PROPAGATION OF SURFACE WAVES IN FCC HALF-SPACES WITHIN SURFACE ELASTICITY

C. Enzevaee[1] and H.M. Shodja[1,2]

[1]*Department of Civil Engineering, Sharif University of Technology, P.O. Box 11155-9313, Tehran, Iran*
[2]*Institute for Nanoscience and Nanotechnology, Sharif University of Technology, P.O. Box 11155-9161, Tehran, Iran*

e-mail: cenzevaee@gmail.com

1. General

Surface wave propagation in solids with semi-infinite extent due to its broad encountering in such field of works as surface science, seismology, acoustoelectronics, and nondestructive evaluations has been of great interest to many researchers and engineers. The inadequacy of classical theory of elasticity in capturing all the propagating waves under certain situations and describing such a phenomenon as dispersion associated to a propagating wave with wavelength comparable to the intrinsic length of the medium of interest is well-known. A remedy to such dilemma is to employ such augmented theories as surface theory of elasticity which has been proposed by [2]. Using this theory, an analytical solution is presented for Love and Rayleigh surface wave propagations in a semi-infinite medium with face-centered cubic (fcc) single crystal structure. The anisotropic nature of the fcc half-spaces along its free surface and within its bulk are incorporated in the formulations accurately [1]. To show the effect of the crystallographic anisotropy and its orientation, two different crystallographic orientations with respect to the incident wave vector are examined: (a) the free surface of the half-space is (010) plane and the surface waves propagate in [100] direction as shown in Fig.1, (b) the free surface of the half-space is (110) plane and the surface waves propagate in [110] direction as shown in Fig.2.

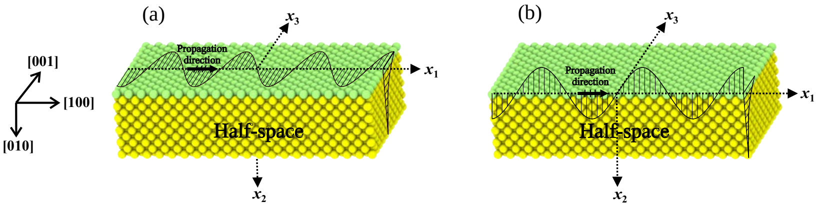

Figure 1: (a) Love and (b) Rayleigh Surface wave propagation in a half-space with (010) free surface.

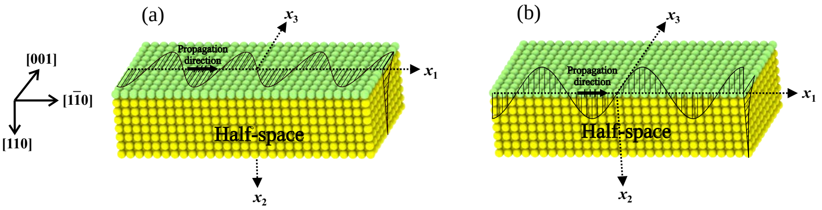

Figure 2: (a) Love and (b) Rayleigh Surface wave propagation in a half-space with (110) free surface.

2. Formulation

In the mathematical framework of surface elasticity presented by [2], surface is considered as a two-dimensional linearly elastic layer, governed by the following equation of motion:

$$\text{div}_{s0}\mathbf{S}^s - \mathbf{S}.\mathbf{n} = \rho^s \ddot{\mathbf{u}}, \tag{1}$$

where $\mathbf{S}^s$ is the first Piola-Kirchhoff surface stress tensor, div_{s0} is the divergence with respect to the undeformed surface, $\mathbf{n}$ is the outward unit normal vector to the free surface, $\mathbf{S}$ is the continuous extension to the surface of the stress tensor in the bulk. Moreover, the constitutive relation for the surface is:

$$\mathbf{S}^s = \mathbf{I}.\mathbf{M} + \mathbf{I}.\mathbf{C}^s.\mathbf{E}^s + \left(\boldsymbol{\nabla}_{s0}\mathbf{u}\right).\mathbf{M}, \tag{2}$$

where $\mathbf{I}$ is the inclusion map from the tangent space of the surface (2D) onto the 3D space, $\mathbf{M}$ is the residual stress tensor, $\boldsymbol{\nabla}_{s0}\mathbf{u}$ is the surface gradient of the displacement field, and $\mathbf{E}^s = \frac{1}{2}\left(\mathbf{Du} + \mathbf{Du}^T\right)$ is the surface strain tensor, and $\mathbf{Du} = \mathbf{P}.\left(\boldsymbol{\nabla}_{s0}\mathbf{u}\right)$ where $\mathbf{P} = \mathbf{I}^T$ is the perpendicular projection matrix from the 3D space onto the tangent space of the surface.

3. Numerical results

Contrary to the predictions based on classical theory, surface elasticity theory predicts Love waves can propagate in a homogeneous elastic fcc half-space and is dispersive which is depicted in 3(a) for Al, Cu, and Ni, and 3(b) for Ag, Au, and Pd. It can be seen that Love wave propagation in Ag, Al, Au, Cu, Ni, and Pd half-spaces associated to either free surface layers, (010) and (110) is normal dispersive.

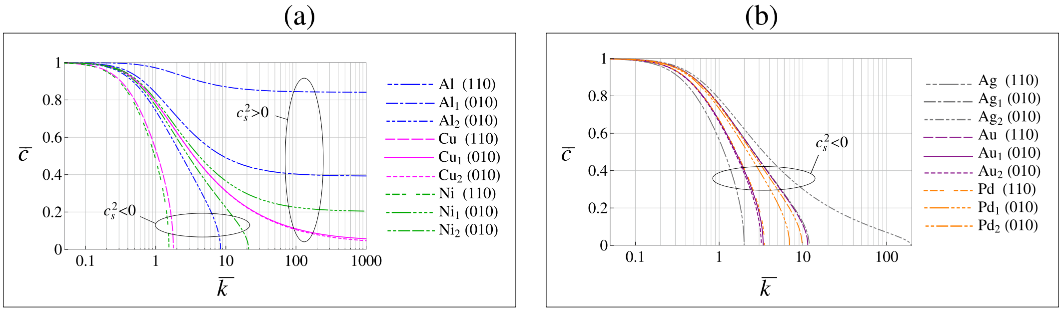

Figure 3: Variation of the normalized Love wave phase velocity, $\bar{c}$ with the normalized wave number $\bar{k}$, for several half-spaces made of different fcc single crystals, (a) Al, Cu, and Ni, and (b)Ag, Au, and Pd.

Rayleigh wave propagation in a homogeneous single crystalline fcc half-space is shown to be dispersive and its propagation in Al half-space with (110) plane as its free surface is anomalous dispersive, whereas in Ag, Au, Cu, Ni, and Pd half-space is normal dispersive.

References

[1] C Enzevaee and HM Shodja. Crystallography and surface effects on the propagation of love and rayleigh surface waves in fcc semi-infinite solids. *International Journal of Solids and Structures*, 2018.

[2] Morton E Gurtin and A Ian Murdoch. A continuum theory of elastic material surfaces. *Archive for Rational Mechanics and Analysis*, 57(4):291–323, 1975.