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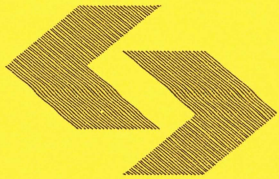

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OPTIMUM STRATEGIES IN A FISCAL-MONETARY GAME. SIMULATION ANALYSIS

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The paper presents the results from the simulation analysis of the *policy-mix*, carried out for the fiscal-monetary game, in which the fiscal and the monetary authorities take decisions on the choice of the optimum strategy from the point of view of realization of their respective economic objectives. In order to represent the interrelations between, on the one hand, the instruments of fiscal policy and of the monetary policy, and, on the other hand – the economic effects resulting from their application, a modified logistic function was used. The study method adopted allows for consideration of the specificity of the effects from these instruments on business cycle, consisting in the limited effectiveness of applying the extreme restrictive and extreme expansive policies, and the respective impact on the economy. The simulation study was meant to show the influence exerted both by the parameters of the function and the priorities of the fiscal and monetary authorities on the Nash equilibrium state, identified with the choice of a definite combination of the budgetary and monetary policies.

1. Introduction

The present paper concerns the problem of choice of policy-mix in the context of game theory and mutual decision conditioning between the fiscal authorities (the government) and the monetary authorities (the central bank). Policy-mix constitutes, in this perspective, a combination of the fiscal and monetary policies, each of them being characterized by the definite degrees of restrictiveness and expansiveness.

The subject of analysis is the game, whose essence is shown in Table 1, presenting the payoffs. The strategies of fiscal authorities are those of the budgetary policy – from the extremely restrictive in the first row to the extremely expansive in the last one. The measure of the degree of restrictiveness / expansiveness of the fiscal policy is constituted here by the level of budgetary deficit in relation to the GDP. Analogously, the strategies of the monetary authorities range from the extremely restrictive one in the first column to the extremely expansive one in the last column, the degree of restrictiveness / expansiveness being equivalent simply to the value of the real interest rate. Payoffs are denoted in the following manner: y_{ij} – payoff for the fiscal authorities (GDP growth rate) in case the government applies the fiscal strategy F_i and the central bank applies the monetary strategy m_j; p_{ij} – payoff of the monetary authorities (inflation) for the same pair of policies. Symbol b_i denotes the budgetary deficit in relation to GDP, corresponding to the i-th fiscal strategy, while r_j denotes the real interest rate, ascribed to the j-th monetary strategy.

It is assumed that the fiscal and monetary authorities take decisions independently, and the Nash equilibrium state in such a game is identified with the choice of a definite combination of the budgetary and monetary policies.

The analysis here presented is a continuation of the investigations by the same author (Woroniecka-Leciejewicz, 2006-2012), concerning the issues of selecting the policy-mix with the use of a game between the government and the central bank. In the earlier papers (Woroniecka-Leciejewicz, 2008-2011) the formulas defining the nonlinear interdependences between the values, corresponding to economic growth and inflation, and the policy-mix instruments, were derived on the basis of the Taylor series expansion, and this was the foundation for the analysis of the game equilibrium and the Pareto optimality of the solutions. In view of the wish of complementing the theoretical considerations, contained in the papers mentioned, with a simulation analysis, allowing for the analysis of various variants, in particular – in the context of effectiveness of the fiscal and monetary policies in their influence on the state of economy – in the last publication (Woroniecka-Leciejewicz, 2012) a modified logistic function was applied in the fiscal-monetary game, and the results from the initial calculations were presented.

Table 1. The fiscal-monetary game – the payoff table

		Central bank – the monetary policy ← restrictive … expansive →			
	Payoffs table	Monetary strategy M_1 (interest rate r_1)	Monetary strategy M_2 (interest rate r_2)	…	Monetary strategy M_n (interest rate r_n)
Government – fiscal policy ← expansive restrictive →	Fiscal strategy F_1 (budgetary deficit b_1)	p_{11} / y_{11}	p_{12} / y_{12}	… / …	p_{1n} / y_{1n}
	Fiscal strategy F_2 (budgetary deficit b_2)	p_{21} / y_{21}	p_{22} / y_{22}	… / …	p_{2n} / y_{2n}
	…			…	
	Fiscal strategy F_m (budgetary deficit b_m)	p_{m1} / y_{m1}	p_{m2} / y_{m2}	… / …	p_{mn} / y_{mn}

Source: Woroniecka-Leciejewicz (2010a), 191

Application of the modified logistic function makes it possible to account for the specific features of the influence exerted by the instruments of fiscal and monetary policies on the state of economy (including the influence on GDP and inflation). The specificity, accounted for, consists in the fact that these instruments are effective only within a definite interval of values of the instrument, their effectiveness falling distinctly outside of this interval. This means, in particular, that the possibilities of lowering inflation by applying increasingly restrictive monetary policy are

limited, similarly as the possibility of stimulating economic growth with increasingly expansive fiscal policy is limited.

The study, based on the use of the logistic function, was continued in the direction of consideration – on the one hand – of the influence exerted by the parameters, characterizing the effectiveness of the policy-mix instruments, and – on the other hand – the impact from the macroeconomic objectives, assumed by the fiscal and monetary authorities, on the outcome from the game. The study conducted resulted in the present article.

2. Assumptions concerning the influence of the policy-mix instruments on the state of economy

The game is analysed under a set of assumptions, concerning the influence, exerted by the instruments of fiscal and monetary policies, on the state of economy, as characterized by the GDP dynamics and inflation. Thus, it is assumed that:

1. the increase of the interest rate, *ceteris paribus*, brings about the decrease of the rate of economic growth ($\frac{\partial y}{\partial r} < 0$) and limits inflation ($\frac{\partial p}{\partial r} < 0$),
2. increase of budgetary deficit, *ceteris paribus*, contributes to the increase of inflation ($\frac{\partial p}{\partial b} > 0$).

A problem is constituted by the adoption of the assumption concerning the impact of the budgetary deficit on real production growth in the economy. Two variants can be considered:

A – increase of the state budget deficit, *ceteris paribus*, causes an increase in the GDP growth rate ($\frac{\partial y}{\partial b} > 0$),

B – increase of the state budget deficit, *ceteris paribus*, limits the growth of the GDP ($\frac{\partial y}{\partial b} < 0$).

The study here presented concentrates on the analysis of the fiscal-monetary game under variant A, which appears to reflect in a more realistic manner the influence of the fiscal policy on the possibilities of economic growth over a short time horizon.

It is, additionally, assumed that $\Delta b_i = b_i - b_{i-1} > 0$, $\Delta r_j = r_j - r_{j-1} < 0$. This assumption constitutes a reflection of the scheme of construction of the payoff table, related to the adopted sequence of the fiscal and monetary strategies – starting with the restrictive ones and passing over to the expansive ones, with the increase of the degree of expansiveness of the fiscal policy being accompanied by the increasing

budgetary deficit, and the increase of expansiveness of the monetary policy – by the decreasing interest rate.

For purposes of reflecting the interdependence between the economic growth and inflation on the one hand and the instruments of the macroeconomic policy on the other a modified logistic function was used, which enables accounting for the specific character of the impact from the fiscal and monetary policy instruments, consisting, in particular, in the fact that the effectiveness of these instruments is high within a certain interval of values, and drops outside of this interval. Application of the logistic function for reflecting the action of the policy-mix instruments on the state of economy in the fiscal-monetary game with consideration of the properties of the function, important in this context, and the interpretation of the parameter values, was presented in the last paper by the same author, Woroniecka-Leciejewicz (2012).

The logistic function has the form $y = f(x) = \frac{\alpha}{1+\beta e^{-\chi x}}$, with the parameter $\beta>0$ reflecting the monotone passage of the function value from the initial $f(x_0)=0$ to the attainment of the ultimate value of $f(x_k)=\alpha$ (for $\chi>0$), or vice versa – from the value $f(x_0)=\alpha$ to the attainment of the ultimate value of zero (for $\chi<0$). Depending upon the values of parameters α and χ, the logistic function shows the increase of the explained variable (for both parameters being either positive or negative) or its decrease (when the two parameters have opposite signs). By complementing the logistic function with the constant δ, i.e. $y = f(x) = \frac{\alpha}{1+\beta e^{-\chi x}} + \delta$, we obtain the possibility of reflecting the passage of the function value from an arbitrary initial value to an arbitrary ultimate value, by adopting appropriate values of α and δ. Simulations were performed for $\chi > 0$.

The influence of the fiscal instrument, that is - of the budgetary deficit, related to the GDP (b) - on the rate of economic growth (y), is reflected by the increasing logistic function $y = f(b) = \frac{\alpha_1}{1+\beta_1 e^{-\chi_1 b}} + \delta_1$, with $\beta_1>0$, $\chi_1>0$, $\alpha_1>0$. This means that the increase of the budgetary deficit, *ceteris paribus*, contributes to stimulation of economic growth ($\frac{\partial y}{\partial b} > 0$), but the effectiveness of the action of budgetary policy on economic growth is limited to a definite interval of values of the instrument. It is assumed that both the possibility of stimulating growth by applying increasingly expansive fiscal policy (increasing the deficit) and the possibility of exerting negative influence on production dynamics by the increasingly restrictive policy of deficit reduction are limited. Under the extremely restrictive fiscal policy the rate of growth of the GDP is the lowest and equal δ_1 ($y_{\min} = \delta_1$), and further increase of the restrictiveness of the policy does not entail any change in production dynamics. Analogously, under the extremely expansive policy the highest rate of GDP growth

is equal $\delta_1 + \alpha_1$ ($y_{max} = \delta_1 + \alpha_1$), and further increase in expansiveness is no longer effective at all.

The influence of the interest rate (r) on the economic growth (y) is described by the decreasing logistic function $y = f(r) = \frac{\alpha_2}{1+\beta_2 e^{-\chi_2 r}} + \delta_2$, with $\beta_2>0$, $\chi_2>0$, $\alpha_2<0$. As the interest rate increases, the dynamics of the GDP drops ($\frac{\partial y}{\partial r} < 0$) from the initial maximum value ($y_{max}= \delta_3$) for the extremely expansive monetary policy and the accompanying extremely low value of the interest rate, down to the lowest value ($y_{min} = \delta_2 + \alpha_2$), when the interest rate attains the upper extreme. The effectiveness of the monetary policy in its influence on the GDP dynamics decreases along with the passage towards the extreme policies – expansive or restrictive. Thus, for instance, the possibility of stimulating economic growth through application of increasingly expansive monetary policy is limited, the incremental effects dropping distinctly.

Analogously, the impact of the fiscal instrument on inflation (p) is described by the increasing logistic function $p = f(b) = \frac{\alpha_3}{1+\beta_3 e^{-\chi_3 b}} + \delta_3$, with $\beta_3>0$, $\chi_3>0$, $\alpha_3>0$. It is, therefore, assumed that the increase of the budgetary deficit of the state, accompanying the expansive fiscal policy, *ceteris paribus*, increases the inflationary tensions in the economy and the level of inflation ($\frac{\partial p}{\partial b} > 0$), while the decrease of deficit brings about the constraining of inflation, but the effectiveness of the budgetary policy decreases with the passage towards the extremes, either expansive of restrictive. For the extremely restrictive fiscal policy inflation is extremely low ($p_{min} = \delta_3$) and further increase of restrictiveness of this policy does not contribute to a decrease of inflation. Similarly, limited effectiveness of the fiscal policy and its instruments in the impact on inflation can be observed for the extremely expansive policy, when inflation is the highest ($p_{max} = \delta_3 + \alpha_3$) and further increase of expansiveness does not cause any further increase of inflation rates.

The influence of the interest rate (r) on inflation (p) is described by the decreasing logistic function, $p = f(r) = \frac{\alpha_4}{1+\beta_4 e^{-\chi_4 r}} + \delta_4$, with $\beta_4>0$, $\chi_4>0$, $\alpha_4<0$. Along with the increase of the interest rate, ceteris paribus, inflation level is constrained ($\frac{\partial p}{\partial r} < 0$), dropping from the initial extremely high ($p_{max} = \delta_4$), under the radically expansive monetary policy, to the extremely low ($p_{min} = \delta_4 + \alpha_4$), under the radically restrictive policy. In this case, as well, application of the logistic function is associated with the limited scope of effectiveness of the monetary policy – for in-

stance, the possibility of pressing down inflation by applying increasingly restrictive monetary policy (raising the interest rate to extremely high values) is limited.

3. Nash equilibrium and the priorities of the fiscal and monetary policies. Simulation results

As indicated before, calculation of payoffs in the fiscal-monetary game was performed with the use of the modified logistic function. Payoffs of both the fiscal authorities (GDP growth rate, y) and the monetary authorities (inflation, p) depend upon the policy-mix instruments: budgetary deficit in relation to the GDP (b_i) and the real interest rate (r_j):

$$y_{ij} = f(b_i, r_j) = \frac{\alpha_1}{1+\beta_1 e^{-\chi_1 b_i}} + \frac{\alpha_2}{1+\beta_2 e^{-\chi_2 r_j}} + \delta_1 , \quad (1)$$

$$p_{ij} = f(b_i, r_j) = \frac{\alpha_3}{1+\beta_3 e^{-\chi_3 b_i}} + \frac{\alpha_4}{1+\beta_4 e^{-\chi_4 r_j}} + \delta_2 . \quad (2)$$

The results of calculations for selected values of parameters (variant 1) are shown in Table 2. For each combination of the fiscal and monetary policies the rate of GDP growth (lower left corner of each cell) and the level of inflation (upper right corner) are provided. The lowest inflation level, accompanied, though, by the lowest economic growth, occurs for the selection of the combination of the extremely restrictive monetary and fiscal policies (upper left corner of the payoff table). Along with the increase of the degree of expansiveness of the monetary policy, *ceteris paribus*, and the lowering of the interest rate (passage to the right) inflation and production dynamics increase. The highest inflation and the highest GDP growth characterize the economy in conditions of the extremely expansive monetary and budgetary policies (lower right corner of the table).

The fiscal and the monetary authorities are driven by their own objectives when choosing the optimum strategies. Two cases are considered in the study: the first one, in which it is assumed that the fiscal authorities aim at maximum rate of GDP growth, and the monetary authorities aim at minimization of inflation, and the second one, in which it is assumed that the monetary and fiscal authorities define concrete goals, meaning a desired inflation level and some planned GDP dynamics.

In the first of these cases, for each monetary strategy j the fiscal authorities select the optimum fiscal strategy $i^*(j)$, which maximizes the rate of GDP growth: $\max_i y_{ij}$. Analogously, monetary authorities choose for each fiscal strategy i the optimum monetary strategy $j^*(i)$., which corresponds to the minimum inflation level. In such a situation, the fiscal authorities dispose of a dominating strategy, that is – the strategy, which is the optimum strategy from the point of view of the government irrespective of the decisions, taken by the central bank with respect to the interest rate. The dominating strategy of the fiscal authorities is in this case the most

expansive budgetary policy. And, in complete analogy, the radically restrictive monetary policy constitutes for the monetary authorities the dominating strategy, that is – the one, which is optimal no matter which fiscal strategy is chosen by the government. Game equilibrium is then determined by the dominating strategies, motivating to the choice of the combination of the extremely restrictive monetary policy and the extremely expansive fiscal policy (as shown in Fig. 1).

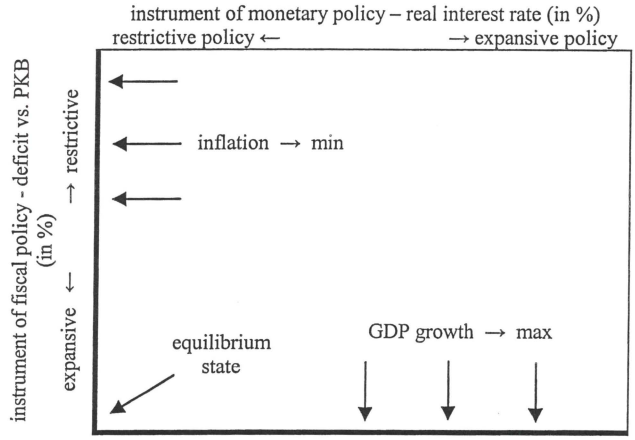

Fig. 1. Dominating fiscal and monetary strategies – the case of maximisation of the real GDP growth and minimisation of inflation rate

In the second case considered it is assumed that the fiscal and monetary authorities aim at minimisation of divergence of, respectively, the GDP growth and inflation, from the desired values. We assume, like before, that for each monetary strategy j fiscal authorities choose the optimum fiscal strategy $i^*(j)$, so as to minimise the square divergence of the GDP growth rate from the desired value: $\min_i (y_{ij} - y^*)^2$. Analogously, monetary authorities select for every fiscal strategy i the optimum monetary strategy $j^*(i)$, which minimises the square divergence between actual inflation and the desired one (the inflation objective), i.e. $\min_j (p_{ij} - p^*)^2$. Thus, the optimum budgetary strategies characterise the reaction of the fiscal authorities to the potential moves of the central bank, and, vice versa, the optimum monetary strategies describe the reactions of the monetary authorities to various fiscal strategies.

For the case, when the fiscal and monetary authorities tend to minimise the squared divergences of, respectively, the real economic growth and the inflation level from the desired values, calculations were performed for alternating assumptions. Location of the equilibrium point is no longer obvious and depends, in par-

ticular, upon the effectiveness of the action of the fiscal and monetary policy instruments, reflected through the values of parameters of the modified logistic function and upon the priorities of the government and the central bank in the policies conducted. In further course of the paper the results from the respective analyses are presented.

Table 2. The payoff table. Optimum fiscal and monetary strategies. Nash equilibrium

	3.0	2.4	2.0	1.8	1.0	0.0
-1.0	-1.13 -0.15	-0.74 0.03	-0.34 0.21	-0.09 0.32	1.30 0.96	3.84 2.11
0.0	0.02 1.24	0.41 1.42	0.81 1.60	1.06 1.71	2.46 2.34	5.00 3.50
1.0	1.18 2.62	1.57 2.80	1.97 2.98	2.22 3.10	3.61 *3.73*	6.16 4.89
1.4	1.53 3.05	1.93 3.23	2.32 3.41	2.57 3.52	3.97 4.16	6.51 5.31
1.6	1.68 3.23	2.07 3.41	2.47 3.59	2.72 3.70	4.12 4.34	6.66 5.49
1.8	1.81 3.39	2.21 3.56	2.60 3.74	2.85 3.86	4.25 4.49	6.79 5.65
2.0	1.93 3.52	2.32 3.70	2.72 3.88	2.96 3.99	4.36 4.63	6.90 5.78
2.4	2.11 3.74	2.50 3.92	2.90 4.10	3.14 4.21	4.54 4.85	7.08 6.00
3.0	2.28 3.95	2.68 4.13	3.07 4.31	3.32 4.42	4.72 5.06	7.26 6.22
6.0	2.51 4.22	2.90 4.40	3.30 4.58	3.55 4.69	4.95 5.33	7.49 6.49

Parameters (variant 1): α_1=6; β_1=1; χ_1=1; α_2=-5; β_2=1; χ_2=1; δ_1=3; α_3=5; β_3=1; χ_3=1; α_4=-11; β_4=1; χ_4=1; δ_2=8. Desired GDP growth = 3.5%, inflationary goal=2.5%.

Notations: optimum strategies: 3.59 – fiscal, 2.47 – monetary, Nash equilibrium – between the solutions inside thick frame boxes

Table 2 shows the optimum fiscal strategies, chosen by the government for each possible monetary strategy, based on minimisation of the squared divergence of the GDP dynamics from the desired values (the planned rate of growth is assumed at the level of roughly 3.5%). Likewise, the optimum monetary strategies are shown, constituting the reaction to various potential fiscal strategies (based on minimisation of the squared divergence between actual inflation and the inflationary goal, assumed at the level of roughly 2.5%). It can be noted that the degree of restrictiveness of the monetary policy depends upon the choice of the fiscal policy by the government. The more expansive the fiscal policy, the more restrictive the monetary policy, applied in response by the central bank, so as to avoid excessive inflation, exceeding the assumed inflationary goal. For higher values of the budgetary deficit the desired value of inflation is attained for the appropriately higher interest rates. Analogously, when the government conducts more restrictive budgetary policy, the central bank, aiming at the attainment of the inflationary goal, may settle on the less restrictive (more expansive) monetary policy, with appropriately lower interest rates.

Likewise, the degree of restrictiveness or expansiveness of the fiscal policy depends upon the monetary policy, applied by the central bank. The more restrictive the monetary policy – the more expansive, in response, the budgetary policy, since attainment of the desired rate of economic growth under higher interest rates requires more pronounced, pro-growth, expansive fiscal policy, characterised by higher budgetary deficit. And conversely, in response to a more expansive monetary policy, the government limits the degree of budgetary expansiveness, conducting an appropriately more restrictive fiscal policy.

Figure 2 presents, in analogy to Table 2, the optimum fiscal and monetary strategies, and the Nash equilibrium state for the same economic goals: GDP dynamics at the level of 3.5% and inflation at the level of 2.5%, with admission of the very small, close to zero, change of the fiscal policy instrument (Δb_i) and of the monetary policy instrument (Δr_j). Purposefully, a broader scope of change in the values of instruments is shown here compared to the consecutive figures, shown in the paper. Owing to this, the specificity of the action of these instruments on the state of economy, including GDP growth and inflation, is better seen. Inside a certain interval of values of the fiscal and monetary policy instruments, which can be called effective, the action of the instruments on the economy is tangible, while outside of this interval, that is – for the extreme values of the instruments, corresponding to radically restrictive or expansive orientation of the policies – the effectiveness distinctly drops off. It can be observed that within the effective range of values of the policy-mix instruments the choice of the optimum fiscal policy depends upon the decision of the monetary authorities – the more restrictive the monetary policy – the more expansive, in response, the fiscal policy, and vice versa. Yet, beyond this range of values, when we tend, for instance, towards the radically restrictive monetary policy, the optimum fiscal strategy does not change any more under the impact of further sharpening of the monetary policy by the central bank. Similarly, if we move towards the extremely expansive monetary policy, the optimum fiscal policy no longer reacts to the further alleviation of the monetary policy. Summing up, in the regions of extremely restrictive or extremely expansive interest rate strategies the optimum fiscal strategy turns into the dominating strategy.

Analogously, for a certain (effective) interval of values of the fiscal policy instruments, the choice of the optimum monetary policy depends upon the budgetary policy selected by the fiscal authorities – the more expansive the fiscal policy, the more restrictive monetary policy is applied, in response, by the central bank (and vice versa). Outside of the interval, further sharpening of the budgetary policy, or its further increase of expansiveness, makes the optimum monetary strategy turn into the dominating strategy.

For the assumed parameter values and the priorities of the fiscal and monetary authorities the Nash equilibrium state corresponds to the combination of the relatively neutral fiscal and monetary policies, which are characterised by the deficit of the state budget in relation to the GDP at the level of approximately 1.42% and the real interest rate at the level of roughly 1.87%.

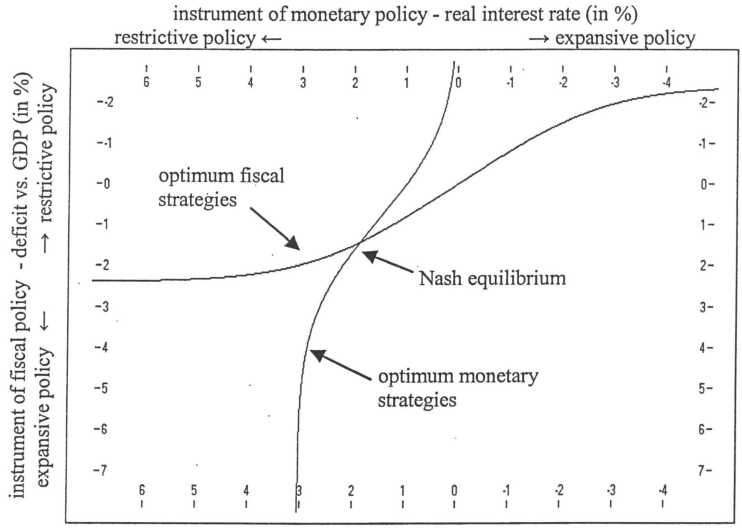

Fig. 2. Optimum fiscal and monetary strategies (desired GDP growth = 3.5%, inflationary goal = 2.5%). Results of simulations for variant 1, broader scope of changes in instrument values

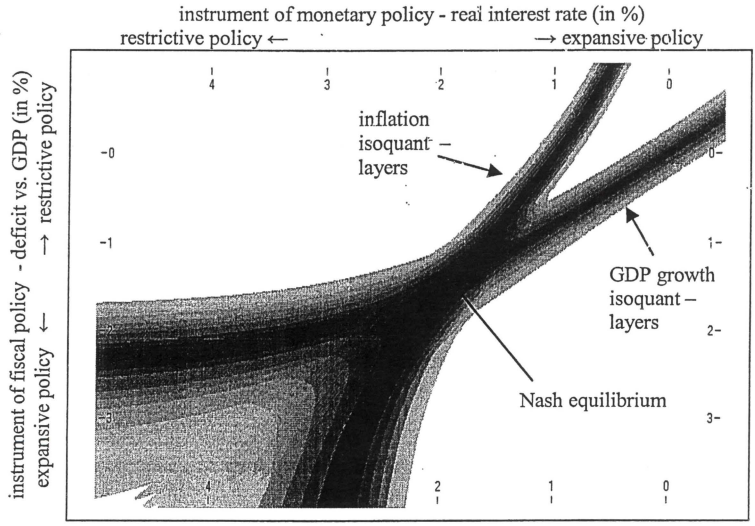

Fig. 3. Isoquants – a layered diagram with admissible deviations from the assumed goals for GDP growth (=3.5%) and for inflation (=2.5%). Results of simulations for variant 1

Figure 3 shows the isoquants of the GDP growth and the isoquants of inflation, demonstrating what alternative policy-mixes (being the combinations of the fiscal and monetary policies, featuring a definite degree of restrictiveness / expansiveness) allow for the attainment of the assumed GDP dynamics and the assumed level of inflation (with variable range of the admissible changes with respect to the desired values). It can be noticed that solutions, contained in the narrow range of admissible deviations from the desired values for, simultaneously, economic growth and inflation, are also contained in a small neighbourhood of the Nash equilibrium. If we admit a broader range of deviations from the assumed macroeconomic objectives, the region of the "sufficiently good" solutions (that is – the ones "sufficiently" close to the desired values, within the admissible deviation from both GDP dynamics and inflation goals) broadens, as well. One can see a distinct asymmetry in the shape of the admissible solutions region – this area extends in a more pronounced manner towards the combinations of the more expansive fiscal policies and the more restrictive ones, compared to those for the Nash equilibrium state.

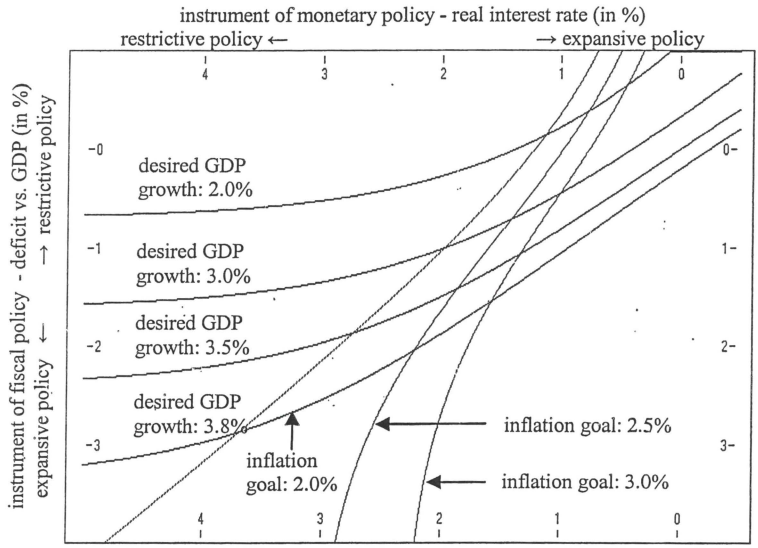

Fig. 4. Optimum fiscal and monetary strategies vs. priorities of the government and the central bank, along with the corresponding Nash equilibrium states. Simulation results – variant 1

The subsequent simulations show the influence of changes in the priorities of the fiscal and monetary policies on the location of the Nash equilibrium, and hence also on the choice of the corresponding policy-mix. The respective results are shown in Figs. 4 and 5. A change in the priorities in the framework of fiscal policy, re-

flected through a change in the desired rate of GDP growth, brings about a shift in the optimum budgetary strategy either in the direction of a more expansive fiscal policy, when more ambitious goals concerning economic growth are adopted, or in the direction of a more restrictive budgetary policy in the opposite case. Likewise, a change in the priorities of the monetary authorities, which is reflected in the shift of the desired level of inflation, entails a shift in the optimum monetary strategies – in the direction of a more expansive monetary policy in the case of less demanding priorities of the central bank, i.e. higher inflationary goal, or in the direction of a more restrictive monetary policy, when the desired inflation level is lowered. Under the impact of changes in the priorities of fiscal and monetary authorities the location of Nash equilibrium accordingly moves (Fig. 4).

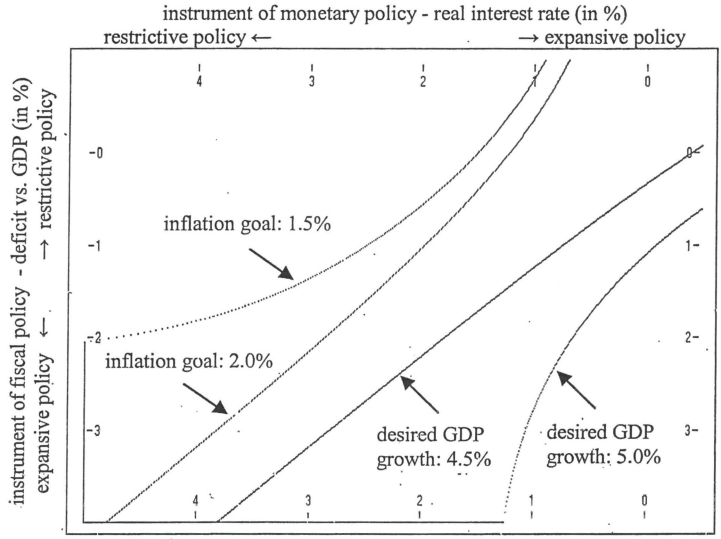

Fig. 5. Optimum fiscal and monetary strategies for the excessively ambitious (unrealistic) macroeconomic goals. Simulation results – variant 1

The diagram of Fig. 5 is an illustration for a different case, namely when the excessively high expectations and non-realistic goals of the fiscal and/or monetary authorities make it impossible to attain the equilibrium state within the effective interval of values of instruments of the macroeconomic policy and thereby hamper a rational choice in the domain of policy-mix. The excessive expectations of the fiscal and monetary authorities incline to choosing the combination of the extremely restrictive monetary policy and the extremely expansive budgetary policy. An instance of calculations for variant 1 shows that definition of the strongly restrictive inflationary goal by the monetary authorities (e.g. inflation at the level of 1.5% or 2%) , coupled with the very high GDP growth dynamics planned by the fiscal authorities (e.g. 4.5% or 5%) results in just this kind of situation. The possibility of occurrence

of such cases would imply a need for some degree of coordination of the monetary and fiscal policies.

4. The influence of parameters of the logistic function on optimum strategies and on equilibria

The subsequent simulation study concerned the analysis of the influence, exerted by the parameters of the modified logistic function, associated with the effectiveness and the scope of action of the policy-mix instruments.

In the context of the assumption, stipulating the limited capacity of exerting influence by the fiscal and monetary policy on the economy, the parameter α_i characterises the maximum interval of changes in the macroeconomic categories, reflecting the state of the economy (economic growth rate and inflation), which take place under the impact from the instruments (the gradual passage in the modified logistic function from the initial value for the radically restrictive / expansive policy to the ultimate value, accompanying, respectively, the radically expansive / restrictive policy – see the considerations in Section 2 and in Woroniecka-Leciejewicz, 2012). Increase of the absolute value of the parameter α_i brings about a widening of the interval of possible changes in production and inflation, being the reaction to the application of the instruments of macroeconomic policy, with the parameter α_1 reflecting the influence of the budgetary deficit on the GDP growth rate, α_2 reflecting the influence of the interest rate on the GDP growth rate, α_3 reflecting the influence of the budgetary deficit on inflation, and α_4 reflecting the influence, exerted by the interest rate on inflation. Analogously, a decrease of the absolute value of the parameter α_i brings about a narrowing down of the interval of changes in the dynamics of GDP and inflation, as resulting from the economic policy.

Tables 3 and 4 show the results of simulations, performed for various values of the parameter α_i. It can be observed that the increase of the absolute values of the parameters α_1 and α_2, reflecting the influence of the policy-mix instruments on the GDP growth rate, brings about the increase of the range of GDP dynamics for the analysed interval of the instrument values. For the values of parameters α_1 and α_2, adopted in variant 1 (see Table 2), the rate of growth of the GDP, lowest for the combination of both radically restrictive policies, fiscal and monetary (negative rate of growth, equal -0.15% for the budgetary surplus of 1% of the GDP and the real interest rate equal roughly 3%), increases gradually with the growing expansiveness of the fiscal and monetary policies – up to 6.49% for the combination of the two extremely expansive policies (budgetary deficit equal 6% of the GDP and real interest rate equal zero). This would mean the possibility of achieving GDP growth by 6.63 percentage point. For the very same range of changes in the values of instruments, but higher values of parameters α_1 and α_2, due to the increased expansiveness of the budgetary and monetary policies, the rate of growth of the GDP rises from -1.04% to 7.73%, that is – by 8.77 percentage points (see Table 3). Analogously, lowering of the absolute values of α_1 and α_2 brings about the narrowing of the interval, in which the values of production dynamics are contained, their changes resulting from the shifts in policy orientations, reflected in the values of the respective

instruments (Table 5). For the parameters α_1 and α_2 with values assumed at lower absolute levels, the increase of expansiveness of the fiscal and monetary policies (within the limits shown in the table) brings about a smaller change in comparison with variant 1 – the GDP growth rate increases from 0.40% for the combination of the two restrictive policies to 5.49% for the combination of the two expansive policies – that is: by 5.09 percentage point (see Table 4).

Table 3. Payoff table for the higher absolute values of the α parameters. Optimum fiscal and monetary strategies. Nash equilibrium

	3.0	2.4	2.0	1.8	1.0	0.0
-1.0	-2.50 -1.04	-2.04 -0.81	-1.57 -0.57	-1.27 -0.43	0.38 0.40	3.38 1.90
0.0	-0.88 0.81	-0.42 1.04	0.05 1.27	0.34 1.42	2.00 2.25	5.00 3.75
1.0	0.73 2.66	1.20 2.89	1.67 3.12	1.96 3.27	3.61 4.10	6.62 5.60
1.4	1.23 3.23	1.70 3.46	2.16 3.69	2.46 3.84	4.11 4.67	7.12 6.17
1.6	1.44 3.46	1.91 3.70	2.37 3.93	2.67 4.08	4.32 4.90	7.32 6.41
1.8	1.62 3.67	2.09 3.91	2.56 4.14	2.85 4.29	4.50 5.11	7.51 6.62
2.0	1.78 3.85	2.25 4.09	2.72 4.32	3.01 4.47	4.66 5.29	7.67 6.80
2.4	2.03 4.14	2.50 4.38	2.97 4.61	3.26 4.76	4.91 5.58	7.92 7.08
3.0	2.28 4.43	2.75 4.66	3.22 4.90	3.51 5.04	5.16 5.87	8.17 7.37
6.0	2.60 4.79	3.06 5.02	3.53 5.26	3.83 5.40	5.48 6.23	8.17 7.37

Parameters: α_1=8; α_2=-6.5; α_3=7; α_4=-13; remaining parameters as in variant 1. Desired GDP growth = 3.5%, inflationary goal=2.5%. Notations: optimum strategies: 3.27 –fiscal, 1.96 – monetary, Nash equilibrium – cells in thick frames

A similar effect can be observed for the influence, exerted by the instruments of policy-mix on inflation. Increase of the absolute values of the parameters α_3 and α_4 causes the broadening of the admissible scope of inflation for the adopted range of values of the instruments. In variant 1 (Table 2) inflation at the lowest level of -1.13% (i.e. actually – deflation), corresponding to the combination of the two extremely restrictive policies, increases under the influence of growing policy expansiveness, both regarding fiscal and monetary policies, to the maximum of 7.49% (increase of inflation altogether by 8.62 percentage points). For the same interval of instrument values, but for the higher absolute values of the parameters, when the policies change their orientation towards the increasingly expansive one, inflation raises from -2.50% (deflation) to 8.48%, that is – by 10.98 percentage points (see Table 4). Along with the decrease of the absolute values of the parameters α_3 and α_4, the interval of changes in inflation rate narrows down (see Table 4).

Table 4. Payoff table for the higher absolute values of the α parameters. Optimum fiscal and monetary strategies. Nash equilibrium

	3.0	2.4	2.0	1.8	1.0	0.0
-1.0	-0.11 0.40	0.23 0.54	0.57 0.69	0.79 0.78	2.00 1.29	4.19 2.21
0.0	0.70 1.44	1.04 1.58	1.38 1.73	1.60 1.82	2.80 2.33	5.00 3.25
1.0	1.51 2.48	1.85 2.62	2.19 2.77	2.41 2.86	3.61 3.37	5.81 4.29
1.4	1.76 2.80	2.10 2.94	2.44 3.09	2.66 3.18	3.86 3.69	6.06 4.61
1.6	1.86 2.93	2.20 3.08	2.54 3.22	2.76 3.31	3.97 3.82	6.16 4.74
1.8	1.95 3.05	2.29 3.19	2.64 3.34	2.85 3.43	4.06 3.94	6.25 4.86
2.0	2.03 3.15	2.37 3.30	2.72 3.44	2.93 3.53	4.14 4.04	6.33 4.96
2.4	2.16 3.32	2.50 3.46	2.84 3.60	3.06 3.69	4.26 4.20	6.46 5.13
3.0	2.28 3.48	2.62 3.62	2.97 3.76	3.18 3.85	4.39 4.36	6.58 5.29
6.0	2.44 3.68	2.78 3.82	3.12 3.97	3.34 4.06	4.55 4.56	6.74 5.49

Parameters: α_1=4.5; α_2=-4; α_3=3,5; α_4=-9,5; remaining parameters as in variant 1. Desired GDP growth = 3.5%, inflationary goal=2.5%. Notations: optimum strategies: 3.46 –fiscal, 2.50 – monetary, Nash equilibrium – cells in thick frames

A change in the values of parameters α_1 and α_2 brings also about a change in the optimum budgetary strategies, being the reaction of the fiscal authorities to the decisions of the central bank, and, quite in analogy, a change in the values of parameters α_3 and α_4 causes a shift in the optimum monetary strategies. Consequently, location of the Nash equilibrium state changes correspondingly, as well.

Parameter χ, appearing in the logistic function, used in the analysis, reflects the speed of transition of the output variable towards the new level of values in reaction to the increase of value of the explanatory (input) variable. The higher the absolute value of χ, the faster the transition, the lower the absolute value of χ, the slower the transition. Simulation calculations were performed for the parameter values χ>0. In reference to the function, describing the state of the economy, characterised by the GDP growth and inflation in dependence upon the instruments of macroeconomic policy, it should be noted that the faster transition might be interpreted as a narrower interval of the values of policy-mix instruments, within which these instruments influence effectively the economy, while the slower transition might be interpreted as a broader interval of effectiveness of the policy instruments. In the analysis here reported the parameters $\chi_1 \div \chi_4$ reflect, respectively:

- χ_1 - influence of the budgetary deficit on the GDP growth rate,
- χ_2 - influence of the interest rate on the GDP growth rate,

- χ_3 – influence of the budgetary deficit on inflation,
- χ_4 – influence of the interest rate on inflation.

The results of simulations for various values of the parameter χ_i are presented in Figs. 6 through 12. The diagram of Fig. 6 indicates that a change in value of the parameters χ_1 and χ_2, corresponding to the influence of the policy-mix instruments on the GDP growth rate, brings about a change in the optimum fiscal strategies in response to the strongly restrictive monetary strategies. The higher the values of parameters χ_1 and χ_2, the narrower the interval of the effective influence, exerted by the macroeconomic instruments on economic growth. The values of these parameters influence also the degree of restrictiveness / expansiveness of the fiscal policy in the region, in which this strategy turns into the dominating strategy for the extremely restrictive monetary policy. Analogously, Fig. 6 shows how the optimum monetary strategies change under the impact from the changes in the values of parameters χ_3 and χ_4, reflecting the influence of the policy-mix instruments on inflation. Here, as well, the higher the values of the parameters, the narrower the interval of effective action of the instruments on the level of inflation in the economy. The values of the parameters χ_3 and χ_4 influence also the degree of restrictiveness / expansiveness of the monetary policy in the region, where it turns into the dominating strategy for the extremely expansive / restrictive fiscal policy.

Consequently, location of the Nash equilibrium depends also upon the values of parameters χ_i. For low values (e.g. for the parameter values $\chi_1=\chi_2=\chi_3=\chi_4=0.5$) the Nash equilibrium state corresponds to the combination of the more expansive fiscal policy and the more restrictive monetary policy (budgetary deficit at the level of roughly 2.84% of the GDP, and real interest rate at the level of around 3.7%) compared to variant 1. For high values (e.g. for the parameter values) the Nash equilibrium state corresponds to the combination of the rather restrictive fiscal policy and the rather expansive monetary policy (budgetary deficit at the level of approximately 0.71% of the GDP and real interest rate at around 0.72%).

The subsequent figures present the results of simulation analyses for higher (Figs. 7, 8 and 9) and lower (Figs. 10, 11 and 12) values of the parameters χ_i. These figures provide both the respective isoquants and the diagrams illustrating the influence of the priorities of the fiscal and monetary authorities on the Nash equilibrium state and hence also on the choice of the corresponding policy-mix.

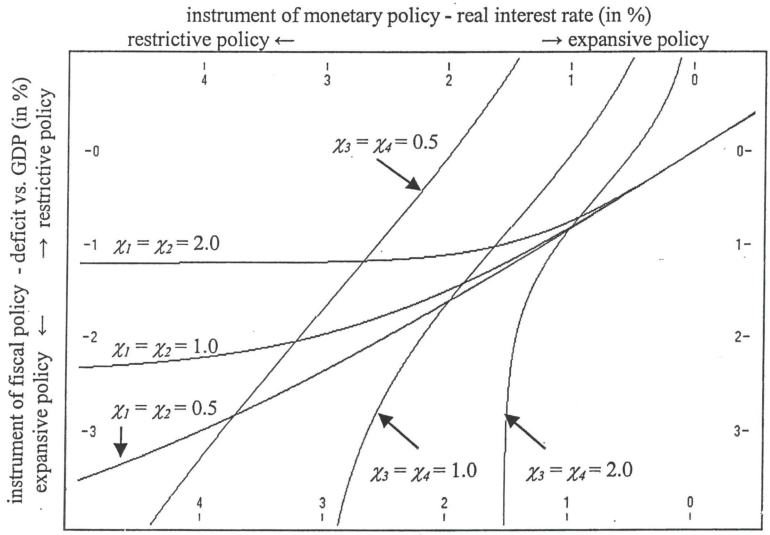

Fig. 6. Optimum fiscal and monetary strategies (desired GDP growth = 3.5%, inflationary goal = 2.5%). Equilibrium states for different $\chi_1 \div \chi_4$; remaining parameters as in variant 1

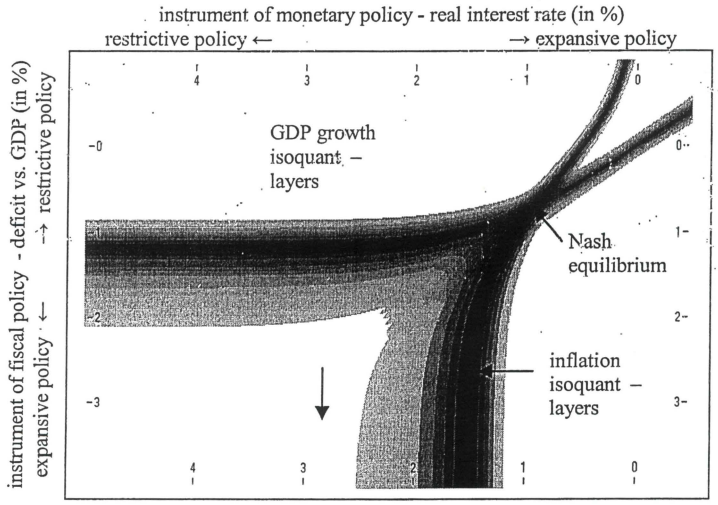

Fig. 7. Isoquants – layered diagram with admissible deviations from the assumed goal values: GDP growth = 3.5%, inflation = 2.5%. Simulation results for: χ_1, χ_2, χ_3, χ_4 = 2.0; remaining parameters as in variant 1

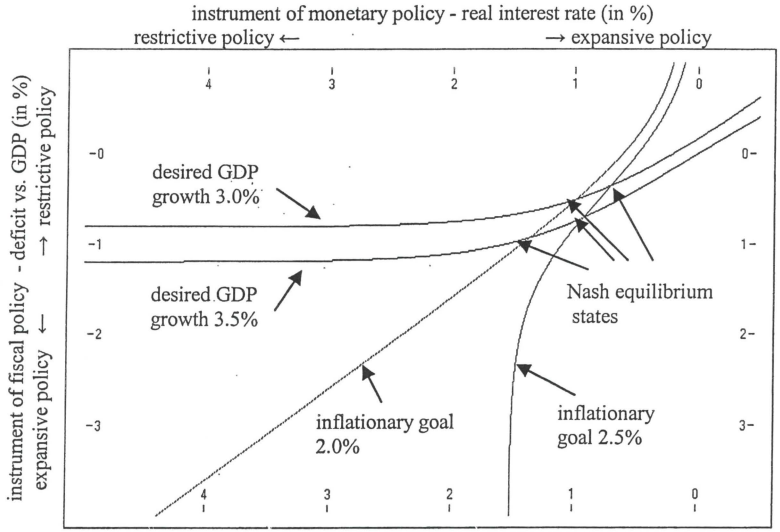

Fig. 8. Optimum fiscal and monetary strategies vs. priorities of the government and the central bank, and the corresponding Nash equilibria. Simulation results for variant 4aa: χ_1, χ_2, χ_3, $\chi_4 = 2.0$; remaining parameters as in variant 1

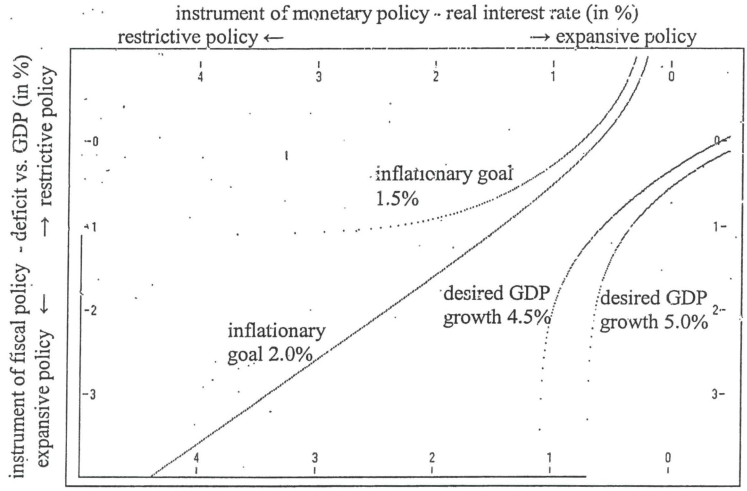

Fig. 9. Optimum fiscal and monetary strategies for the excessively ambitious (unrealistic) macroeconomic goals. Simulation results for variant 4aa: χ_1, χ_2, χ_3, $\chi_4 = 2.0$; remaining parameters as in variant 1

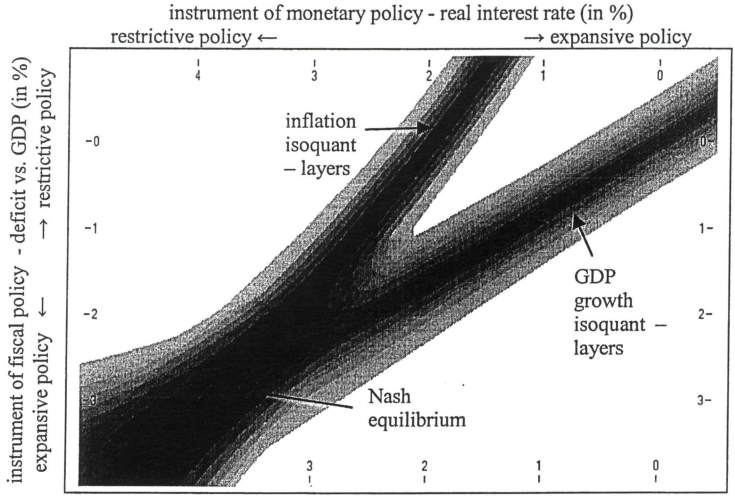

Fig. 10. Isoquants – layered diagram with admissible deviations from the assumed goal values: GDP growth = 3.5%, inflation = 2.5%. Simulation results for variant 4bb: χ_1, χ_2, χ_3, χ_4 = 0.5; remaining parameters as in variant 1

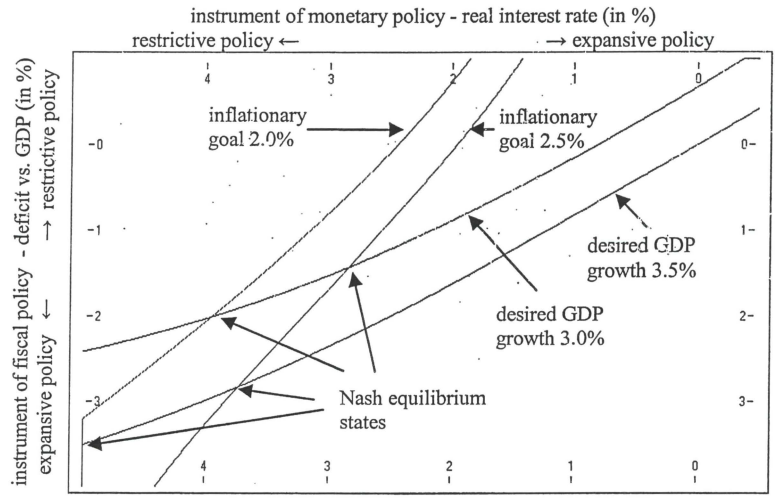

Fig. 11. Optimum fiscal and monetary strategies vs. priorities of the government and the central bank, and the corresponding Nash equilibria. Simulation results for variant 4bb: χ_1, χ_2, χ_3, χ_4 = 0.5; remaining parameters as in variant 1

The influence of the changes in the priorities of the fiscal and monetary authorities, exerted on the Nash equilibrium state and, consequently, on the choice of the policy-mix, is shown in Figs. 8 and 9, and then 11 and 12. Similarly as for the initially adopted values of parameters χ_i, taken as equal 1.0, the direction of impact from the economic objectives on the degree of restrictiveness / expansiveness of the optimum budgetary and monetary strategies is analogous. Determination of the more ambitious goals related to the GDP dynamics results in the increased expansiveness of the optimum strategies of fiscal policy, while acceptance of a lower growth dynamics leads to the limitation of the expansion of the budgetary policy (Figs. 8 and 11). Likewise, a change in the inflationary goal influences the character of the monetary policy – raising of inflationary goal (acceptance of a higher inflation) leads to an alleviation in the monetary policy, while lowering of the inflationary goal increases the restrictiveness of the policy. Under the influence of changes in the priorities of the fiscal and monetary authorities, the location of the Nash equilibrium of the game also shifts. In addition, one can observe that the higher the values of the parameters χ_i, the narrower the area, within which the equilibrium states are contained for the macroeconomic goals that can change within a given interval. In other words, for higher values of the parameters χ_i the same change in the priorities of the government and the central bank results in smaller changes as to the values of the fiscal and monetary policy instruments.

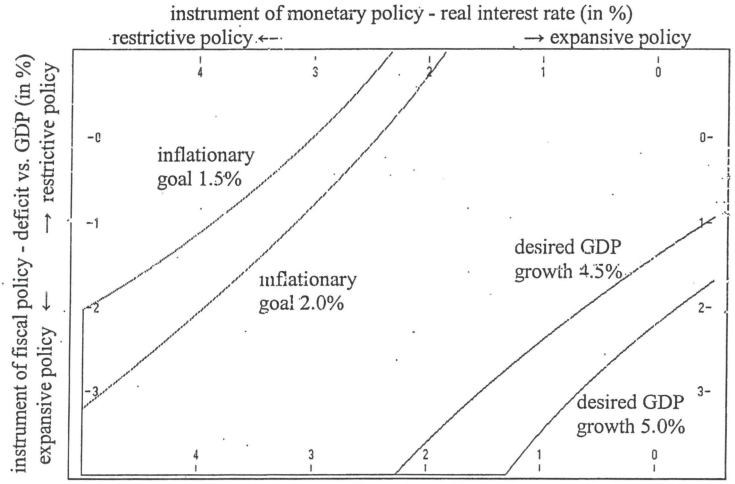

Fig. 12. Optimum fiscal and monetary strategies for the excessively ambitious macroeconomic goals. Simulation results for variant 4bb: χ_1, χ_2, χ_3, χ_4 = 0.5; remaining parameters as in variant 1

Similarly as for the earlier simulations (variant 1), the calculations performed for other values of the parameters χ_i indicate the possibility of occurrence of the

situations, in which the overly ambitious, unrealistic goals of the fiscal and monetary authorities make it impossible to attain the equilibrium state inside the effective interval of the policy-mix instruments, see Figs. 9 and 12. This leads both decision makers, the government and the central bank towards the dominating strategies, proper for the ineffective region of values of the economic policy instruments, and, consequently – under lack of coordination – the necessity of choosing the combination of the extremely restrictive monetary policy with the extremely expansive fiscal policy.

The subsequent simulations concerned the influence of the parameter β on the optimum strategies and the Nash equilibrium state. It can be observed that as the value of this parameter increases, the modified logistic function is shifted towards the higher values of the instrument, which means that also the range of changes of the instrument, within which it effectively influences the economy, is pushed to the higher values of the instrument. In particular, the inflexion point of the modified logistic function is also shifted, this being equivalent to the increase of the value of the instrument, for which its effectiveness (strength of influence on the economy) is the highest. Quite analogously, movement of the function in the opposite direction takes place along with the decrease of the value of parameter β. Then, the instrument remains effective for its correspondingly lower values. The parameters β_1 - β_4 reflect, respectively: β_1 - the influence of the budgetary deficit on the GDP growth rate; β_2 - the influence of the interest rate on the GDP growth rate; β_3 - the influence of the budgetary deficit on inflation; and β_4 – the influence of the interest rate on inflation.

The results of simulations for various values of the parameters β_i are shown in Fig. 13. It can be observed that a change in the values of parameters β_1 and β_2, reflecting the influence of the policy-mix instruments on the rate of growth of the GDP, brings about a change in the optimum fiscal strategies. In particular, there is a shift of the effective interval of values for the optimum fiscal strategy, within which it is truly effective with regard to economic growth, towards the higher values of the instrument, that is – towards the bigger fiscal expansion. The diagram also shows that the higher the values of β_1 and β_2, the more expansive the optimum budgetary policy, primarily as a response to the strongly restrictive monetary policy. The values of these parameters exert influence, as well, on the degree of restrictiveness / expansiveness of the fiscal policy in the region, where it turns into the dominating strategy for the extremely restrictive monetary policy. The Figure shows also how the optimum monetary strategies change under the influence of changes in the values of parameters β_3 and β_4, reflecting the influence of the macroeconomic policy instruments on the level of inflation. The higher values of the parameters bring about the shift of the interval of effective action of the instruments upon inflation towards higher values of the monetary policy instrument, that is – towards the higher interest rates, implying an increased degree of restrictiveness of the effective monetary policy. The values of parameters β_3 and β_4 influence, as well, the degree of restrictiveness / expansiveness of the monetary policy in the region, where it becomes the dominating strategy in the conditions of the extremely expansive / restrictive fiscal

policy. The location of the Nash equilibrium point changes in accordance with the changes in the optimum fiscal and monetary strategies.

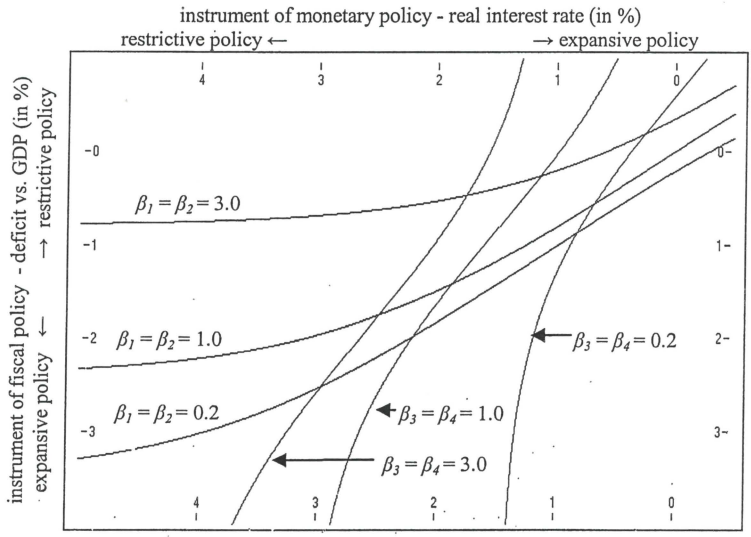

Fig. 13. Optimum fiscal and monetary strategies (the desired GDP growth = 3.5%; the inflationary goal = 2.5%). Equilibrium states for various values of parameters β_1, β_2, β_3, β_4; remaining parameters as in variant 1

5. Summary of the study

The present paper provides the results of simulations for the fiscal-monetary game, in which the players' payoffs were determined with the use of a modified logistic function, which implied the assumption of a limited effectiveness of the instruments of fiscal and monetary policy in their action upon the economic situation, including GDP growth and inflation. This is equivalent to both limited capacity of stimulating growth of economy through increasingly expansive fiscal policy and a barrier to effective lowering of inflation through sharpening of the already restrictive monetary policy. The assumption was made in the study that the monetary authorities aim at a desired level of inflation, the so called inflationary goal, while the fiscal authorities – to the attainment of a desired (planned) economic growth.

The results of simulations indicate that the optimum fiscal and monetary strategies depend both upon the parameters of the logistic function, reflecting the effectiveness and the scope of tangible action of the policy-mix instruments, and upon the priorities, adopted by the government and the central bank in the shaping of the macroeconomic policies.

Within a certain interval of values of the fiscal and monetary policy instruments, which can be referred to as the effective one, the action of these instruments on the economy is effective, while outside of it, under extremely low / high values of the instruments, corresponding to the radically restrictive or expansive orientation of the policies – the effectiveness of the instruments drops very distinctly. Within the effective interval of instrument values the choice of the optimum fiscal policy depends upon the decision of the central bank – the more restrictive the monetary policy, the more expansive, in response, the fiscal policy, and vice versa. Outside of the interval of effective action, as we move towards the radically restrictive monetary policy – the optimum fiscal strategy stops to change under the influence of further sharpening of the policy by the central bank, similarly as this occurs when we move towards the extremely expansive monetary policy – the optimum fiscal strategy stops to react to further relaxation of the monetary policy. Thus, one can observe a tendency towards a dominating fiscal strategy for the passage towards the extremely restrictive / expansive monetary policy. Analogously, for the monetary policy, one can show the interval of its effective action, within which the degree of restrictiveness of the monetary policy depends upon the policy that is conducted by the fiscal authorities. The central bank reacts to the increase of expansiveness of the budgetary policy by the sharpening of the monetary policy, in order to avoid inflation exceeding the inflationary goal. Outside of this interval it is, again, characteristic, that the monetary policy tends towards the dominating one, constituting the response to the radically restrictive / expansive orientation of the fiscal policy.

The results indicate that under the influence of changes in the priorities of the central bank and the government the optimum fiscal and monetary strategies do change, and, consequently, also the Nash equilibrium shifts, this equilibrium being identified with the choice of the policy-mix. A change in the objectives of the fiscal policy, finding its reflection in the change of the planned GDP growth, causes a shift in the optimum budgetary strategies in the direction of the more expansive ones in connection with determination of more ambitious goals concerning economic growth, or in the direction of increasing restrictiveness of this policy in the opposite case. Likewise, a change in the priorities of the monetary authorities, finding its reflection in the change of the desired inflation levels, causes a shift in the optimum monetary strategies: in the direction of the more expansive ones, when the priorities of the policy are milder, and so the allowed or desired inflation is higher, or in the direction of a harder monetary policy, when the inflationary goal is set a lower, more restrictive level. The simulations conducted indicate also the possibility of occurrence of the case, when – in conditions of lack of coordination – the unrealistically defined goals of the fiscal and monetary authorities hamper the attainment of the equilibrium state and thus also the rational choice concerning the policy-mix.

On the basis of the results obtained the isoquants were presented of the GDP growth and of inflation, showing what alternative policy-mixes (being the combinations of the fiscal and monetary policies of a definite degree of restrictiveness / expansiveness) allow for the attainment of the assumed GDP dynamics and inflation rate. This enables the observation of the optimum solutions and the solutions near to optimum (with an admissible range of deviations from the desired values).

6. References

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