APPLICATION OF THE FUZZY LATTICE BOLTZMANN METHOD FOR A NUMERICAL MODELLING OF 2D THIN METAL FILMS IRRADIATED BY ULTRASHORT LASER PULSES

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1. Introduction

In the paper, the numerical modelling of heat transfer in two-dimensional metal films is considered. The fuzzy coupled lattice Boltzmann equations for electrons and phonons supplemented by the adequate boundary and initial conditions have been applied to analyse the thermal process proceeding in a thin metal film. The two-dimensional 9-speed model (D2Q9) with fuzzy trapezoidal values of relaxation times and boundary conditions is proposed. The problem considered is solved by the fuzzy lattice Boltzmann method using α -cuts and the rules of directed interval arithmetic [1]. The application of α -cuts allows one to avoid complicated arithmetical operations in the set of fuzzy numbers. In the final part of the paper the results of numerical computations are shown.

2. The fuzzy Boltzmann transport equation

The fuzzy Boltzmann transport equations for the 2D coupled model with two kinds of carriers (*e*-electrons and *ph*-phonons) can be written using the following formulas [2]

(1) $$\frac{\partial \tilde{e}_e}{\partial t} + \mathbf{v}_e \cdot \nabla \tilde{e}_e = -\frac{\tilde{e}_e - \tilde{e}_e^0}{\tilde{\tau}_{re}} + \tilde{Q}_e$$

(2) $$\frac{\partial \tilde{e}_{ph}}{\partial t} + \mathbf{v}_{ph} \cdot \nabla \tilde{e}_{ph} = -\frac{\tilde{e}_{ph} - \tilde{e}_{ph}^0}{\tilde{\tau}_{r\,ph}} + \tilde{Q}_{ph}$$

where $\tilde{e}_e, \tilde{e}_{ph}$ are the fuzzy energy densities, $\tilde{e}_e^0, \tilde{e}_{ph}^0$ are the equilibrium fuzzy energy densities, $\mathbf{v}_e, \mathbf{v}_{ph}$ are the frequency-dependent propagation speeds, $\tilde{\tau}_{re}, \tilde{\tau}_{r\,ph}$ are the fuzzy relaxation times, *t* denotes the time and $\tilde{Q}_e, \tilde{Q}_{ph}$ are the fuzzy energy sources related to a unit of volume for electrons and phonons respectively.
The fuzzy values of the electron and phonon energy densities at their equivalent nonequilibrium temperatures are given by the formulas

(4) $$\tilde{e}_e(\tilde{T}_e) = \left(n_e \frac{\pi^2}{2} \frac{k_b^2}{\varepsilon_F} \right) \tilde{T}_e^2$$

(5) $$\tilde{e}_{ph}(\tilde{T}_{ph}) = \left(\frac{9\eta_{ph} k_b}{\Theta_D^3} \int_0^{\Theta_D/\tilde{T}_{ph}} \frac{z^3}{\exp(z) - 1} \mathrm{d}z \right) \tilde{T}_{ph}^4$$

where Θ_D is the Debye temperature of the solid, k_b is the Boltzmann constant, ε_F is the Fermi energy, $\tilde{T}_e, \tilde{T}_{ph}$ are the fuzzy lattice temperatures for electrons and phonons respectively, while n_e is the electron density and η_{ph} is the phonon density.
The fuzzy electron and phonon energy sources are calculated using the following expressions [2]

(6) $$\tilde{Q}_e = Q' - G(\tilde{T}_e - \tilde{T}_{ph}), \quad \tilde{Q}_{ph} = G(\tilde{T}_e - \tilde{T}_{ph})$$

where Q' is the power density deposited by the external source function and G is the electron-phonon coupling factor which characterizes the energy exchange between electrons and phonons. The equations (1) and (2) should be supplemented by the initial and boundary conditions [3]. The temporal variation of laser output pulse is treated as source term in the energy equation and may be approximated by the form of exponential function [3]

$$Q'(x,t) = I_0 \delta e^{-\delta y - \beta t} e^{-\frac{2x^2}{r^2}} \tag{7}$$

where I_0 is the peak power intensity of the laser pulse, δ is the absorption coefficient, β is the laser pulse parameter, r is the radius of the laser beam, x and y are the coordinates.

3. Results of computations

As a numerical example, the heat transport in a gold thin film of the dimensions 1000 nm × 200 nm has been analysed. The following input data have been introduced: $\tilde{q}_{b1} = \tilde{q}_{b2} = \tilde{q}_{b3} = \tilde{0}\,\text{W/m}^2$, $\tilde{T}_{b4} = (285, 292.5, 307.5, 315)\text{K}$, $T_0 = 3$ 0 ($\Delta t = 0.01$ ps, fuzzy relaxation times for phonons $\tilde{\tau}_{ph} = (\tau_{ph} - 0.05\tau_{ph}, \tau_{ph} - 0.025\tau_{ph}, \tau_{ph} + 0.025\tau_{ph}, \tau_{ph} + 0.05\tau_{ph})$ and electrons $\tilde{\tau}_e = (\tau_e - 0.05\tau_e,\ \tau_e - 0.025\tau_e,\ \tau_e + 0.025\tau_e, \tau_e + 0.05\tau_e)$, $r = 160\,\text{nm}$, the other material and laser properties are defined in Table 1.

	τ_{ph} [ps]	τ_e [ps]	Θ_D [K]	$n_e(\times 10^{28})$ [1 / m^3]	ε_F [eV]	$I_0(\times 10^{13})$ [W/m^2]	$\beta(\times 10^{13})$ [1/s]	$\delta(\times 10^{7})$ [1/m]
Au	0.8	0.04	170	5.9	5.53	2	0.5	7.55

Table 1: Material and laser properties.

Figure 1 illustrates the fuzzy electrons heating curves obtained for $\alpha = 0.5$ in the nodes: (100, 80) – 1, (100, 100) – 2 and (100,120) – 3.

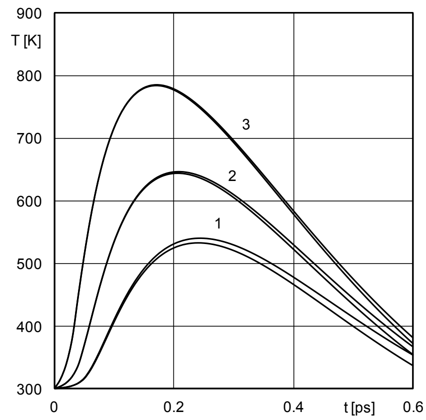

Fig. 1. Fuzzy electrons heating curves in chosen nodes.

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References

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