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ON A SYSTEM OF QUADRIC SURFACES.

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THE following theorem was communicated to me by Dr Klein; "given in regard to a quadric surface two sibi-reciprocal line-pairs, the two tractors (or lines meeting each of the four lines) form a sibi-reciprocal line-pair." This may be presented under a more general form as a theorem relating to the tractors of any two line-pairs. In fact, if a given line-pair is taken to be sibi-reciprocal in regard to a quadric surface, we thereby establish only a four-fold relation between the coefficients of the surface, and the surface will still depend on five arbitrary parameters. Whence if two given line-pairs are taken to be each of them sibi-reciprocal in regard to one and the same quadric surface, we thereby establish only an eight-fold relation and the surface will still depend upon one arbitrary parameter. The theorem thus is: given any two line-pairs, then each of these, and also the pair of tractors, are sibi-reciprocal in regard to a singly infinite system of quadric surfaces.

The question arises, what is this system of quadric surfaces? It is, in fact, the system of surfaces having in common a skew quadrilateral constructed as follows: starting from the two given line-pairs, construct the two tractors, each of them intersected by the given line-pairs in two point-pairs; and on each tractor construct the double or sibi-reciprocal points of the involution thus determined; these double points are the vertices (those on the same tractor being opposite vertices) of the skew quadrilateral; which is consequently at once obtained by joining the two double points on the one tractor with the two double points on the other tractor. The construction is an immediate consequence of the following theorem: consider a skew quadrilateral, and drawing its two diagonals, take a pair of lines cutting each diagonal harmonically; these will be sibi-reciprocal in regard to any quadric surface through the skew quadrilateral.

The condition of passing through a skew quadrilateral is that of passing through a certain system of eight points; in fact, the eight points may be taken to be the four vertices and any four points on the four sides respectively. But observe that the system of the quadric surfaces through *any* eight points has the characteristics (1, 2, 3); viz. there are in the system 1 surface passing through a given point, 2 touching a given line, 3 touching a given plane; the system of surfaces through the same skew quadrilateral has the characteristics (1, 2, 1).