ISOGONAL AND ISOTOXAL HEXAGONS AS EXTREMAL YIELD FIGURES

H. Altenbach[1], P.L. Rosendahl[2], W. Becker[2], and V.A. Kolupaev[3]

[1] *Otto-von-Guericke-Universität Magdeburg, Universitätsplatz 2, D-39106 Magdeburg, Germany*
[2] *Technische Universität Darmstadt, Franziska-Braun-Straße 7, D-64287 Darmstadt, Germany*
[3] *Fraunhofer Institute for Structural Durability and System Reliability, Schloßgartenstr. 6, D-64289 Darmstadt, Germany*

e-mail: holm.altenbach@ovgu.de

1. Area of Interest

In the theory of plasticity the existence of the yield surface is assumed. In the past, several yield surfaces were formulated. However, choosing an appropriate surface for a particular material remains challenging. The extremal yield figures for isotropic material behavior will be generalized in this paper for universal practical application. Measured data for concrete will be approximated with the help of the proposed criterion.

2. Introduction

Yield criteria for isotropic material behavior are built up using invariants of the symmetric second-rank stress tensor. Three stress invariants – the trace (axiator) I_1 of the stress tensor and the invariants I'_2, I'_3 of the stress deviator expressed by principal stresses [1, 2] – are used here. The stress angle θ

(1) $$\cos 3\theta = \frac{3\sqrt{3}}{2}\frac{I'_3}{(I'_2)^{3/2}}, \qquad \theta \in \left[0,\, \frac{\pi}{3}\right]$$

is sometimes preferred over I'_3 because it allows for a simple geometrical interpretation of the stress state. Yield criteria as function of the equivalent stress σ_{eq} are formulated according to

(2) $$\Phi(I_1,\, I'_2,\, I'_3,\, \sigma_{\text{eq}}) = 0 \qquad \text{or} \qquad \Phi(I_1,\, I'_2,\, \theta,\, \sigma_{\text{eq}}) = 0,$$

where the dependence on I_1 can be neglected for pressure-insensitive materials.

The criteria are subjected to special requirements. Among others, such so-called plausibility assumptions are the explicit resolvability of the criterion with respect to σ_{eq}, wide range of possible convex shapes in the π-plane, continuous differentiability except at the border, and no additional outer contours surrounding the physically reasonable shape of the surface. Fulfilling all of the above yields reliable criteria that are easy to handle. Unfortunately, criteria satisfying all requirements are not known.

3. Extremal Yield Figures

The lower and upper bounds of the convexity restriction for isotropic criteria give extremal yield figures of isotoxal and isogonal hexagons (Figs. 1 and 2). The polynomial formulations of these hexagons are known as CAPURSO and HAYTHORNTHWAITE criterion, respectively [2]. However, their polynomial forms feature intersections surrounding the physically reasonable shape of the surface and their application is intricate.

Isotoxal hexagons as function of stress angle (lower bound) can be described using the PODGÓRSKI criterion [3]. A criterion for isogonal hexagons (upper bound) as function of stress angle without case discrimination is missing. Both hexagons degenerate to the same equilateral triangles in border cases.

The present work aims at finding a universal yield criterion which contains the extremal yield figures and satisfies all plausibility assumptions.

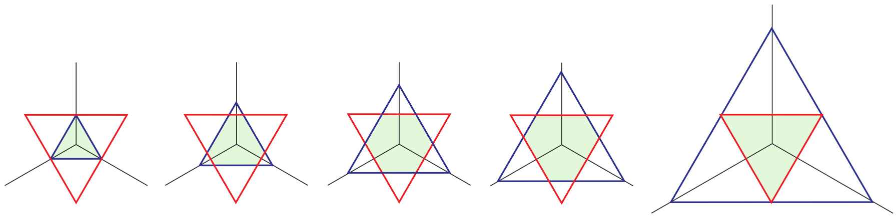

Figure 1: Isogonal hexagons (upper bound) consist of two intersecting triangles in the π-plane.

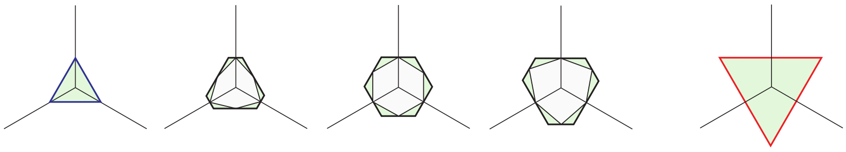

Figure 2: Isotoxal (lower bound) and isogonal hexagons in the π-plane.

4. Universal Yield Criterion

Consider the minimum function $\Omega = \min[\kappa, \lambda]$. In order to avoid case discrimination, the $\min$ operator may be replaced according to [4]:

(3) $$\Omega = \frac{1}{2}\left[\kappa + \lambda + \sqrt{(\kappa - \lambda)^2}\right].$$

Using the IVLEV [2] shape function

(4) $$\kappa(\theta,\, \beta,\, \gamma) = \cos\left[\frac{1}{3}\left(\pi\,\beta - \arccos\left[\sin\left[\gamma\,\frac{\pi}{2}\right]\,\cos[3\,\theta]\right]\right)\right]$$

and the scaled MARIOTTE [2] shape function

(5) $$\lambda(\theta,\, \alpha,\, \beta,\, \gamma) = \frac{1}{2}\,(3\,\alpha + 1)\,\cos\left[\frac{1}{3}\left(\pi\,\beta - \arccos\left[-\sin\left[\gamma\,\frac{\pi}{2}\right]\,\cos[3\,\theta]\right]\right)\right],$$

we obtain the universal yield function

(6) $$\sigma_{\mathrm{eq}} = \sqrt{3\,I_2}\,\frac{\Omega(\theta,\, \alpha,\, \beta,\, \gamma)}{\Omega(0,\, \alpha,\, \beta,\, \gamma)},$$

normalized with respect to the unidirectional tensile stress $\sigma_{\mathrm{eq}} = \sigma_+$. The parameters are restricted as follows:

(7) $$\alpha \in [0, 1], \qquad \beta \in [0, 1], \qquad \gamma \in [0, 1].$$

This yield function (6) contains the HAYTHORNTHWAITE criterion (isogonal hexagons) with $\alpha \in [0, 1]$, $\beta = 0$, and $\gamma = 1$, the CAPURSO criterion (isotoxal hexagons) with $\alpha \in \{0, 1\}$, $\beta \in [0, 1]$, and $\gamma = 1$, and the regular dodecagon in the π-plane according to SOKOLOVSKY [2] with $\alpha = 1/3$, $\beta = 1/6$, and $\gamma = 1$ without plane intersecting. The VON MISES criterion follows with $\alpha \in [0, 1]$, $\beta \in [0, 1]$, and $\gamma = 0$.

References

[1] H. Altenbach, J. Altenbach, and A. Zolochevsky. *Erweiterte Deformationsmodelle und Versagenskriterien der Werkstoffmechanik*. Deutscher Verlag für Grundstoffindustrie, Stuttgart, 1995.
[2] V. A. Kolupaev. *Equivalent Stress Concept for Limit State Analysis*. Springer, Cham, 2018.
[3] J. Podgórski. General failure criterion for isotropic media. *J. of Engineering Mechanics*, 111(2):188–201, 1985.
[4] H. Walser. Isogonal polygons (in German: Isogonale Vielecke). http://www.walser-h-m.ch, Frauenfeld, 2018.