639.

AN ELEMENTARY CONSTRUCTION IN OPTICS.

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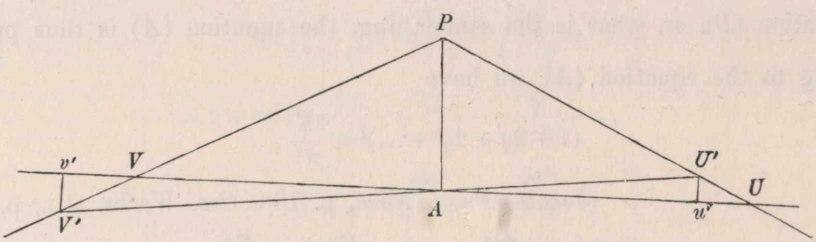
CONSIDER two lines meeting at a point P, and a point A; through A, draw at right angles to AP, a line meeting the two lines in the points U, V respectively; and through the same point A draw any other line meeting the two lines in the



points U', V' respectively; also let the points u', v' be the feet of the perpendiculars let fall from U', V' respectively on the line UV; then we have

$$\frac{1}{Au'}+\frac{1}{Av'}=\frac{1}{AU}+\frac{1}{AV}.$$

The theorem can be proved at once without any difficulty. It answers to the optical construction, according to which, if UPV represents the path of a ray through a convex lens AP, then the thin pencil, axis $U'P$ and centre U', converges after refraction to the point V', where $U'V'$ are *in lineâ* with A the centre of the lens; considering as usual the inclinations to the axis as small, we have approximately $AV'=Av'$, $AU'=Au'$, and the theorem is

$$\frac{1}{AU'}+\frac{1}{AV'}=\frac{1}{AU}+\frac{1}{AV},\ =\frac{1}{AF},$$

if AF is the focal length of the lens.

In the original theorem, the line UV need not be at right angles to AP, but may be any line whatever; the projecting lines $U'u'$ and $V'v'$ must then be parallel to AP, and the theorem remains true.