672.

ON THE GAME OF MOUSETRAP.

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In the note "A Problem in Permutations," *Quarterly Mathematical Journal,* t. I. (1857), p. 79, [161], I have spoken of the problem of permutations presented by this game.

A set of cards—ace, two, three, &c., say up to thirteen—are arranged (in any order) in a circle with their faces upwards; you begin at any card, and count one, two, three, &c., and if upon counting, suppose the number five, you arrive at the card five, the card is thrown out; and beginning again with the next card, you count one, two, three, &c., throwing out (if the case happen) a new card as before, and so on until you have counted up to thirteen, without coming to a card which has to be thrown out. The original question proposed was: for any given number of cards to find the arrangement (if any) which would throw out all the cards in a given order; but (instead of this) we may consider *all* the different arrangements of the cards, and inquire how many of these there are in which all or any given smaller number of the cards will be thrown out; and (in the several cases) in what orders the cards are thrown out. Thus to take the simple case of four cards, the different arrangements, with the cards thrown out in each, are

1, 2, 3, 4	1,
1, 2, 4, 3	1, 3, 4, 2,
1, 3, 2, 4	1,
1, 3, 4, 2	1,
1, 4, 2, 3	1, 2, 3, 4,
1, 4, 3, 2	1,
2, 1, 3, 4	3, 4,
2, 1, 4, 3	—
2, 3, 4, 1	—
2, 3, 1, 4	4,
2, 4, 1, 3	—
2, 4, 3, 1	3, 2,
3, 1, 2, 4	4,
3, 1, 4, 2	—
3, 2, 1, 4	4, 2, 1, 3,
3, 2, 4, 1	2, 3,
3, 4, 1, 2	—
3, 4, 2, 1	—
4, 1, 2, 3	—
4, 1, 3, 2	3,
4, 2, 1, 3	2, 1, 3, 4,
4, 2, 3, 1	3, 1, 2, 4,
4, 3, 1, 2	—
4, 3, 2, 1	—.

Classifying these so as to show in how many arrangements a given card or permutation of cards is thrown out, we have the table

No.	Thrown out.
9	none
4	1
1	3
2	4
1	3, 2
1	2, 3
1	3, 4
1	1, 3, 4, 2
1	1, 2, 3, 4
1	4, 2, 1, 3
1	2, 1, 3, 4
1	3, 1, 2, 4,
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viz. there are nine arrangements in which no card is thrown out, four arrangements in which only the card 1 is thrown out, one arrangement in which only the card 3 is thrown out, and so on.

It will be observed that there are five arrangements in which all the cards are thrown out, each throwing them out in a different order; there are thus only five orders in which all the cards are thrown out.

The general question is of course to form a like table for the numbers 5, 6, ..., or any greater number of cards.