ON THE SCREW DISLOCATION CONSIDERING SURFACE ENERGY: STRAIN-GRADIENT ELASTICITY VS. SURFACE ELASTICITY

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1. Introduction

Nowadays it is well-known that the surface phenomena are almost responsible for the mechanical and physical properties of micro- and nanostructured materials. In particular, they are responsible for the size-effect observed at the nano-scale. Among the theories of continuum which can describe such surface-related behavior it is worth to mention the Gurtin-Murdoch surface elasticity model [5] and the first and second strain gradient elasticity presented by Toupin [11], Mindlin [7,8], see also [9,10], and Aifantis [1,2]. The characterization of the surface elasticity within the strain-gradient elasticity was performed in [4,6] considering anti-plane surface waves. Here we compare the both models considering stress concentration in the vicinity of the linear defect such as a screw dislocation. We analyze here a deformation of a hollow circular cylinder with a screw dislocation considering the both theories of strain-gradient elasticity and of surface elasticity.

2. Strain-gradient elasticity

In what follows we consider infinitesimal deformations of an elastic solid which are described by the displacement field $\mathbf{u} = \mathbf{u}(\mathbf{x})$, where $\mathbf{x}$ is the position vector. Strain energy density W is given by [7]

$$W = W_1 + W_2, \quad W_1 = \frac{1}{2}\mathbf{e} : \mathbf{C} : \mathbf{e}, \quad W_2 = \frac{1}{2}\nabla\mathbf{e} \,\vdots\, \mathbf{A} \,\vdots\, \nabla\mathbf{e}, \quad \mathbf{e} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right), \tag{1}$$

where $\mathbf{C} = C_{ijkl}\mathbf{i}_i \otimes \mathbf{i}_j \otimes \mathbf{i}_k \otimes \mathbf{i}_l$ and Einstein's summation convention is used, $\mathbf{A} = A_{ijklmn}\mathbf{i}_i \otimes \mathbf{i}_j \otimes \mathbf{i}_k \otimes \mathbf{i}_l \otimes \mathbf{i}_m \otimes \mathbf{i}_n$ are the fourth- and six-order tensors of elastic moduli, respectively, $\mathbf{i}_k$, $k = 1, 2, 3$, are vectors of Cartesian orthonormal basis. For an isotropic strain gradient solid the elastic moduli tensors are given by [3]

$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right), \tag{2}$$

$$\begin{aligned} A_{ijklmn} = {}& a_1\left(\delta_{ij}\delta_{kl}\delta_{mn} + \delta_{ij}\delta_{km}\delta_{ln} + \delta_{ij}\delta_{kn}\delta_{lm} + \delta_{in}\delta_{jk}\delta_{lm}\right) + a_2\left(\delta_{ij}\delta_{kn}\delta_{lm}\right), \\ &+ a_3\left(\delta_{ik}\delta_{jl}\delta_{mn} + \delta_{ik}\delta_{jm}\delta_{ln} + \delta_{il}\delta_{jk}\delta_{mn} + \delta_{im}\delta_{jk}\delta_{ln}\right) + a_4\left(\delta_{il}\delta_{jm}\delta_{kn} + \delta_{im}\delta_{jl}\delta_{kn}\right), \\ &+ a_5\left(\delta_{il}\delta_{jn}\delta_{km} + \delta_{im}\delta_{jn}\delta_{kl} + \delta_{in}\delta_{jl}\delta_{km} + \delta_{in}\delta_{jm}\delta_{kl}\right), \end{aligned} \tag{3}$$

where δ_{ij} is the Kronecker symbol, λ, μ, a_1, a_2, a_3, a_4, and a_5 are elastic moduli.

The equilibrium equation takes now the form

$$\nabla \cdot (\boldsymbol{\sigma} - \nabla \cdot \boldsymbol{\tau}) = \mathbf{0}, \tag{4}$$

where the tensors $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are defined by $\boldsymbol{\sigma} = \mathbf{C} : \mathbf{e}$, $\boldsymbol{\tau} = \mathbf{A} \,\vdots\, \nabla\mathbf{e}$, which are the second-order stress tensor and third-order hyperstress tensor, respectively. Aifantis' strain-gradient model [1,2] utilizes more simple constitutive equation with one additional length-scale parameter ℓ such that $\boldsymbol{\tau} = \ell^2\nabla\boldsymbol{\sigma}$. Eq. (4) should be complemented by proper boundary conditions which we omit here.

3. Surface elasticity

For an isotropic material the Gurtin-Murdoch model results in the classic constitutive equation in the bulk $W = W_1$ and an additional constitutive relation for the surface strain energy $\mathcal{W}_s$ [5]

$$\mathcal{W}_s = \mu_s \boldsymbol{\epsilon} : \boldsymbol{\epsilon} + \frac{1}{2}\lambda_s(\mathrm{tr}\boldsymbol{\epsilon})^2, \quad \boldsymbol{\epsilon} = \frac{1}{2}\left(\mathbf{P}\cdot(\nabla_s \mathbf{u}) + (\nabla_s \mathbf{u})^T \cdot \mathbf{P}\right), \quad \mathbf{P} \equiv \mathbf{I} - \mathbf{n}\otimes\mathbf{n}$$

where λ_s and μ_s are the surface Lamé moduli, tr is the trace operator, ∇_s is the surface nabla operator, $\mathbf{P}$ is the surface unit second-order tensor, $\mathbf{n}$ is the unit vector of outer normal. For a free surface the static boundary condition takes the following form

$$\mathbf{n}\cdot\boldsymbol{\sigma} = \nabla_s \cdot \mathbf{s}, \quad \mathbf{s} \equiv \frac{\partial \mathcal{W}_s}{\partial \boldsymbol{\epsilon}} = \mu_s \boldsymbol{\epsilon} + \lambda_s \mathbf{P}(\mathrm{tr}\boldsymbol{\epsilon}).$$

Here $\mathbf{s}$ is the surface stress tensor.

4. Screw dislocation

In order to compare the both theories we consider the hollow circular cylinder of radius a with a screw dislocation. It is known that the strain-gradient elasticity and surface stresses affect the singularity near defects. Using the semi-inverse approach the deformation of a cylinder with a screw dislocation is given as a mapping [12]

$$r = r(R), \quad \phi = \Phi, \quad z = \frac{b}{2\pi}\phi + Z, \tag{5}$$

where r, ϕ, z and R, Φ, Z are the polar coordinates in the actual and reference placements, respectively, and b is the magnitude of the Burgers vector. For small deformations mapping (5) gives an example of an antisymmetric deformations such as in [4]. We discuss the solutions behaviour for $a \to 0$. We demonstrate that the both theories give similar qualitative results for the displacement amplitudes. Nevertheless, there are some quantitative differences which will be discussed during SOLMECH2018 in all details.

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