Raport Badawczy
Research Report

RB/5/2009

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Warszawa 2009

Decision interactions of the monetary and fiscal authorities in the choice of policy mix

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The paper presents an analysis of the states of equilibrium and Pareto-optimality of the solutions in the monetary-fiscal games between the fiscal and monetary authorities each having either two or three qualitatively different strategies: expansive, neutral and restrictive. Two sets of assumptions about the influence, exerted by the instruments of the monetary policy (real interest rate) and of the fiscal policy (budgetary deficit related to the GDP) on the state of economy (rate of economic growth and inflation) are considered. The results obtained indicate that, along with the case of the prisoner's dilemma, to which the discussion in (Blinder, 1983; Bennett and Loayza, 2001) is limited, other situations may also occur, where the independent decisions of the central bank and the government do not necessarily lead to the choice of a Pareto non-optimal solution.

1. Introduction

This paper studies the application of game theory in the analysis of macroeconomic policy and the choice of policy mix, being a combination of the monetary and fiscal policy. The study analyzes the interactions and mutual influence between the decisions made by the monetary and fiscal authorities, which apply qualitatively different strategies, defined by the degrees of their restriction / expansion orientation. A two-person game between the central bank and the government is developed, whose equilibrium states are studies for Pareto optimality. The present paper extends (Woroniecka-Leciejewicz, 2008) by presenting new results in the analysis of the equilibrium states and Pareto optimality of solutions under different assumptions.

In the discussions, taking place around the problem of selection of the policy mix, understood as the combination of the monetary and fiscal policy, the arguments are quoted both for and against the independence of the central bank. The arguments for the independence of the central bank include higher effectiveness in countering inflation, lower variability of inflation, as well as positive impact on the levels and variability of production (see, e.g., Eijffinger, DeHaan, 1996; Wojtyna, 1996, 1998; Marszałek, 2005). On the other hand, though, independence of the central bank may give rise to definite difficulties in coordinating the monetary and fiscal policy. The source of these difficulties lies, first of all, in the different objectives of the monetary and fiscal authorities, and in the different assessments of the potential effects of the macroeconomic policies. The studies, concerning interdependence between the monetary and fiscal policies, have been started by Sargent and Wallace (1981), who formulated the concept of the so-called "unpleasant monetarist arithmetic". In the discussion of coordination of the macroeconomic policy the significance is also emphasised of the reliability and clarity of the policy conducted (Blinder, 1999; Blackburn, Christensen, 1989; Wojtyna, 1998; Walsh; Gjedrem, 2001).

In trying to resolve the issue of choice between the independence of the central bank and coordination of the monetary and fiscal policy one can refer to the analysis based on game theory. Consideration of Pareto optimality of the equilibrium solutions in the monetary-fiscal game shall provide the basis for the assessment of the benefits and losses of the independent conduct of the budgetary and monetary policies compared to their coordination.

The selection of policy mix, including monetary and fiscal policy, is modeled here by a two-person game between the central bank and the government. It is a single-stage non-zero-sum game. Each of the players takes decision independently, taking into account the probable reaction of the other player. The governmental strategies are here equivalent to the fiscal policies, differing by the degree of restrictiveness. This degree of restrictiveness is measured in the model here adopted by the level of budgetary deficit in relation to the GDP. The strategies of the central bank differ also by the degree of restrictiveness, and the measure of this restrictiveness is the real interest rate value. The payoff of the central bank in the game is related to inflation rate – it is assumed that the monetary authorities tend to the possibly low inflation level. The payoff, maximised by the fiscal authorities (government) is the rate of increase of the GDP.

Table 1. Game with a finite number of fiscal and monetary strategies.

	Payoff table	Central bank: Monetary strategy M_1 (real interest rate r_1)	Monetary strategy M_2 (real interest rate r_2)	...	Monetary strategy M_n (real interest rate r_n)
Government	Fiscal strategy F_1 (budgetary deficit b_1)	p_{11} / y_{11}	p_{12} / y_{12}	... / ...	p_{1n} / y_{1n}
	Fiscal strategy F_2 (budgetary deficit b_2)	p_{21} / y_{21}	p_{22} / y_{22}	... / ...	p_{2n} / y_{2n}
	...			...	
	Fiscal strategy F_m (budgetary deficit b_m)	p_{m1} / y_{m1}	p_{m2} / y_{m2}	... / ...	p_{mn} / y_{mn}

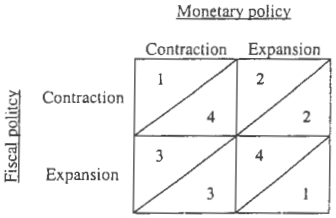
Table 1 presents the payoffs for the here defined game. Payoffs are denoted as follows: y_{ij} – payoff of the government (GDP growth rate) in case of fiscal strategy F_i, and by the central bank – of monetary strategy M_j; p_{ij} – payoff of the central bank (inflation rate) in the same situation. We denote with r the real interest rate, while with b – budget deficit in relation to the GDP. The concept of such a game between the central bank and the government, with a finite number of fiscal and monetary strategies was presented already in the earlier work of the author (Woroniecka, 2006).

An analogous game was considered, in particular, in the studies of Blinder (1983) and Bennett and Loayza (2001). They suggest that independently acting monetary and fiscal authorities would tend (conform to the Nash equilibrium[1]) to a restrictive monetary and expansive

[1] The Nash equilibrium concept is defined as follows. Each player does what he would if he knew what the other player was going to do. It is an equilibrium in the sense that the two resulting strategies are consistent with one another; once the game is played, neither player has any desire to change his decision. Not all games have a unique Nash equilibrium. The fiscal-monetary game to be considered here does.

budgetary policies, which means a Pareto non-optimal solution, similarly as in the prisoner's dilemma.

Table 2. Preference ordering in the monetary-fiscal game according to Blinder



Source: Blinder (1983, p. 23)

The prisoner's dilemma was originally framed in 1950 by Merrill Flood and Melvin Dresher working at RAND Corporation. The title "prisoner's dilemma" and the version with prison sentences as payoffs are due to Albert Tucker. Than Anatol Rapoport and Albert Chammah formalized the game as a general form of the prisoner's dilemma (Rapoport, Chammah 1970). In its simplest form the prisoner's dilemma is a game described by the payoff matrix:

	cooperate (C)	defect (D)
cooperate (C)	*R, R*	*S, T*
defect (D)	*T, S*	*P, P*

satisfying the following chain of inequalities: $T>R>P>S$.

There are two players, Row and Column. Each has two possible moves, "cooperate" or "defect," corresponding, respectively, to the options of remaining silent or confessing in the illustrative anecdote above. For each possible pair of moves, the payoffs to row and column (in that order) are listed in the appropriate cell. R is the "reward" payoff that each player receives if both cooperate. P is the "punishment" that each receives if both defect. T is the "temptation" that each receives if he alone defects and S is the "sucker" payoff that he receives if he alone cooperates. We assume here that the game is symmetric, i.e., that the reward, punishment, temptation or sucker payoff is the same for each player, and payoffs have only ordinal significance, i.e., they indicate whether one payoff is better than another, but tell us nothing about how much better. It is now easy to see that we have the structure of a dilemma like the one in the story. Suppose Column cooperates. Then Row gets R for cooperating and T for defecting, and so is better off defecting. Suppose Column defects. Then Row gets S for cooperating and P for defecting, and so is again better off defecting. The move D for Row is said to strictly dominate the move C: whatever his opponent does, he is better off choosing D than C. By symmetry D also strictly dominates C for Column. Thus two "rational" players will defect and receive a payoff of P, while two "irrational" players can cooperate and receive greater payoff R. In standard treatments, game theory assumes rationality and common knowledge. Each player is rational, knows the other is rational, knows that the other knows he is rational, etc. Each player also knows how the other values the outcomes. But since D strictly dominates C for both players, the argument for dilemma here requires only that each player knows his own payoffs. (The argument remains valid, of course, under the stronger standard assumptions.) It is also worth noting that the outcome (D, D) of both players defecting is the game's only strong Nash equilibrium, i.e., it is the only

outcome from which each player could only do worse by unilaterally changing its move. Flood and Dresher's interest in their dilemma seems to have stemmed from their view that it provided a counterexample to the claim that the Nash equilibria of a game constitute its natural "solutions".

In the opinion of Blinder and Bennett and Loayza only coordination of both policies can bring a better choice. This is illustrated by Table 2, containing the preferences of the monetary and fiscal authorities according to Blinder, for various policy mix combinations.

Table 3. Payoffs in monetary-fiscal game according to Bennett and Loayza.

		Central bank	
		Strict monetary policy	Loose monetary policy
Fiscal authority	Strict fiscal policy	*Outcome:* low inflation low employment *Payoffs:* Central bank: 6+1=7 Fiscal authority: 3+1=4	*Outcome:* medium inflation medium employment *Payoffs:* Central bank: 4+2=6 Fiscal authority: 2+4=6
	Loose fiscal policy	*Outcome:* medium inflation medium employment *Payoffs:* Central bank: 4+2=6 Fiscal authority: 2+4=6	*Outcome:* high inflation high employment *Payoffs:* Central bank: 1+3=4 Fiscal authority: 1+6=7

	Payoffs		
	low level	*medium level*	*high level*
	Inflation		
Central bank	6 3	4 2	1 1
	Employment		
Fiscal authority	1 1	2 4	3 6

Source: Bennett and Loayza (2001), p. 301.

Blinder assumed that association of the restrictive monetary and budgetary policy is the most desirable variant from the point of view of the monetary authorities, while it is the least so in the eyes of the government. The exactly opposite preferences are ascribed to the expansive character of both policies. It appears, though, that quite arbitrary preferences were assumed for the two remaining variants of the policy mix, namely that both decision making subjects prefer the state of economy, in which the policy conducted is the combination of the restrictive monetary and expansive fiscal policy, rather than vice versa.

Bennett and Loayza approach this issue from a slightly different angle (see Table 3). They do not decide as to which of the two solutions is more advantageous from the point of view of the monetary and fiscal authorities: rigid monetary and loose fiscal policy or vice versa, assuming that they bring about similar inflation and employment levels. Hence, they ascribe these two mixes the same level of preference.

It is interesting to analyse whether actually the values of inflation and unemployment, designated as "medium" for two combinations of restrictive (strict) and expansive (loose)

policies, are identical, because if not, then the fact that they can be higher or lower might have a significant impact on Pareto-optimality of solutions and on appearance – or not – of the prisoner's dilemma. For this purpose, the monetary-fiscal game introduced here shall be analysed from the point of view of Pareto-optimality of solutions, but, in distinction from the report by Bennett and Loayza, the economic standing shall be measured with the rate of GDP growth, and not the level of employment.

2. Equilibrium in the monetary-fiscal game with two strategies

Before we start analysing game with three strategies, we shall shortly present an instance of a game incorporating two qualitatively different strategies – restrictive and expansive, on both sides – of the fiscal and monetary policy (Table 4). Central bank, aiming at decreasing inflation (p), chooses between amore restrictive policy, characterised by a higher real interest rate (r) and the less restrictive one, characterised by a lower interest rate. Government, when taking decisions concerning the budgetary policy, chooses between a more restrictive policy, involving a lower level of deficit in the state budget (b), or a more expansive policy (with higher deficit) aiming to attain the possibly high real growth of the GDP (y). Left column in the table reflects the restrictive monetary policy, while the right column – expansive one, and, analogously, upper row corresponds to a restrictive policy, while the lower row – to a more expansive fiscal policy.

Table 4. The monetary-fiscal game with two strategies

			Central bank	
Payoff table			Strategy M_1	Strategy M_2
			r_1	r_2
Government	Strategy F_1	b_1	p_{11} / y_{11}	p_{12} / y_{12}
	Strategy F_2	b_2	p_{21} / y_{21}	p_{22} / y_{22}

It is assumed that:

$$\Delta b = b_2 - b_1 > 0, \quad \Delta r = r_2 - r_1 < 0. \tag{3}$$

The game is analysed under alternative assumptions concerning the influence of the real interest rate and the budgetary deficit on the rate of growth of GDP and the level of inflation, with the general simplifying assumption that these dependencies are linear. Two variants are considered.

Variant A: the first partial derivatives of both the rate of growth of the GDP and inflation with respect to the real interest rate are negative, while the first partial derivatives of both the rate of growth of the GDP and inflation with respect to the budgetary deficit are positive:

$$\frac{\partial y}{\partial r} < 0, \quad \frac{\partial p}{\partial r} < 0, \quad \frac{\partial y}{\partial b} > 0, \quad \frac{\partial p}{\partial b} > 0. \tag{1}$$

Variant B: the first partial derivatives have the same signs as in variant A, except for the first partial derivative of the rate of growth of the GDP with respect to the budgetary deficit, which, in distinction from the variant A, is negative:

$$\frac{\partial y}{\partial r}<0,\quad \frac{\partial p}{\partial r}<0,\quad \frac{\partial y}{\partial b}<0,\quad \frac{\partial p}{\partial b}>0. \tag{2}$$

The assumption that the increase of the real interest rate, *ceteris paribus*, causes a decrease in the rate of growth of the GDP and limitation of inflation is analogous in both variants, similarly as the assumption that the increase of the budgetary deficit contributes to the increase of inflation. The difference concerns the influence of the budgetary deficit on the real increase of production in the economy. It is assumed in variant A that increase of the state budget deficit, *ceteris paribus*, causes the increase of the rate of growth of the GDP, while in variant B – that it limits the rate of growth of production value.

In this paper the analysis of the game between the central bank and the government concentrates on variant A, which, in a short time horizon, appears to reflect more realistically the interactions considered between the monetary and the budgetary policies, although we shall also treat to some extent the second variant of assumptions (B).

Table 5 contains the payoffs of the game for variant A. The lowest inflation and, at the same time, the lowest economic growth occur for the choice of restrictive policies, both monetary and fiscal (the upper left corner of the payoff matrix). As the interest rate decreases (passage to the right-hand column), inflation increases and the rate of growth of the GDP as well. Similarly, owing to the increase of the budgetary deficit an increase of inflation and production takes place (passage to the lower row). The highest inflation, but also the highest increase of production, are observed for the economy when both monetary and fiscal policies have expansive character (lower right corner of the table).

The optimum strategy of the central bank in the case of application by the government of the restrictive fiscal policy (lower deficit b_1) is the choice of the restrictive monetary policy (higher interest rate r_1), since inflation in the first row is minimised ($p < p + \frac{\partial p}{\partial r}\Delta r$). In case the fiscal authorities choose an expansive budgetary policy, the optimum strategy of the monetary authorities is still restrictive, since minimum inflation in the second row is selected ($p + \frac{\partial p}{\partial b}\Delta b < p + \frac{\partial p}{\partial b}\Delta b + \frac{\partial p}{\partial r}\Delta r$). A conclusion can be drawn therefrom that restrictive monetary policy is the dominating strategy for the central bank, optimal irrespective of what strategy of the fiscal policy is selected by the government – restrictive or expansive.

By carrying out an analogous reasoning for the alternative decisions of the fiscal authorities, which aim at maximisation of the real GDP growth (y), the following conclusion can be drawn: the optimum budgetary strategy in response to the restrictive monetary policy is the expansive policy, since selection is oriented at the maximum rate of growth of the GDP in the first column ($y + \frac{\partial y}{\partial b}\Delta b > y$). Likewise, in the opposite case, if the central bank conducts an expansive monetary policy, the optimum fiscal strategy is the expansive one – as oriented at the maximisation of the GDP growth in the second column ($y + \frac{\partial y}{\partial b}\Delta b + \frac{\partial y}{\partial r}\Delta r > y + \frac{\partial y}{\partial r}\Delta r$). Hence, the government, similarly as the central bank, has a dominating strategy, which, in this

case, is the expansive fiscal policy. It is, from the point of view of the government, an optimum strategy, irrespective of the decision, concerning the interest rates, taken by the central bank.

Table 5. Equilibrium in the monetary-fiscal game with two strategies

Payoff table		Central bank: higher interest rate r_1	Central bank: lower interest rate r_2
Government	lower deficit b_1	y / p	$y+\frac{\partial y}{\partial r}\Delta r$ / $p+\frac{\partial p}{\partial r}\Delta r$
	higher deficit b_2	$y+\frac{\partial y}{\partial b}\Delta b$ / $p+\frac{\partial p}{\partial b}\Delta b$	$y+\frac{\partial y}{\partial b}\Delta b+\frac{\partial y}{\partial r}\Delta r$ / $p+\frac{\partial p}{\partial b}\Delta b+\frac{\partial p}{\partial r}\Delta r$

The equilibrium is located in the lower left corner of the payoff matrix. This is not only a Nash equilibrium, it is an equilibrium determined by the dominating strategies. It leads to the choice, made by the entities responsible for the macroeconomic policy, of the restrictive monetary policy, on the one hand, and the expansive budgetary policy on the other hand. In fact, over many years it has been observed in Poland that in response to an excessively expansive fiscal policy and a high level of budgetary deficit central bank would conduct restrictive policy, maintaining real interest rates at a high level.

3. Pareto-optimality of solutions in the game with two strategies

It is now interesting to know, whether this equilibrium, determined by the dominating strategies of both players, the government and the central bank, is Pareto-optimal or not. In this context it is particularly important to compare two variants of the strategic solutions: the state of equilibrium, corresponding to the dominating strategies, i.e. the expansive fiscal policy, characterised by a high budgetary deficit and a restrictive monetary policy, featuring high levels of real interest rates (lower left corner of the payoff matrix), and the alternative solution – the restrictive budgetary policy and the expansive monetary policy (upper right corner of the payoff matrix).

For this purpose we should consider several distinct cases:

Case A1:

$$\frac{\partial y}{\partial b}\Delta b > \frac{\partial y}{\partial r}\Delta r\,, \quad \frac{\partial p}{\partial b}\Delta b < \frac{\partial p}{\partial r}\Delta r\,. \tag{4}$$

The above conditions means that, first, a change in the rate of economic growth, caused by the difference of the budgetary deficit in the two fiscal strategies is bigger than the change, caused by the difference of the real interest rates of the two analysed monetary strategies. The change of the rate of growth of the GDP, brought about by the difference of the budgetary

deficit, is influenced both by the force of impact – *ceteris paribus* – measured by the partial derivative $\frac{\partial y}{\partial b}$, and the very value of the difference between the magnitudes of the budgetary deficit in both variants analysed of the fiscal policy, Δb. Analogously, the change in the rate of growth of the GDP, caused by the difference of the interest rates, depends upon the force of influence, measured with the partial derivative $\frac{\partial y}{\partial r}$, and upon the very difference of the interest rates, Δr. Second, the change of inflation, caused by the difference of the budgetary deficit in the two fiscal strategies is smaller than the change, brought about by the difference of the real interest rates of the two analysed monetary strategies.

Case A2:

$$\frac{\partial y}{\partial b}\Delta b > \frac{\partial y}{\partial r}\Delta r\,, \quad \frac{\partial p}{\partial b}\Delta b > \frac{\partial p}{\partial r}\Delta r\,. \tag{5}$$

This assumption can be interpreted in the manner analogous to the case A1. This means that, first, the change in the rate of economic growth, caused by the difference of the budgetary deficit, is bigger than the change, caused by the difference of the real interest rates, and, second, the change of inflation, caused by the difference of the budgetary deficit, is bigger than the change, caused by the difference of the interest rates.

Case A3:

$$\frac{\partial y}{\partial b}\Delta b < \frac{\partial y}{\partial r}\Delta r\,, \quad \frac{\partial p}{\partial b}\Delta b < \frac{\partial p}{\partial r}\Delta r\,. \tag{6}$$

This assumption means that the change of rate of economic growth, caused by the difference of the budgetary deficit, is smaller than the change, caused by the difference of the real interest rates, and, analogously, the change of inflation, caused by the difference of the budgetary deficit is smaller than the change cause by the difference of the interest rates.

Case A4:

$$\frac{\partial y}{\partial b}\Delta b < \frac{\partial y}{\partial r}\Delta r\,, \quad \frac{\partial p}{\partial b}\Delta b > \frac{\partial p}{\partial r}\Delta r\,. \tag{7}$$

This condition means that the change of the rate of economic growth, caused by the difference of the budgetary deficits is smaller than the change, brought about by the difference of the real interest rates, while the change of inflation, caused by the difference of the budgetary deficits is bigger than the change, caused by the difference of the interest rates.

Tables 6 and 7 show the preferences in the monetary-fiscal game for the above four cases of assumptions. In case A2 the equilibrium state (lower left corner of the preference table, combination of the expansive fiscal and restrictive monetary policies) constitutes a Pareto-optimal solution. The rate of growth of the GDP is higher, and inflation lower in comparison with the combination of the restrictive fiscal and the expansive monetary policies (upper right corner of the table). Adoption of the dominating strategies by the two players is a more advantageous solution both from the point of view of the government and the central bank. Similarly, in cases A2 and A3 equilibrium is Pareto-optimal. Under the assumptions A2 the rate of growth of the GDP is higher, but inflation is also higher in comparison with the combination of strategies from the upper right corner of the table. The equilibrium point is a more advantageous solution from the point of view of the government, but worse from the

point of view of the central bank. The case A3 is a mirror reflection of the case A2. In the case A3 the state of equilibrium is more advantageous from the point of view of the monetary authorities (lower inflation), but less advantageous from the point of view of the fiscal authorities (lower rate of GDP growth) in comparison with the combination of the restrictive budgetary and the expansive monetary policies.

Table 6. Preference ordering in the monetary-fiscal game. Cases A1-A3

A1		Central bank	
		r_1	r_2
Government	b_1	1 4	3 3
	b_2	2 2	4 1

A2		Central bank	
		r_1	r_2
Government	b_1	1 4	2 3
	b_2	3 2	4 1

A3		Central bank	
		r_1	r_2
Government	b_1	1 4	3 2
	b_2	2 3	4 1

Table7. Case A4 – the prisoner's dilemma

A4		Central bank	
		r_1	r_2
Government	b_1	1 4	2 2
	b_2	3 3	4 1

In the case A4 (Table 7) the equilibrium state is characterised by the worse economic indicators (lower growth of the GDP and higher inflation) than the state in the upper right corner. In this case the choice of the dominating strategies by both subjects shaping the macroeconomic policy does not result in a Pareto-optimal solution. This case represents the situation, known in the literature as the prisoner's dilemma, when conflict arises between the individual rationality, represented by the criterion of the dominating strategy and the group rationality, represented by the criterion of Pareto optimality. The subjects, caring for their individual interests, end up with the outcome disadvantageous for all and for each of the subjects individually.

On the basis of the above analysis conditions can be formulated, which decide, whether the Nash equilibrium (in the problem here analysed determined by the dominating strategies) constitutes, at the same time, a Pareto-optimal solution. Pareto-optimality of solutions depends on which policy – monetary or fiscal – is more effective in affecting economic growth, and which of them is more effective in influencing inflation, and also which of the instruments undergoes bigger shifts: the interest rate (the instrument of the monetary policy) or the budgetary deficit (the instrument of the fiscal policy). After having carried out the analysis of Pareto optimality of solutions for the game, accounting for two qualitatively different strategies of the fiscal and monetary policies, the possibility of existence of four different cases was indicated, of which three concern the situation, when Nash equilibrium constitutes, at the same time, a Pareto optimal solution. In these cases independent formulation of the monetary and fiscal policies by the central bank and the government leads to effective solutions. Only one of the variants analysed involves the existence of the case, known from literature, of the prisoner's dilemma. This particular case is also referred to by the authors of studies from the domain of coordination of the monetary and fiscal policies.

Here, a conflict arises between the individual rationality, when the resolving criterion is the possession of the dominating strategies by the players, and the group rationality, when Pareto optimality of solutions is decisive. This situation takes place when monetary policy acts more effectively on the economic growth than on inflation, and vice versa – the fiscal policy influences more effectively inflation than the GDP growth. In this case the necessity of coordinating the policies of the central bank and the government arises.

4. Game equilibrium and Pareto optimality under other assumptions

Until now the game between the subjects responsible for the monetary and fiscal policies was analysed for the assumptions based on variant A (equation 1), which, in a short time horizon, seems to reflect more realistically the interactions considered between the monetary and budgetary policies. One might, however, consider also other assumptions, concerning the signs of the partial derivatives of the payoff functions of the central bank and the government (i.e. inflation and GDP growth rate) depending upon the instruments of the monetary and fiscal policies – presented previously, at the beginning of Section 2, as variant B of assumptions (equation 2). In this variant it is assumed, contrary to variant A, that the increase of the budgetary deficit causes, *ceteris paribus*, limitation of the rate of growth of the GDP.

Table 8. Equilibrium in the game with two strategies. Variant B

Payoff table		Central bank: higher interest rate r_1	Central bank: lower interest rate r_2
Government	lower deficit b_1	p $y+\frac{\partial y}{\partial b}(-\Delta b)$	$p+\frac{\partial p}{\partial r}\Delta r$ $y+\frac{\partial y}{\partial b}(-\Delta b)+\frac{\partial y}{\partial r}\Delta r$
	higher deficit b_2	$p+\frac{\partial p}{\partial b}\Delta b$ y	$p+\frac{\partial p}{\partial b}\Delta b+\frac{\partial p}{\partial r}\Delta r$ $y+\frac{\partial y}{\partial r}\Delta r$

Table 8 is the table of payoffs conform to the variant B. The lowest inflation takes place in the case of selection of the combination of the restrictive policies, both monetary and fiscal (upper left corner of the payoff table). Along with the lowering of the interest rate (passage to the right-hand column), but also due to the increasing budgetary deficit (passage to the lower row) an increase of inflation takes place. The highest inflation arises in the economy, when both monetary and fiscal policies are expansive (lower right corner of the table). In this respect the situation, compared to variant A, has not changed. The change concerns the government payoffs. The lowest rate of growth of the GDP characterises the combination of the restrictive monetary and expansive fiscal policy (lower left corner of the table). Along with the decrease of the interest rate (passage to the right hand column) and/or curbing of the budgetary deficit (passage to the upper row) an increase of production takes place. The highest rate of growth is guaranteed by the association of the expansive monetary policy and the restrictive fiscal policy (upper right corner of the table).

The central bank, driven by the minimisation of inflation (p), shall, irrespective of the decision of the fiscal policies, select the restrictive monetary policy, since: $p < p+\frac{\partial p}{\partial r}\Delta r$ and $p+\frac{\partial p}{\partial b}\Delta b < p+\frac{\partial p}{\partial b}\Delta b+\frac{\partial p}{\partial r}\Delta r$. The restrictive monetary policy constitutes for the central bank,

similarly as in variant A, the dominating strategy. On the other hand, the government, aiming at maximisation of the real growth of the GDP (y), shall, irrespective of the decision of the monetary authorities, select the restrictive fiscal policy, since: $y+\frac{\partial y}{\partial b}(-\Delta b)+\frac{\partial y}{\partial r}\Delta r > y+\frac{\partial y}{\partial r}\Delta r$ and $y+\frac{\partial y}{\partial b}(-\Delta b) > y$. In this case the dominating strategy of the government is restrictive, and not expansive, as in variant A, fiscal policy.

Equilibrium in the game, corresponding to variant B, is determined by the dominating strategies and leads to the selection, by both the monetary and fiscal authorities, of the restrictive policies. This amounts to the policy of high real interest rates and low budgetary deficit of the state. The equilibrium state is in the upper left corner of the table (Table 8).

The Nash equilibrium in variant B is a Pareto optimal solution, which is obvious, if we analyse the payoff table. The equilibrium state guarantees the lowest level of inflation, since other solutions yield higher production, but always at the cost of increased inflation. We can, though, like in variant A, track individual cases:

Case B1: $$\frac{\partial y}{\partial b}(-\Delta b) > \frac{\partial y}{\partial r}\Delta r, \quad \frac{\partial p}{\partial b}\Delta b < \frac{\partial p}{\partial r}\Delta r, \tag{8}$$

Case B2: $$\frac{\partial y}{\partial b}(-\Delta b) > \frac{\partial y}{\partial r}\Delta r, \quad \frac{\partial p}{\partial b}\Delta b > \frac{\partial p}{\partial r}\Delta r, \tag{9}$$

Case B3: $$\frac{\partial y}{\partial b}(-\Delta b) < \frac{\partial y}{\partial r}\Delta r, \quad \frac{\partial p}{\partial b}\Delta b < \frac{\partial p}{\partial r}\Delta r, \tag{10}$$

Case B4: $$\frac{\partial y}{\partial b}(-\Delta b) < \frac{\partial y}{\partial r}\Delta r, \quad \frac{\partial p}{\partial b}\Delta b > \frac{\partial p}{\partial r}\Delta r \tag{11}$$

and, on the basis of assumptions adopted, determine for them the preferences of the central bank and the government. These preferences are contained in Table 9.

Table 9. Preference ordering in the monetary-fiscal game. Cases B1-B3

B1	Central bank	
Government	r_1	r_2
b_1	1 2	3 1
b_2	2 4	4 3

B2	Central bank	
Government	r_1	r_2
b_1	1 2	2 1
b_2	3 4	4 3

B3	Central bank	
Government	r_1	r_2
b_1	1 3	3 1
b_2	2 4	4 2

In all these cases equilibrium has a Pareto optimal character. There is no solution among the remaining three that could improve the performance in terms of one criterion (GDP growth or inflation) without worsening the other. In the cases B1 and B2 both solutions in the second row, that is – containing expansive fiscal policy – are not Pareto optimal. The state of

equilibrium is by all means a better solution in comparison with them, both in terms of the inflation criterion and economic growth.

Table 10. Preference ordering in the monetary-fiscal game. Case B4

B4		Central bank r_1	Central bank r_2
Government	b_1	3 \ 1	1 \ 2
	b_2	4 \ 2	3 \ 4

An interesting instance is constituted by the Case B4 (Table 10) – under the assumption of pure payoffs, i.e. inflation for the central bank, rate of GDP growth for the government, equilibrium is, of course, Pareto optimal. Yet, if we adopt as the payoff function weighted average with the weights ascribed by the two players, respectively, the inflation-related and the economic growth objectives, then there would be a possibility of selecting such weights that the equilibrium state, reflecting the choice of combination of the two policies of restrictive character would not correspond to a Pareto optimal decision. Under asymmetric weights, putting higher preference on the GDP growth, it may turn out that a better solution is the combination of the restrictive fiscal policy and the expansive monetary one (upper right corner of the table) – see the numerical example presented in Table 11.

Table 11. The example of payoff function as the weighted average with the asymmetric weights putting for the GDP growth and the inflation. Case B4

Payoff table (in %)		Central bank r_1	Central bank r_2
Government	b_1	1,33 \ 0,94	2,07 \ 1,36
	b_2	-0,24 \ -0,72	0,50 \ -0,30

Preference ordering		Central bank r_1	Central bank r_2
Government	b_1	2 \ 2	1 \ 1
	b_2	4 \ 4	3 \ 3

In the presented example are assumed the following values of weights in the payoff function x:
for central bank
$x_B = w_{B1} y - w_{B2} p$, where: $w_{B1} + w_{B2} = 1$, $w_{B1} = 0{,}6$, $w_{B2} = 0{,}4$,
and the following values for government
$x_G = w_{G1} y - w_{G2} p$, where: $w_{G1} + w_{G2} = 1$, $w_{G1} = 0{,}7$, $w_{G2} = 0{,}3$.

5. The monetary-fiscal game with three strategies

The game was also analysed between the central bank and the government, with consideration of three qualitatively different strategies of the fiscal and monetary policies: restrictive, neutral and expansive (Table 12). Simplifying assumptions have been adopted, analogous to those adopted in Section 1 for the game with two strategies. Side by side with the assumption, concerning the signs of the partial derivatives, conform to variant A (equation 1) two

additional assumptions have been adopted. First, it was assumed that the analysed changes in the values of the instruments of monetary and fiscal policies are equal:

$$b_2 - b_1 = b_3 - b_2 = \Delta b > 0, \quad r_2 - r_1 = r_3 - r_2 = \Delta r < 0. \tag{12}$$

Second, it was assumed that the dependencies of inflation and production value increase upon the values of the instruments of monetary and fiscal policy – the interest rate and the budgetary deficit – are linear.

Table 12. Game between the central bank and the government with three strategies

			Central bank		
	Payoff table		Strategy M_1	Strategy M_2	Strategy M_3
			r_1	r_2	r_3
Government	Strategy F_1	b_1	p_{11} / y_{11}	p_{12} / y_{12}	p_{13} / y_{13} ← restrictive fiscal policy
	Strategy F_2	b_2	p_{21} / y_{21}	p_{22} / y_{22}	p_{23} / y_{23} ← neutral fiscal policy
	Strategy F_3	b_3	p_{31} / y_{31}	p_{32} / y_{32}	p_{33} / y_{33} ← expansive fiscal policy
			↑ restrictive	↑ neutral	↑ expansive
			monetary policy		

Table 13 contains the payoffs for the game with the above assumptions. The lowest inflation and at the same time the lowest economic growth occur for the choice of restrictive monetary and fiscal policies (upper left corner of the payoff table). As the interest rate decreases (passage to the right towards the neutral, and then expansive monetary policy) inflation and rate of growth of the GDP increase. Likewise, due to the increasing budgetary deficit, inflation and production increase (passage downwards to the neutral, and then expansive fiscal policy). The highest inflation, but also the fastest growth characterise the economy, when both monetary and fiscal policies are expansive (lower right corner of the table).

The central bank selects the strategy aimed at minimising inflation (p). This means choosing the lowest inflation in each row, that is – always the restrictive policy. Hence, restrictive monetary policy is the dominating strategy for the central bank, which is optimal irrespective of the fiscal policy strategy, chosen by the government. Analogously, the government maximises in each column the real growth of the GDP (y) and so chooses always the expansive policy. The conclusion is as follows: the government, similarly as the central bank, has a dominating strategy, the expansive fiscal policy, which is the optimum strategy from the point of view of the government irrespective of the decisions, taken by the central bank in the framework of the interest rate policy.

Similarly as in the case of analysis of two strategies, the state of equilibrium in the game with three strategies is located in the lower left corner of the payoff table. This equilibrium is determined by the dominating strategies, and it leads to the selection of the restrictive monetary and expansive fiscal policies.

Table 13. Payoff table and equilibrium in the game with three strategies

Payoff table		Central bank – monetary policy: restrictive	neutral	expansive
Government – fiscal policy: restrictive		p / y	$p+\frac{\partial p}{\partial r}\Delta r$ / $y+\frac{\partial y}{\partial r}\Delta r$	$p+2\frac{\partial p}{\partial r}\Delta r$ / $y+2\frac{\partial y}{\partial r}\Delta r$
	neutral	$p+\frac{\partial p}{\partial b}\Delta b$ / $y+\frac{\partial y}{\partial b}\Delta b$	$p+\frac{\partial p}{\partial b}\Delta b+\frac{\partial p}{\partial r}\Delta r$ / $y+\frac{\partial y}{\partial b}\Delta b+\frac{\partial y}{\partial r}\Delta r$	$p+\frac{\partial p}{\partial b}\Delta b+2\frac{\partial p}{\partial r}\Delta r$ / $y+\frac{\partial y}{\partial b}\Delta b+2\frac{\partial y}{\partial r}\Delta r$
	expansive	$p+2\frac{\partial p}{\partial b}\Delta b$ / $y+2\frac{\partial y}{\partial b}\Delta b$	$p+2\frac{\partial p}{\partial b}\Delta b+\frac{\partial p}{\partial r}\Delta r$ / $y+2\frac{\partial y}{\partial b}\Delta b+\frac{\partial y}{\partial r}\Delta r$	$p+2\frac{\partial p}{\partial b}\Delta b+2\frac{\partial p}{\partial r}\Delta r$ / $y+2\frac{\partial y}{\partial b}\Delta b+2\frac{\partial y}{\partial r}\Delta r$

The analysis was carried out of a Pareto optimality of the solutions, considering, analogously as in the game with two strategies, four different cases of assumptions, A1 through A4 (equations 4 through 7). Tables 14-17 show the preferences in the game with three strategies for these assumptions.

Table 14. Preference ordering in the game with three strategies. Case A1

A1		Central bank: r_1	r_2	r_3
Government	b_1	1 / 9	3 / 8	6 / 6
	b_2	2 / 7	5 / 5	8 / 3
	b_3	4 / 4	7 / 2	9 / 1

In case A1 (Table 14) the equilibrium state constitutes, at the same time, a Pareto optimal solution. The rate of GDP growth is higher, and inflation is lower in comparison with

solutions on the diagonal of the matrix. Adoption of the dominating strategies by both players is a solution that is more advantageous from the point of view of both government and central bank.

Table 15. Preference ordering in the game with three strategies. Case A2

A2		Central bank		
		r_1	r_2	r_3
Government	b_1	1 / 9	2 / 8	4 / 6
	b_2	3 / 7	5 / 5	7 / 3
	b_3	6 / 4	8 / 2	9 / 1

Likewise, in variants A2 and A3 (Tables 15 and 16) the equilibrium state constitutes a Pareto optimal solution. In one case selection of the restrictive monetary and expansive fiscal policy is more advantageous from the point of view of the government, as guaranteeing higher rate of GDP growth than under the adoption of the expansive monetary and restrictive fiscal policy, while it is less advantageous from the point of view of the central bank – inflation in the equilibrium point is higher. In the second case the situation is reversed – equilibrium is a more advantageous solution from the point of view of the monetary policy, while it is less advantageous in terms of the criteria of fiscal policy.

In the case A4 (Table 17) equilibrium does not provide for a Pareto optimal solution. The rate of GDP growth is lower, and inflation is higher in comparison with solutions on the diagonal. Adoption of the dominating strategies by both players is a solution worse from both the point of view of the government and of the central bank. Here, we can observe a conflict between the individual rationality in the form of the dominating strategy criterion and the group rationality in the form of Pareto optimality. This case represents the situation known as prisoner's dilemma.

Table 16. Preference ordering in the game with three strategies. Case A3

A3		Central bank		
		r_1	r_2	r_3
Government	b_1	1 / 9	3 / 7	6 / 4
	b_2	2 / 8	5 / 5	8 / 2
	b_3	4 / 6	7 / 3	9 / 1

Table 17. Preference ordering in the game with three strategies. Case A4

A4		Central bank		
		r_1	r_2	r_3
Government	b_1	1 / 9	2 / 7	4 / 4
	b_2	3 / 8	5 / 5	7 / 2
	b_3	6 / 6	8 / 3	9 / 1

6. Summary

The paper presents and analyses the monetary-fiscal game as an instance of a two-person non-zero-sum game, in which monetary and fiscal authorities take their decisions independently. The game is analysed for either two qualitatively different strategies in terms of fiscal and monetary policies, i.e. restrictive and expansive, or three different strategies, where, side by side with the restrictive and expansive policies, also the neutral one is considered. The degree of restrictiveness / expansion of the fiscal policy is measured with the ratio of the budgetary deficit to the GDP, while of the monetary policy – with the real interest rate. When constructing the payoff table it was assumed that the central bank aims at minimisation of inflation, while the government – at maximisation of the real economic growth.

It is shown that the Nash equilibrium in the analysed game is determined by the dominating strategies. Two variants (designated A and B) of assumptions concerning the underlying model of economic processes have been considered, with the differences applying mainly to the influence exerted by the budgetary deficit on the GDP growth. The results obtained indicate that the dominating strategies lead to selection of the restrictive monetary and expansive fiscal policies (in variant A), or of restrictive policies, both monetary and fiscal (in variant B). The equilibrium state in variant A, i.e. the choice of the combination of restrictive monetary and expansive fiscal policies, corresponds, in fact, to the situation, which existed over the recent years in Poland, when in response to an excessively expansive fiscal policy and high levels of budgetary deficit the central bank conducted a restrictive policy, maintaining real interest rates at a high level.

For both variants considered the analysis was carried out of Pareto optimality of the solutions in the monetary-fiscal game, with indication of the possibility of existence of four different cases. As the assumptions conform to the variant A are adopted, for which the equilibrium state means the combination of the restrictive monetary and expansive fiscal policies – in three out of four cases analysed the Nash equilibrium of the game is equivalent to the Pareto optimal solution, and in only one the situation of the prisoner's dilemma, known from literature, was encountered. In this latter case a conflict arises between the individual rationality, reflected through the criterion of domination, and the group rationality, taking the form of Pareto optimality. This situation takes place when monetary policy acts more effectively on the economic growth than on inflation, and vice versa – the fiscal policy influences more effectively inflation than the GDP growth. In this case the necessity of

coordinating the policies of the central bank and the government arises. Conditions are presented, as well, for the existence of the four cases analysed.

It was further shown that when a different set of assumptions is adopted, concerning the underlying economic processes (variant B), the Nash equilibrium in the game is also determined by the dominating strategies, but it leads to selection of both restrictive policies – the monetary and the fiscal one. This means the policy of high real interest rates and low budgetary deficit, which is, at the same time, a Pareto optimal solution. The state of equilibrium provides for the lowest level of inflation, with other solutions yielding the possibility of higher production value, but always at the cost of higher inflation. As we analyse, exactly like for the variant A, individual cases, and determine, on the basis of assumptions adopted, the preferences of the government and the central bank for these cases, we reach the conclusion that in each of the cases considered equilibrium in variant B is a Pareto optimal solution. Attention was drawn to an interesting case, in which, under pure payoffs of inflation for the central bank and the GDP growth rate for the government, the equilibrium is Pareto optimal, but if we assumed as the payoff function a weighted average of inflation and GDP growth for both players, then, given a appropriate choice of weights, the possibility could arise that the state of equilibrium, reflecting the choice of the combination of both restrictive policies, would not be Pareto optimal. For asymmetric weights, with higher preference for economic growth, it may turn out that a better solution would be the combination of the restrictive fiscal and expansive monetary policies.

Against this background, the results obtained by Blinder and by Bennett and Loayza for a similar problem, were discussed. The authors referred to show that the independently acting monetary and fiscal authorities shall tend, just like in the prisoner's dilemma, to the Nash equilibrium state, determined by the dominating strategies: the restrictive monetary policy and the expansive fiscal policy, which not optimal in Pareto sense. In the opinion of these authors, such a conclusion constitutes a sufficient prerequisite for stating that the two kinds of policies must necessarily be coordinated, since this leads to better solutions than the independent decisions of the central bank and the government. The results reported here, though, demonstrate that the prisoner's dilemma is but one of several feasible cases, that might occur. In the majority of these cases game equilibrium is Pareto optimal, which means that the independence of the central bank in shaping the monetary policy may lead to effective solutions. The concrete indication of the case we deal with in practice, and hence whether the coordination of the policy mix in the two domains considered is needed, shall depend upon the course of the background economic processes and the use of instruments of the macroeconomic policy. In particular, of key importance is – which policy, monetary or fiscal, influence more effectively economic growth, and which of them influences more effectively inflation, as well as – which of the instruments is subject to stronger changes, the interest rate or the budgetary deficit.

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