal filling and flow in range (0; $2\pi{\cdot}\beta$) has only one solution and that is why this range is highly relevant.

4.3 The algorithm for planning canal diameters for given flow values

The calculation procedure shown below concerns the following cases (see Fig. 12):

- flow Q exceeds the upper boundary of values domain for $\beta{\cdot}6.7586936$; then a change of values for given canal diameters d and slopes J have to be considered.

- new segments must be added to the existing network; then the diameters and slopes must be defined for the new canals under the assumption that sewage inflows Q into the canals have been forecasted and they are known.

In both cases while calculating diameters and slopes for the new canals for given flows Q the inequality $2\pi \cdot \beta - Q > 0$ has to be considered. The fulfilling of the inequality warrants the existence of only 1 solution of the equation describing the relation between canal slope J and canal flow Q.

The calculation procedure consists of the following steps which are realized for the forecasted and fixed flow values Q:

Step 1. Determination of canal slope value J. The value can be determined according to the existing technical standards or calculated regarding the relations for minimal slopes which are known from literature [4, 6, 10, 9]:

$$J = \frac{a}{d} \tag{12a}$$

where a – parameter depending on the art of sewage system, or:

$$J = \frac{\tau_{\min}}{\rho \cdot R} = \frac{4 \cdot \tau_{\min} \cdot (\pi - 0{,}5 \cdot \varphi)}{\rho \cdot \left(\pi - 0{,}5 \cdot \varphi + 0{,}5 \cdot \sin \varphi\right)} \cdot \frac{1}{d} \tag{12b}$$

with $\varphi = 2 \cdot \arccos\left(2 \cdot \frac{H}{d} - 1\right)$, where: J – minimal canal bottom slope ensuring the occurrence of canal self-purification, τ_{min} – tangential tension [kg/m^2], with τ_{min} > 0,225 [kg/m^2] for communal and industrial wastewater, ρ – specific gravity of sewage kg/m^3, R – hydraulic radius.

The canal slope shall be calculated for 60% – 70% of the canal filling height.

The bottom slope is a limit slope and is expressed as

$$J_g = \frac{3{,}778 \cdot 10^{-3}}{d^{1/3}} \tag{12c}$$

Step 2. Solving the following equations:

$$\varsigma \cdot d^{\frac{8}{3}} - Q = 0 \tag{13}$$

$$\varsigma = \frac{\pi}{n} \cdot \left(\frac{1}{4}\right)^{\frac{5}{3}} \cdot J^{\frac{1}{2}}$$

a) For $J = \frac{a}{d}$:

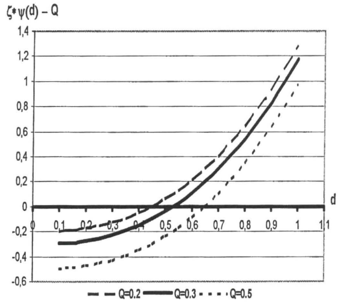
$$\alpha_1 \cdot d^{\frac{13}{6}} - Q = 0 \qquad \alpha_1 = \frac{\pi}{n} \cdot \left(\frac{1}{4}\right)^{\frac{5}{3}} \cdot a^{\frac{1}{2}} \qquad \text{(14a)}$$

b) For J ensuring canal self-purification:

$$\alpha_2 \cdot \mathrm{d}^{\frac{13}{6}} - \mathrm{Q} = 0 \qquad \alpha_2 = \frac{2\pi}{n} \cdot \left(\frac{1}{4}\right)^{\frac{5}{3}} \cdot \left(\frac{\tau_{\min}}{1{,}1106 \cdot \rho}\right)^{\frac{1}{2}} \qquad \text{(14b)}$$

c) For the bottom slope J:

$$\alpha_3 \cdot d^{\frac{5}{2}} - Q = 0 \qquad \alpha_3 = \frac{\pi}{n} \cdot \left(\frac{1}{4}\right)^{\frac{5}{3}} \cdot \left(3{,}778 \cdot 10^{-3}\right)^{\frac{1}{2}} \qquad \text{(14c)}$$



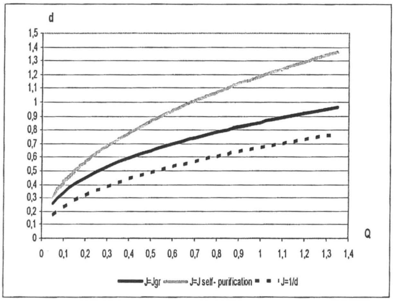


Fig.11. Pictures of function $\xi \cdot d^{\frac{5}{2}} - Q$ for the different values of Q(top) and relations between canal diameter d and canal flow Q for different canal slopes J (down).

From the above equations one can calculate canal diameter *d* for forecasted flow value *Q*. Solving equations (13) for known *Q* we get the minimal bottom diameter d_* and above it the nonequality $\varsigma \cdot d^{\frac{8}{3}} - Q > 0$ is fulfilled. The diameters values less or equal the bottom value are forbidden. For the bottom slope (case 3) the resulted relations are presented in Fig. 11 (*top*).

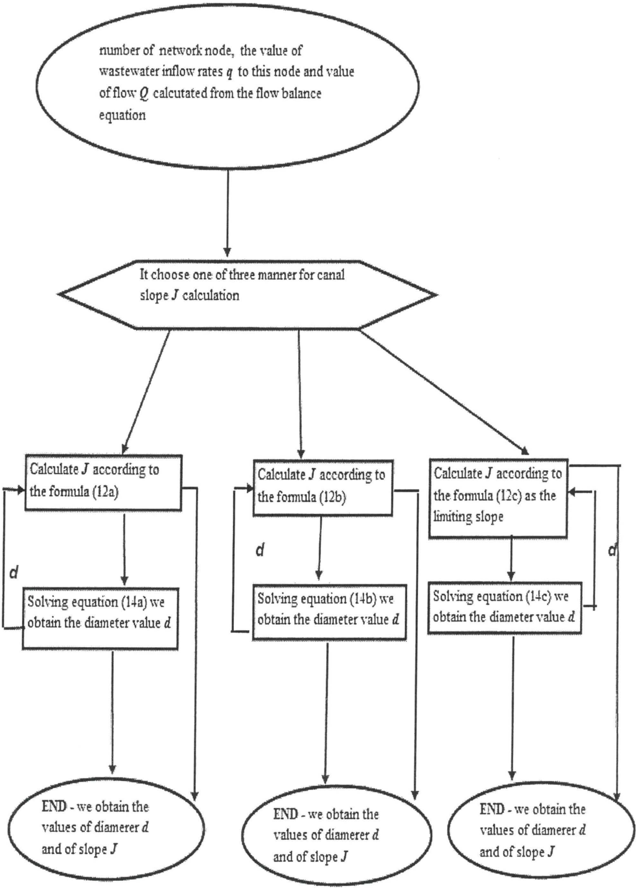


Fig. 12. Scheme of the algorithm for calculating canal diameters for given flow values.

If a solution d_* of the equation (13) exists, then inequality $\varsigma \cdot d^{\frac{8}{3}} - Q > 0$ is valid for all values $d > d_*$. If canal slope J has been calculated from relations (12a-12c) and value d greater than d_* will be taken into account then one shall pass to **Step 1** and the canal slope has to be calculated again. If a solution of equation (13) does not exist then one shall return to **Step 1**, change value J and solve once again equation (13). In Fig. 11*(down)* relations between the solution of equation (13) (concerning canal diameter d) and canal flow Q for different canal slopes J are shown.

4.4. Simulation of an exemplary wastewater network

The algorithms presented for modellinsg and planning the sewage networks have been tested on an exemplary housekeeping network consisting in general of 27 nodes connected by 26 segments (Fig. 13). The net consits of 14 input nodes (W_6, W_7, W_8, W_{10}, W_{11}, W_{14}, W_{15}, W_{16}, W_{19}, W_{20}, W_{21}, W_{23} , W_{25}, W_{26}, W_{27}) and of 1 output node W_1. Other nodes constitute the connections between different segments of the network. The arrows in Fig. 13 show the sewage flow direction.

The sewage flow rates values for the input nodes are given in advance. The flow rates in the connection nodes have been calculated according to the balance equations. For the respective segments diameters $d = 0,2$ and the canal slopes $J = 0,5‰$ have been given. For such the structure of the net the filling hights H/d and flow velocities v in the respective segments have been calculated.

The network investigated was also calculated by means of MOSKAN system worked out in IBS PAN [8]. This system is based on hydraulic model SWMM5 developed by EPA [11].

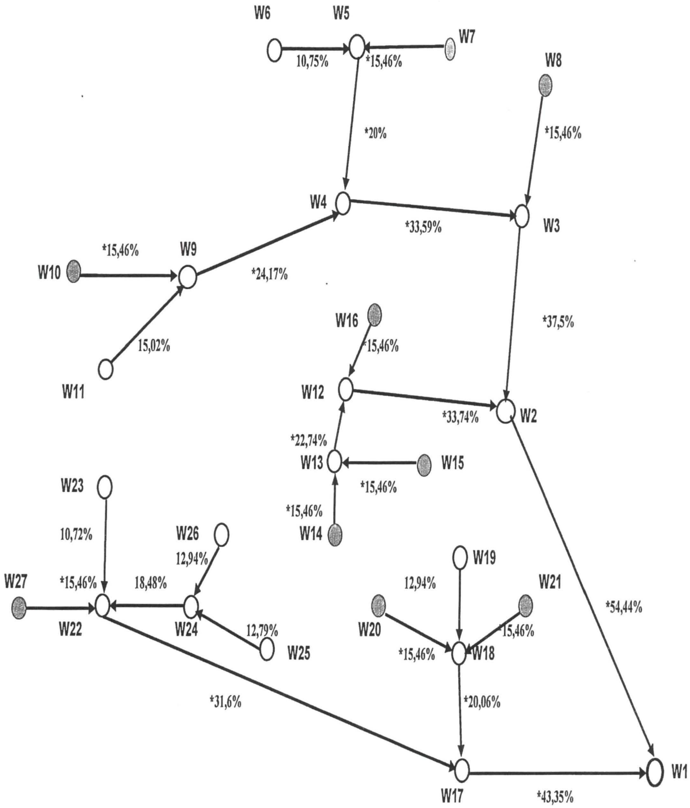


Fig. 13. Structure of the sewage net investigated.

Table 1. Results of modelling computations for the exemplary net shown in Fig. 13.

Upper node	Lower node	Segment	input flows in node	flows in segments Q [dm3/s]	H/d [%]	v [m/s]	H/d [%] MOSKAN	v [m/s] MOSKAN
W6	*W5*	1	0,56	0,56	10,72	0,309	11	0,29
W7	*W5*	*2*	*0,31*	*0,31*	*8,09*	*0,259*	*8*	*0,26*
W5	*W4*	*3*	*0,27*	*1,14*	*15,08*	*0,383*	*15*	*0,38*
W10	*W9*	*4*	*0,36*	*0,36*	*8,69*	*0,271*	*9*	*0,27*
W11	*W9*	5	1,13	1,13	15,02	0,382	14,6	0,39
W9	*W4*	6	0,64	2,13	20,48	0,460	20	0,46
W4	*W3*	7	0,64	3,91	27,78	0,549	28	0,55
W8	*W3*	8	0,11	0,11	4,98	0,189	5	0,19
W3	*W2*	9	0,1	4,12	28,53	0,557	29	0,56
W14	*W13*	10	0,11	0,11	4,98	0,189	5	0,19
W15	*W13*	11	0,32	0,32	8,22	0,261	8	0,26
W13	*W12*	12	0,23	0,66	11,59	0,325	12	0,33
W16	*W12*	13	0,24	0,24	7,17	0,240	7	0,24
W12	*W2*	14	1,86	2,76	23,29	0,497	23	0,49
W2	*W1*	15	0,73	7,61	39,42	0,661	39	0,66
W23	*W22*	16	4,56	4,56	30,06	0,574	30	0,58
W27	*W22*	17	4,4	4,4	29,51	0,568	30	0,57
W25	*W24*	18	4,81	4,81	30,90	0,582	31	0,58
W26	*W24*	19	3,53	3,53	26,37	0,533	26	0,53
W24	*W22*	20	3,69	12,03	51,10	0,745	51	0,75
W22	*W17*	21	1,53	22,52	79,47	0,841	79	0,84
W19	*W18*	22	0,83	0,83	12,94	0,348	13	0,35
W20	*W18*	23	0,3	0,3	7,97	0,256	8	0,26
W21	*W18*	24	0,19	0,19	6,43	0,223	6	0,22
W18	*W17*	25	0,22	1,54	17,46	0,419	17	0,42
W17	*W1*	26	0,57	24,63	89,27	0,832	89	0,83
W1	*Sewage plant*							

The results obtained from the modelling run are presented in Table 1 for both cases of modelling, using the algorithm proposed in 4.1 and using MOSKAN system.

The analysis of the results shows that for two network segments 21 and 26 the canal filling degrees are too large and they exceed the given allowable value of 75% of filling high. The sewage inflows are fixed and then some new canal diameters and new canal slopes have to be calculated what has been done using the planning algorithm proposed in 4.3.

Three cases of calculation of bottom slope *J* have been applied and they are:

a. *J* is the inverse of diameter *d* according to (12a);

b. *J* is the minimal slope securing the self-purification process in the sewage canal according to (12b);

c. *J* is the limit slope according to (12c).

The values of *d* have been obtained by the optional calculation of *J* from equations (14a), (14b) or (14c). For the new values of *d* and *J* the new filling degrees *H/d* and flow velocities *v* can be computed. The results obtained from the planning run are presented Table 2.

Table 2. Results of planning computations for the exemplary net shown in Fig. 13.

			Case *a* of calculation				**Case *b* of calculation**				**Case *c* of calculation**			
Upper node	**Lower node**	**Q**	**d**	**J [‰]**	**H/d [%]**	**v [m/s]**	**d**	**J [‰]**	**H/d [%]**	**v [m/s]**	**d**	**J [‰]**	**H/d [%]**	**v [m/s]**
W22	W17	22,52	0,25	4	55,76	0,771	3,5	6,2	30,25	0,632	3	5,6	38,23	0,632
W17	W1	24,63	0,25	4	59,01	0,787	3,5	6,2	35,43	0,687	3	5,6	40,13	0,687

The analysis of the results in Table 2 shows that for the given flow values the least filling degree is obtained for the bottom slope being the least slope securing the canal self-purification process. For the bottom slope being the inverse of diameter *d* the filling degree *H/d* is the greatest one and exceeds 50%.

The calculation results for modelling and planning runs obtained while using the algorithms proposed and MOSKAN software are very similar and practically comparable. The differences existing are caused by the rounding of numbers used in MOSKAN. It means that the algorithms presented are more reliable and dependable than these classical ones that use the nomograms and not less reliable than the complicated approach that uses SWMM algorithm for sewage networks hydraulic modelling.

5. ALGORITHM FOR THE CALCULATION OF WASTE-WATER NETWORKS IN DYNAMIC STATE

In the above descriptions two algorithms for modelling and planning of sewage networks working in steady state have been presented and in the following an algorithm for dynamic modelling of such the networks is proposed. The goal of the algorithm is to calculate the filling heights of sewage in the network canals and of sewage flow velocities for fixed network structure and slowly changing sewage inflows. The forecasted inflow values are given in advance and the investigation presented concerns the sanitary and mixed gravitational sewage networks.

5.1. Simplified flow models of sewage

The commonly known dynamic flow models of wastewater networks are based on two Saint-Venant equations, i.e. on continuity and dynamic equations [12, 1, 13, 14, 10]. The model presented below concerns the housekeeping, i.e. sanitary, or combined, i.e. sanitary connected with rain sewage networks consisted of segments and nodes. The nodes are the points in which few segments join together or into/from which the wastewater inflows/outflows. The equations of flow continuity hold in the nodes and the conditions of concordane of sewage surface levels hold in the canals which are combining each other. It is assumed that the main hydraulic parameters of the network such as shape, canal dimension, canal slope and roughness are constant at any one time, the sewage inflows are slow-changing in time and the nets investigated are of gravitational type.

The formulation of the wastewater net model proposed in the paper is based on the Saint-Venant continuity equation and on the Manning formula which have the following forms [13]:

a) continuity equation

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}-\zeta=0 \tag{15}$$

b) Manning formula

$$Q=\frac{1}{n}R^{2/3}\cdot J^{1/2}\cdot A \tag{16}$$

where: A – cross sectional area of a canal [m^2], Q – flow rate [m^3/s], ζ – sewage inflow calculated for the canal length unit, $v=\frac{Q}{A}$ – mean velocity of the sewage flow [m/s], J – canal slope [%], R – canal hydraulic radius [m], n – roughness coefficient [$s/m^{1/3}$].

In the following there is assumed that the network is divided by nodes into N segments (canals) and each j-th canal is divided into M_j subsegments with the relative lengths $\Delta x_1, \Delta x_2, \Delta x_3, \ldots, \Delta x_{M\,j}$ as is shown in Fig. 14. In the following relations j means the canal index and i means the subsegment index of j-th canal.

The flow changes in respective subsegments of j-th canal can be written in form of equations (for $j = 1,\ldots,N$):

$$\Delta Q_{1\,j}=Q_{1\,j}-W_j-\zeta_{1\,j}$$

$$\Delta Q_{i\,j}=Q_{i\,j}-Q_{i-1\,j}-\zeta_{i\,j} \qquad i=1,\ldots,M_j \tag{17a}$$

$$\Delta Q_{M_j\,j}=Q_{M_j\,j}-Q_{M_j-1\,j}-\zeta_{i\,j}$$

$$\zeta_{i\,j}(t)=\zeta_j(t)\cdot\Delta x_i$$

where: N – number of segments, M_j – number of subsegments in j-th canal, $Q_{i\,j}$ – flow in i-th subsegment of j-th canal given by the Manning formula, $\zeta_{i\,j}$ – sewage inflow to j-th canal calculated for the length unit of i-th subsegment, W_j – sewage inflow to j-th canal being the sum of the outputs from other canals combined with canal j.

$$W_j = \sum_{k \neq j}^{N} P_{kj} \cdot Q_{M_j j} + \gamma_j \qquad j=1,\ldots,N \qquad (17b)$$

where: P_{kj} – matrix of elements 0 or 1 describing the connections between the network segments, $Q_{M_j j}$ – outflow from j-th segment of the network, γ_j – sewage inflow to j-th network segment.

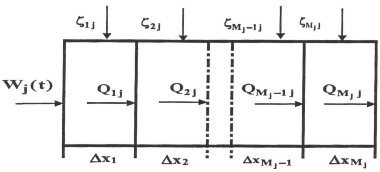


Fig. 14. Division of j-th canal segment into subsegments.

Under assumption that roughness coefficient n and canal slope J are constant along the whole length of the segment considered we can write the formula describing the flow rate:

$$Q_{ij}(t) = \frac{1}{n} R_{ij}(t)^{2/3} \cdot J_j^{1/2} \cdot A_{ij}(t) \qquad (17c)$$

After rearranging equation (15) to the form:

$$\frac{\Delta Q}{\Delta x} + \frac{\Delta A}{\Delta t} = \zeta \qquad (18)$$

and after taking into account equations (17a) one can receive equations set determining the change of cross sectional area ΔA in time Δt:

$$\frac{\Delta A_{ij}(t)}{\Delta t} = \frac{Q_{i-1j}(t) - Q_{ij}(t)}{\Delta x_i} + \zeta_j(t) \qquad (19)$$

The calculated changes are used to determine A_{ij} for the next time step. From (19) for each j-th segment and for $j = 1,\ldots,N$ the equations result:

$$A_{1j}(t+\Delta t) = A_{1j}(t) + \frac{\Delta t}{\Delta x_i} \cdot \left(\sum_{k \neq j}^{M_j} P_{kj} \cdot Q_{M_j j}(t) + \gamma_j(t) - Q_{1j}(t) \right) + \zeta_j(t) \qquad (20a)$$

$$A_{i\,j}(t+\Delta t)=A_{i\,j}(t)+\frac{\Delta t}{\Delta x_i}\cdot\left(Q_{i-1\,j}(t)-Q_{i\,j}(t)\right)+\zeta_j(t) \quad i=1,\ldots,M_j \tag{20b}$$

where: Δt – time step, Δx_i – lenght of i-th canal subsegment.

Solving equations (20a – 20b), beginning from the moment $t = 0$ up to time T with given sewage inflow $\zeta_i(t)$, we will receive the set of values $A_{i\,j}$ (for $j=1,\ldots,N$, $i = 1,\ldots,M_j$, and $t= 0,\ldots,T$) for each Δx_i and each time step Δt, where T is the total simulation time. While solving equations (20a – 20b) the initial conditions for $t = 0$, for cross sectional area A and flow rate Q have to be given.

The flow model is described by relations (17a-17c) and (20a - 20b). From (17a-17c) flows $Q_{i\,j}(t)$ can be computed for each moment t and each j-th network segment ($j = 1,\ldots,N$). Knowing these flow values we can calculate cross sectional area A for next time period $t+\Delta t$ using relations (20a, 20b).

According to Manning formula the sewage flow depends on the hydraulic radius R and on the cross sectional area A whereas R and A depend on the canal filling height H.

To simplify the description of following relations (21a-22c) indexes i and j in them will be omitted but the relations concern each section of the relative network segment.

From Manning formula and taking into account canal geometry one can formulate the following relations:

For $H <= 0.5d$:

$$A = \frac{d^2}{8}\cdot(\varphi - \sin\varphi) \tag{21a}$$

$$\varphi = 2\cdot\arccos\left(1 - 2\cdot\frac{H}{d}\right) \tag{21b}$$

$$R = \frac{1}{4}d\left(1 - \frac{\sin\varphi}{\varphi}\right) \tag{21c}$$

For $H > 0.5d$:

$$A = \frac{\pi d^2}{4} - \frac{d^2}{8}\cdot(\varphi - \sin\varphi) \tag{22a}$$

$$\varphi = 2 \cdot arccos\left(2 \cdot \frac{H}{d} - 1\right) \tag{22b}$$

$$R = \frac{d}{4} + \frac{d}{8} \cdot \frac{\sin \varphi}{\pi - 0.5\varphi} \tag{22c}$$

where: H – canal filling height, φ – canal central angle, d – canal inside diameter.

From the above relations one can see that for circular canals cross sectional area A and hydraulic radius R depend on canal filling height H and these relations can be described as $A=F_1(H)$ and $R=F_2(H)$. In this way, while knowing cross sectional area A, one can determine filling height H and hydraulic radius R.

In Fig. 15 the relations between A and canal filling degree H/d for different diameter values d are shown.

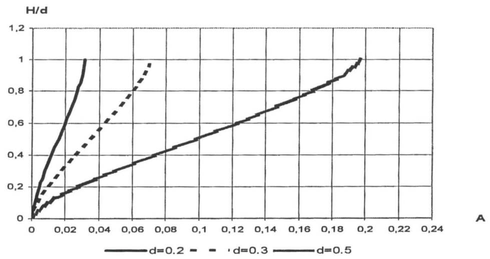


Fig. 15. Relations between canal filling degree H/d and cross sectional area A for different values d.

5.2. Algorithm for the first version of the networks model

The algorithm proposed for calculating the sewage flow dynamic model consists for given time t of the following steps (see Fig.16):

Step 1. Determining cross sectional area $A_{ij}(t)$ by solving equations (20a–20b) at moment t for each j-th network segment and i-th canal subsegment $i=1,\ldots,M_j$.

Step 2. Calculating height $H_{ij}(t)$ by solving equation $F(H_{ij}) - A_{ij} = 0$ for two following options (for j-th network segment, $j = 1,\ldots,N$, and for all canal subsegments $i = 1,\ldots,M_j$):

For $H_{ij} \le 0.5 d_j$:

$$F(H_{ij}) = \frac{d_j^2}{8} \cdot \left(2\arccos\left(1 - 2\frac{H_{ij}}{d_j}\right) - \sin\left(2\arccos\left(1 - 2\frac{H_{ij}}{d_j}\right)\right)\right) \qquad (23a)$$

For $H_{ij} > 0.5 d_j$:

$$F(H_{ij}) = \frac{\pi d_j^2}{4} - \frac{d_j^2}{8} \cdot \left(2\arccos\left(2\frac{H_{ij}}{d_j} - 1\right) - \sin\left(2\arccos\left(2\frac{H_{ij}}{d_j} - 1\right)\right)\right) \qquad (23b)$$

Step 3. Calculating hydraulic radius $R_{ij}(t)$ from relations (21c) or (22c).

Step 4. Determining flow $Q_{ij}(t)$ from Manning formula (17c).

Function $F(H)$ is continuous and for $H = 0.5d$ it has the value $\frac{\pi d^2}{8}$. Equation $F(H_{ij}) - A_{ij} = 0$ for calculating the canal filling degree is nonlinear and the standard numerical methods for solving nonlinear algebraic equations can be here applied. The algorithm presented is rather complicated and to calculate it the solution of additional equation $F(H) - A = 0$ must be solved. Function $F(\cdot)$ has the same form as functions (23a–23b). The calculations are to do sequentially for each segment of the network, beginning from the furthest segment and ending by the segment closest to the wastewater treatment plant.

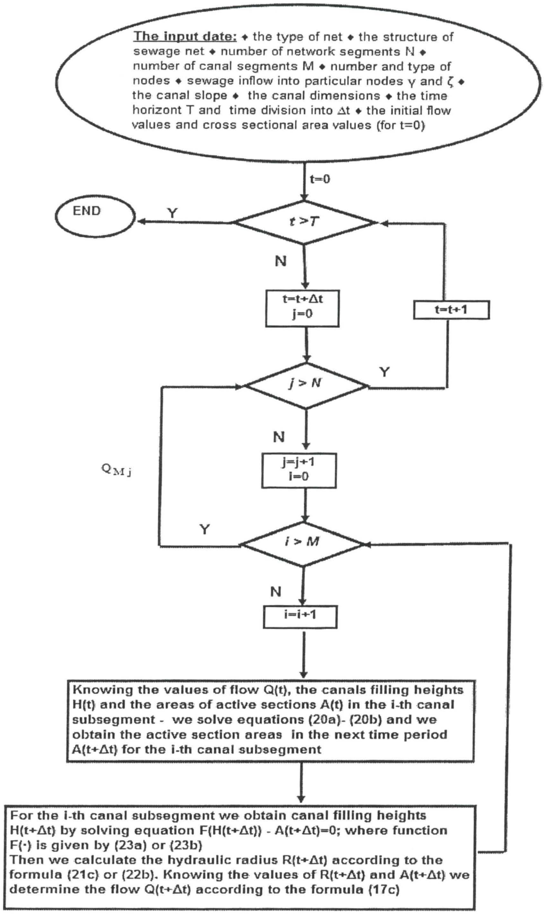


Fig. 16. Schema of the algorithm for sewage network dynamic modelling.

5.3. Second model for dynamic modelling the sewage networks

The algorithm for the first version of model is rather complicated and to calculate it the solution of the additional equation $F(H)$-A=0 must be solved. Function $F(\cdot)$ has the same form as functions (23a)–(23b). The calculations are done in sequence for each segment of network, beginning from the furthest segment and completing for the segment closest to wastewater treatment. The scheme of the algorithm is shown in Fig. 16.

The second version of the network model takes into account the calculation of canal filling height H. For canals with the circular cross-sections the relations between active cross sectional area A and filling height H can be used. As function $F(\cdot)$ jest continues and differentiable then we can write down:

$$\frac{\partial A}{\partial t} = \frac{\partial F}{\partial H} \cdot \frac{\partial H}{\partial t} \qquad (24a)$$

where $F(\cdot)$ is given by (23a) or (23b).

After a transformation of (10a) we obtain:

$$\frac{\partial F}{\partial H} = \frac{d}{4} \cdot \frac{1-\cos\varphi}{\sqrt{\frac{H}{d} - \left(\frac{H}{d}\right)^2}} \qquad (24b)$$

where φ has the form (21b) or (22b).

From equation (18) and from relation (24b) transformed to the difference form we receive:

$$\frac{\Delta Q_{ij}(t)}{\Delta x_i} + \frac{d_j}{4} \cdot \frac{1-\cos(\varphi)}{\sqrt{\frac{H_{ij}(t)}{d_j} - \left(\frac{H_{ij}(t)}{d_j}\right)^2}} \cdot \frac{\Delta H_{ij}}{\Delta t} = 0 \qquad (25a)$$

$$\frac{\Delta H_{ij}}{\Delta t} = \frac{H_{ij}(t+\Delta t) - H_{ij}(t)}{\Delta t} \qquad (25b)$$

To transform the above relation let us determine the change of the filling height H_{ij} during time Δt:

$$H_{1j}(t+\Delta t)=H_{1j}(t)+4\frac{\sqrt{\frac{H_{1j}(t)}{d_j}-\left(\frac{H_{1j}(t)}{d_j}\right)^2}}{d_j\cdot\left(1-\cos(\varphi_{1j}(t)\right)}\cdot\left(\sum_{k\neq j}^{M_j} P_{kj}\cdot Q_{M_j j}(t)+\gamma_j(t)-Q_{1j}(t)+\zeta_{ij}(t)\right)\cdot\frac{\Delta t}{\Delta x_i}$$

(26a)

$$H_{ij}(t+\Delta t)=H_{ij}(t)+4\frac{\sqrt{\frac{H_{ij}(t)}{d_j}-\left(\frac{H_{ij}(t)}{d_j}\right)^2}}{d_j\cdot\left(1-\cos(\varphi_{ij}(t)\right)}\cdot\left(Q_{i-1j}(t)-Q_{ij}(t)+\zeta_{ij}(t)\right)\cdot\frac{\Delta t}{\Delta x_i}$$

(26b)

where: $Q_{ij}(t)$ – flow rate in i-th canal subsegment calculated from Manning formula (17c) for the *j*-th network segment, d_j –internal diameter of *j*-th network segment, φ_i – canal central angle given by formula (21b) or (22b), Δx_i – length of *i*-th canal segment, Δt – time step.

The second version of the network model is described by relations (17c), (21b) or (22b) and (26a)– (26b). In this model flow $Q_{ij}(t)$ is calculated for moment *t* and for each *j*-th network segment (from *j*=1 to *j=N-1*) in all canal segments *i*=1,……,M_j. Then knowing the flows and filling heights H_{ij} for time period *t* we can calculate the filling heights for next period $t+\Delta t$ according to (26a) - (26b).

5.4. Calculation of sewage inflows into combined wastewater networks

The housekeeping as well as industrial or rain sewages are flowing into the combined wastewater network. Depending on the kind of sewage its inflow rate is calculated in different way. For the housekeeping and industrial sewage its inflow towards a given canal is considered as maximal hourly flow γ_{dj} defined by the relation:

$$\gamma_{dj}=\frac{N_{hmax}M\cdot q_{mv}}{24} \qquad (27)$$

where: M – number of inhabitants assigned to the given network canal, q_{mv} – mean value of the sewage outflow from a house unit, N_{hmax} –

coefficient of daily unevenness regarding the housekeeping sewage production.

An exact description of methods calculating the inflows of rain sewage into combined wastewater networks can be found in numerous literature [5], [10], [20], [21], [22].

The inflow of rain sewage into the network canals can be calculated by determining the functions describing the rainfalls and the drainage basin on which the wastewater network is located. For calculating the rainfall sewage the following formula can be used:

$$\gamma_d(t) = q_d \cdot \psi \cdot F \cdot \tau \tag{28}$$

where: γ_d – drift of the rain sewage from the terrain on which the wastewater network is located, [dm^{3}l/s], F – surface of drainage basin from which the sewage is drifting towards the given canal section, [ha], Ψ – coefficient of surface drift being the quotient between the rain sewage amount reaching the net canal and the total rainfall amount that dropped at the regarded soil part, τ – delay coefficient, q_d – rainfall intensity in [dm^3/s ha], being the rainfall amount in dm^3 that dropped at the soil surface of 1 ha in the time of 1 s.

Not all amounts of rain water runs off of the drainage basin towards the net canals and the process occurs gradually with regard to the phenomenon of local retention. It depends on the form of drainage basin, on canals situation, on field slope etc. The amount of water that does not reach the canals but will seep into the ground or will steam away can be estimated by means of the runoff coefficient Ψ calculated from the Reinhold formula:

$$\psi = M \cdot q^{0.567} \cdot t_d^{0.228} \tag{29}$$

where: q – rainfall intensity [dm^3/h], t_d – rainfall duration [min], M – factor characterizing the drainage basin and climatic conditions.

For the given network structure the surface of the drainage basin must be determined regarding the shape and configuration of the ground. In this way the real directions of water runoff towards the canals can be defined. Then the values of Ψ for individual field parts can be calculated. These values depend on the density of buildings situated at the land and on the kind of land covering.

The intensity of significant rainfall is calculated on the base of long term meteorological observations. This rainfall is described by the following parameters:

- duration t, [min],
- rainfall height h_o, [mm],
- intensity $I = h_o/t$, [mm/min],
- range F, [ha],
- probability of appearance p, [%] or
- incidence $c = 100/p$, [years].

There are several relations combining the rainfall intensity, rainfall duration and probability of rainfall appearance. One of the most used relation for calculating the runoff of significant rainfall is the following Błaszczyk formula:

$$q = \frac{6{,}63\sqrt[3]{h_o^{\,2} c}}{t_d^{0{,}67}} \tag{30}$$

where: h_o – mean value of yearly rainfall, [mm], q – rainfall intensity [dm^3/h], c – rainfall incidence [years], t_d – rainfall duration [min], p – appearance probability (%), p=100/c.

Duration of significant rainfall t_d can be calculated from the formula:

$$t_d = 1{,}2 \cdot \sum t_p + t_k = \frac{1}{50} \sum_{i=1}^{N} \frac{L_i}{v_i} + t_k \tag{31a}$$

There is also an another formula for calculating the rainfall duration t_w in which the network and drainage basin retentions are taken into consideration:

$$t_w = \frac{\alpha}{60} \sum_{i=1}^{N} \frac{A_i L_i + V_i F_i}{q_{pi}} \tag{31b}$$

where: N – number of network segment, t_p – time of sewage flow through individual canal segments beginning from the upper network node to the point in which the calculation is currently doing, [min], L_i – length of i-th canal segment, [m], v_i – mean value of flow velocity in i-th canal segment, t_k – time of soil concentration, A_i – surface of lateral canal section, F_i – surface of drainage basin part belonging to the i-th canal segment, V_i – factor of canal volume and of land retention regarding the i-th canal segment, q_{pi} – assumed flow of the rain sewage in i-th segment of the network, α – factor of used capacity of the network retention.

The total sewage flow in a canal is calculated as the sum of housekeeping wastewater, industrial sewage and rainfall water. From this united sewage flow the wastewater outlet taking place in overflow points which are situated above the canal segment investigated has to be subtracted.

Another way of determining the intensity of significant rainfall is the method of constant intensities in which the rainfall duration $t_d = 10$ min and the rainfall incidence $c = 2$ are defined.

Delay coefficient τ depends on the surface of drainage basin, on its shape and slope and it can be calculated from the following Burklieg-Ziegler formula:

$$\tau = \frac{1}{\sqrt[m]{F}}$$

Coefficient τ can take values from 2 up to 8 and these values are bigger for larger drainage basins with bigger slopes.

The appearance of soil retention can be considered also while doing the calculation by using the function $f(t)$ depending on time (Fig. 17). In this picture the following function parameters are specified:

- t_r – duration of soil retention,
- t_d – rainfall duration,
- t_k – total time of the runoff of rainfall water towards the canal investigated.

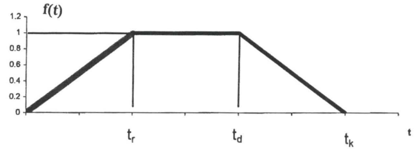


Fig. 17. Function f(t) of wastewater inflow to the sewage network canals.

For estimated function $f(t)$ the wastewater inflow to the sewage net canal $\gamma_d(t)$ can be calculated with the following formula:

$$\gamma_d(t) = q_d \cdot \psi \cdot F \cdot f(t) \tag{32}$$

with F – surface of the drainage basin.

Inflow γ_d calculated by means of (32) can be considered by the modelling of a sewage network as the point-wise inlet introduced into the network nodes, but the better approach from the computational point of view, is to regard it as the sectional inlet assigned to the canal unit.

5.5. Algorithm for the second model of wastewater network simulation

Taking into account the forecasted values of sewage inflow, the velocities v_i and the canal fillings H_i for each interval Δt and for each j-th network segment can be calculated. Using the calculated canal outflow $Q_{M_j j}$ as the additional inflow to the next canal segment we can simulate with this method any part of the net investigated.

The algorithm shown in Fig. 18 is based on the second version of the network model given by equations (26a) – (26b) and (17c) describing the change of filling height H during time Δt and on the Manning formula for determining the flow Q. In this model the heights of segment fillings are determined for individual time periods. To build the model we have to define the following network parameters:

- type of the net – housekeeping or combined sewage network;
- structure of the net – number of network segments N, number of canal segments M, types of canals, number and type of nodes;
- network segment parameters, i.e. canal dimensions, slopes, lengths and roughness coefficients;
- initial data for computing, i.e. initial flows and initial filling height;
- date describing the simulation process, i.e. simulation time, time steps, network division into segments.
- sewage inflows into particular nodes

The inflows of rainfall water to the canals can be given directly according to the functions $\zeta_i(t)$ determined as a result of soil investigations or indirectly using some approximating functions [3].

The task of the algorithm is to determine the values of the following parameters for each net segment and for fixed time period:

- the filling height;
- the flow velocity;
- the flow rates.

It is assumed that the hydraulic parameters of segments, namely canal shape, canal dimension and roughness are constant. The sewage inflows occur in the network nodes.

In the following the main elements of the algorithm will be desribed:

Step 1. The net structure defined by: number of nodes NW, number of segments N, M_j – number of subsegments in the j-th canal, the set of nodes $W = \{k\}$, the set of segments $\{j\}$, j=1,......,N, the set of subsegments in the j-th canal $\{i\}$, i=1,......,M_j , the set of diameters $\{d_i\}$, slopes for the segments J_i, j=1,......,N, roughness n_i for each segment, the initial flow values Q_{ij} and canals filling height values H_{ij} (for t=0) for each segment j=1,......,N and each canal subsegnent

i=1,……,M_j, the time horizont T and time division into Δt, ζ_{ij} – sewage inflow to the j-th canal calculated for the length unit of the i-th subsegment, γ_j – sewage inflow to the j-th network segment should be entered into the algorithm.

Step 2. The sewage inflow $\zeta_{ij}(t)$ the j-th canal calculated for the length unit of the i-th subsegment, γ_j - sewage inflow for individual net nodes are calculated for the given time period $t_k = t_{k-1} + \Delta t$.

Depending on the kind of the network (housekeeping or combined sewage net) the rate of inflow for each segment is calculated using the relations given in point 2.

Step 3. For cosidered time t_k the cross sectional areas A_{ij} and hydraulic radiuses R_{ij} for the known filling heights $H_{ij}(t_k)$ (calculated according to (26a)-(26b)) are determined (for the particular j-th net segments j=1,……,N and for each i-th canal subsegment i=1,……,M_j) as follows:

for $H_{ij} \leq 0.5 d_j$: j=1,……,N i=1,……,M_j

$$A_{ij}(t_k) = \frac{d_j^2}{8} \cdot \left(\varphi_{ij}(t_k) - \sin\left(\varphi_{ij}(t_k)\right)\right) \tag{33a}$$

$$R_{ij}(t_k) = \frac{d_j \cdot \left(\varphi_{ij}(t_k) - \sin(\varphi_{ij}(t_k))\right)}{4 \cdot \varphi_{ij}(t_k)} \tag{33b}$$

$$\varphi_{ij}(t_k) = 2 \cdot \arccos\left(1 - 2 \cdot \frac{H_{ij}(t_k)}{d_j}\right) \tag{33c}$$

for $H_{ij} > 0.5 d_j$: j=1,……,N i=1,……,M_j

$$A_{ij}(t_k) = \frac{\pi d_j^2}{4} - \frac{d_j^2}{8} \cdot \left(\varphi_{ij}(t_k) - \sin\left(\varphi_{ij}(t_k)\right)\right) \tag{34a}$$

$$R_{ij}(t_k) = \frac{d_j}{4} + \frac{d_j}{8} \cdot \frac{\sin\left(\varphi_{ij}(t_k)\right)}{\pi - 0.5 \cdot \varphi_{ij}(t_k)} \tag{34b}$$

$$\varphi_{ij}(t_k) = 2 \cdot \arccos\left(2 \cdot \frac{H_{ij}(t_k)}{d_j} - 1\right) \tag{34c}$$

where: d_j – diameter of j-th network segment.

Step 4. Knowing hydraulic radiuses $R_{ij}(t_k)$ and active areas of canal segments $A_{ij}(t_k)$ we can calculate (for individual segments j=1,……,N and for the i-th canal subsegment where i=1,……,M_j):

a) flow rates Q_i:

$$Q_{ij}(t_k) = \frac{1}{n_j}\left(R_{ij}(t_k)\right)^{2/3} \cdot J_j^{1/2} \cdot A_{ij}(t_k) \tag{35}$$

b) flow velocities v_i:

$$v_{ij}(t_k) = \frac{1}{nj}(R_{ij}(t_k))^{2/3} \cdot J_j^{\frac{1}{2}} \tag{36}$$

where: n_j – roughness coefficient of j-th segment, J_j –canal slope of j-th segment of the network.

The calculations are done sequentially for each segment of the network, beginning from the furthest segment and completing the computing for the segment which is closest to the wastewater treatment plant. Each segment is divided subsequently into M_j subsegments for which the calculations shown are repeated.

Step 5. In each followed node the relation is calculated:

$$W_j(t_k) = \sum_{k \neq j}^{N} P_{kj} \cdot Q_{M_j j}(t_k) + \gamma_j(t_k) \tag{37}$$

where: P_{kj} - matrix consisting of 0 and 1 elements describing the connections between the network segmenst, $Q_{M_j j}$ - outflow from the j-th segment of the network, γ_j - sewage inflow to the j-th network segment, W_j - sewage inflow to the j-th canal being the sum of the outflowws form other canals connected with the j-th canal.

Step 6. Knowing the values of flow rates $Q_{ij}(t_k)$ in all segments of the net we can determine the canal filling heights for the next time period $t_{k+1} = t_k + \Delta t$ as:

$$H_{1j}(t_k+\Delta t)=H_{1j}(t_k)+4\frac{\sqrt{\frac{H_{1j}(t_k)}{d_j}-\left(\frac{H_{1j}(t_k)}{d_j}\right)^2}}{d_j\cdot\left(1-\cos(\varphi_{1j}(t_k))\right)}\cdot\left(W_j(t_k)-Q_{1j}(t_k)+\zeta_{ij}(t_k)\right)\cdot\frac{\Delta t}{\Delta x_i} \tag{38a}$$

$$H_{ij}(t_k+\Delta t)=H_{ij}(t_k)+4\frac{\sqrt{\frac{H_{ij}(t_k)}{d_j}-\left(\frac{H_{ij}(t_k)}{d_j}\right)^2}}{d_j\cdot\left(1-\cos(\varphi_{ij}(t_k))\right)}\cdot\left(Q_{i-1j}(t_k)-Q_{ij}(t_k)+\zeta_{ij}(t_k)\right)\cdot\frac{\Delta t}{\Delta x_i} \tag{38b}$$

After calculating the canal filling heights in all segments of the net for the time period $t_{k+1} = t_k + \Delta t$ we calculate active areas of segments A, hydraulic radiuses R, flow rates Q and velocities v_i for the whole network and for the whole time od simulation. Using the simplified flow models the sewage networks of any kind can be calculated for dynamical case.

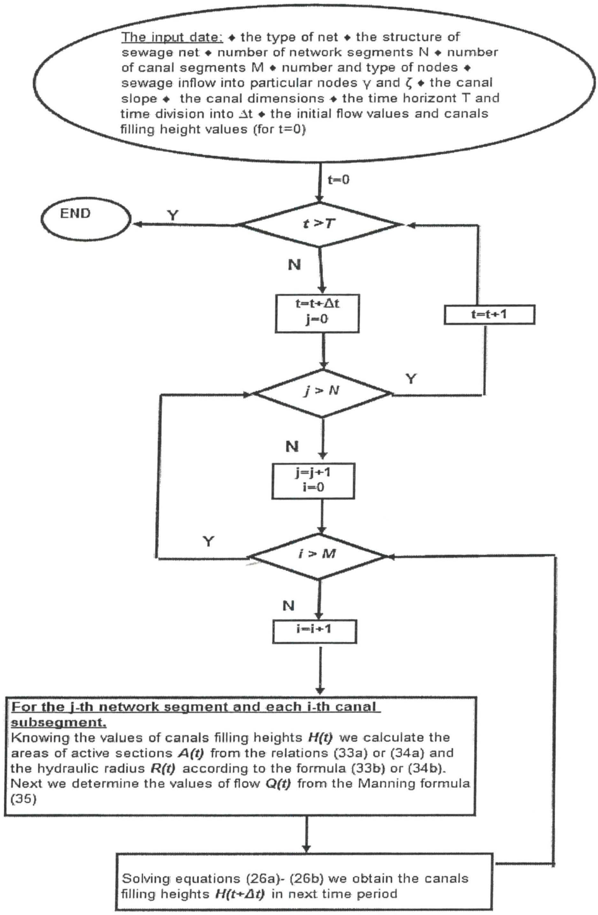


Fig.18. Schema of the computing algorithm for the second version of the network model.

6. CONCLUSIONS

In the paper a new practical approach for computing sewage networks is proposed. It differs from the approaches commonly used in the today's practice of sewage nets operation. The standard and mostly applied method of sewage network calculation consists in using nomograms which enable to calculate in a pure mechanical way the basic parameters of the designed nets such as diameters and canal slopes on the base of estimated sewage inflow values. The nomogram schemes have to be drawn before the process of network designing is started and their drawing occurs on the base of appropriate hydraulic equations and relations. The results received depend strongly on the quality of the schemes used.

The modern approach in this field consists of applying advanced computer programs like SWMM [7, 15] developed by EPA [11] or MIKE URBAN developed by DHI [16]. They use in their computations hydraulic models of sewage networks. This approach requires an advanced computer knowledge from the program users and although the programs mentioned are already commonly used by university scientists then there is lack of their applications in waterworks. The most important obstacle in using this software in operational practice in waterworks is the necessity of having a calibrated hydraulic model of the network investigated. To calibrate the model a GIS system to generate the numerical map of the network and a properly dense monitoring system to collect the measurements data have to be installed on the sewage net what generates expensive costs [17]. The most of Polish waterworks are municipal enterprises and they commonly have not enough money for buying such the costly systems.

It seems that the approach for modelling sewage networks presented in the paper being an indirect method between the standard and modern ones can be currently an ideal tool for computing the networks: it keeps all advantages of the both approaches and it has not got their drawbacks. It uses the analytical relations concerning the hydraulics and geometry of sewage networks and it transforms them to nonlinear equations from which – depending on the requirements – demanded canal fillings and sewage speeds or canal diameters and slopes can be directly calculated. The analysis of the equations performed enables to determine available maximal sewage inflows ingoing to the network nodes. The calculation can be done quickly and exactly avoiding the using of the complicated network hydraulic model.

The computational example presented for steady state modelling and planning of sewage networks is rather simple but the algorithms proposed can be either used unproblematic for modeling and designing more complex municipal sewage systems.

In the paper also an algorithm for dynamic modelling of communal sewage networks is proposed although it does not have been used until now for network calculation; this task waits for realization what will be done in the next future.

For the algorithms presented an essential problem is to determine models of sewage inflows into individual network segments. In the case of communal and industrial sewage these inflows are relatively simple to define knowing the data of the water consumption regarding the end users of the water network connected with the sewage one. These inflows can be modelled as the curves with constant values for subsequent time sections.

A problem arises by modelling the rain fall water flowing into the sewage canals. The rain water inflows can be defined directly by means of some wastewater functions resulted from special field investigation or indirectly by means of the functions describing the rainfall and the referred drainage basin. In the second case several parameters describing the soil like surface and shape of the terrain, field decrease, buildings density on the drainage basin, soil covering etc. must be defined what complicates essentially the problem of inflow modelling [17].

In the paper two algorithms for dynamic modelling and planning of communal sewage networks are proposed. In the first algorithm the network investigated is described with the relations (17a - 17c) and (20a-20b) from which the lateral area A for each canal segment j and for the simulation time t can be determined. Subsequently an additional equation in form $F(H)$-A=0 is formulated with function $F(H)$ defined by (23a) or (23b). From this equation canal filling height H and canal hydraulic radius R can be calculated and using them the sewage flow Q can be determined. This approach seems to be simple but the necessity to formulate and solve the equation *F*(*H*)-*A*=0 complicates the process of the modelling.

In the second algorithm the canal filling height *H* for given simulation times *t* are determined directly by solving the difference equation (38a-38b). Afterwards the sewage flows *Q* can be calculated from (35). The network modelled can be calculated step by step for all network segments taking subsequently the outflows from some canal segments as the inflows to other ones.

For both algorithms the essential problem is to determine the models of sewage inflows to the individual network segments. In the case of communal and industrial sewage these inflows are relatively simple to define knowing the data of the water consumption regarding the end users of the water network connected with the sewage one. These inflows can be modelled as the curves with constant values for subsequent time sections. A problem arises by modelling the rain fall water flowing into the sewage canals. The rain fall sewage inflows can be defined directly by means of some wastewater functions resulted from special field investigation or indirectly by means of the functions describing the rainfall and the referred drainage basin. In the second case several parameters describing the soil like surface and shape of the terrain, field decrease, buildings density on the drainage basin, soil covering etc. must be defined what complicates essentially the problem of inflow modelling.

The algorithms for modelling and planning the sewage networks presented in the paper are in our opinion an indirect approach between the standrad method using the nomogramms and the more sophisticated method using the hydraulic models of the networks like SWMM developed by EPA (*US Environment Protection Agency*). The modelling with the nomogramms is very simple but not very exact and its application is pure mechanical without any need to understand the process of modelling. On the other side the modelling with hydraulic models is very exact and also very difficult because of the need to determine many network and terrain parameters. In the case of our algorithms the exactness is better than by the nomogramms and the complications are lower than by the hydraulic models.

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