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Compliance and emission trading under Kyoto Protocol: Rules for uncertain inventories *

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Abstract. A solution for compliance proving and emission trading in case of uncertainties in reported emission inventories is proposed. It is based on the undershooting concept, from which both mathematical conditions for proving compliance with a risk α, and for calculation of effective emission for trading is derived. With respect to the reported emission unit the effective permit is reduced proportionally to the inventory uncertainty measure. This way a country with higher uncertainty of its inventory is allotted less of effective permits than another country with the same inventory but smaller uncertainty.

Keywords: greenhouse gas inventory uncertainty, compliance with Kyoto Protocol, risk of noncompliance, undershooting, emission trading, effective tradable permits.

1. Introduction

Uncertainty in the greenhouse gas (GHG) inventories has been estimated to be in the range 5-20%, depending on the scope and methodology used (Rypdal, 2001; Monni, 2004a). Even if some of the computations need unification of assumptions and possibly recalculation, the uncertainty is still believed to be about 10-12% or more for most countries (Winiwarter, 2004) and therefore is typically larger than the reduction commitments. Thus, the uncertainty seems to become a big problem both in the compliance proving and in implementation of the flexible mechanisms: emission trading, joint implementation and clean development mechanism.

Uncertainty varies between parties taking part in the Kyoto agreement. They also vary considerably between different emission activities. Having this in mind one can think of better or poorer quality inventories or more or less credible reduction of the GHG emissions. When dealing with the flexible mechanisms, better or poorer quality goods are offered for sale or exchange. Should they be treated on the equal basis? Without explicit rules of maintaining this problem it is rather

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doubtful that it will be solved by the market itself. And leaving this problem unsolved may undermine credibility of the whole reduction process.

This problem found rather inadequate attention in the literature. Assessments of uncertainty were done and compared for several countries, see e. g. (Amstel, 2000; Charles, 1998; Gawin, 2002; Jonas, 2001; Monni, 2004b; Nilsson, 2000; Rypdal, 2001; Rypdal, 2000), see also (Gugele, 2003). Some as far vague considerations on excluding most uncertain activities from the emission trading were mentioned (Victor, 1991; Monni, 2004a). In (Godal, 2000; Godal, 2003a) undershooting as the basis for proving compliance was proposed. Similar ideas were formulated in (Gupta, 2003) and (Gillenwater, 2005). Especially the latter one presents a solution close to those of the present paper. A review of other methods, particularly related to detectability of emission changes, can be found in (Jonas, 2004a; Jonas, 2004b). Also the argumentation in this paper uses undershooting concept as far as the compliance proving is concerned. But in contrast to the earlier papers we consider uncertainty in both the commitment and the basic years as contributing to the overall uncertainty when comparing the involved emission reduction. Our proposition starts with setting this shifted down value to maintain some predefined risk that the real (unknown) emission may happen not to satisfy the reduction obligation. This may be also interpreted that apart of observed (calculated in the inventory) greenhouse gas emission also some unobserved emission, proportional to the inventory uncertainty measure, is added to the inventory before checking compliance with the committed obligations. This approach allows us to treat in a similar way uncertainty of different types, like interval or stochastic ones. To avoid bigger changes in the reduction level connected with undershooting, we propose to appropriately adjust (shift) the reference obligation levels for each party, taking into account the difference between its own uncertainty and an arbitrarily chosen reference level of uncertainty.

The idea of permit trading has been established in order to contribute to achieving environmental goals (Montgomery, 1972). It rests on the heterogeneity of emission reduction costs among the market participants, including differences in technology, experience, as well as availability of natural resources, etc. Our aim is to explicitly include in it also the inventory uncertainty. In the long run this would stimulate further improvements in the field.

Thus, the compliance proving rule proposed in the paper is a starting point for reevaluation of the traded units of emissions. This is done by assuming that the uncertainty of the purchased emissions contributes to the buyer's overall uncertainty. Big uncertainty in the sold emissions increases the uncertainty of the buyer's emission balance and therefore these emissions must be of smaller value for the buyer.

This idea is transfered to definition of an emission permit under observation uncertainty. The proposed effective emission permit includes uncertainty in the following way: a party with a big inventory uncertainty is allocated less emission permits than a party with the same emission and a smaller uncertainty. The effective permits are subject to ordinary trading, as in the case of permits with exact knowledge of emissions.

An idea of changing the trading rules due to the different uncertainties in reported emissions of the trading parties appears also in (Gillenwater, 2005). The starting point is, however, different. Preservation of some common probabilistic characteristics of the trading parties is required there. In a consequence our solution features quite different properties.

In the paper we assume either a deterministic interval distribution of the uncertainty or a stochastic one. Solutions for proving the compliance are provided for both cases, with a given risk. However, nonlinearities inherent in the algebra for the stochastic case did not allow us to fully design the market rules for the emission permits. Therefore, only a solution for the interval type uncertainty has been provided. Recent findings from the Monte Carlo analysis, (Vreuls, 2004; Winiwarter, 2004), indicate that the distribution of uncertainty resembles well that of the stochastic normal distribution. This strongly suggests that normal distribution should be considered in derivations. The choice of the distribution is not only of a theoretical question. The procedure proposed finally results in valuation of the uncertainty and the values obtained depend on choice of the uncertainty distribution. Free parameters, and specifically the risk taken, can help in solving this question.

A preliminary proposition of the above solution was presented at the workshop held in IIASA (Nahorski, 2002). A more elaborated ideas were presented at a conference in Poland (Nahorski, 2003), and their simplified presentation in the IIASA Interim Report (Jonas, 2004a). This paper is an extended and revised version of the papers presented at the Workshop on *Uncertainty in Greenhouse Gas Inventories: Verification, Compliance & Trading* held in Warsaw, Poland in September 2004 (Horabik, 2004; Nahorski, 2004). Apart of smaller amendments, this paper contains treatment of stochastic uncertainty and a proposition to include in the theory dependencies of inventories prepared in the basic and commitment years.

2. Notation and problem formulation

By $x(t)$ we denote the real, unknown emission of a party in a year t. It can be only estimated, basically through emission inventory. Let $\hat{x}(t)$ denote the best available estimate of $x(t)$. This estimate is subject to estimation error connected with inventory uncertainty. Mainly interval type uncertainty will be discussed here, while stochastic type will be only presented in a limited scope.

Table I. Some available uncertainty estimates, in [%].

Country	δ	Level uncert.	Trend uncert.	GHGs	LUCF*	Ref.
AT	8	12	7.5	CO_2	included	(Winiwarter, 2001)
		9.8	5.1	CH_4	excluded	
		15		N_2O	included	(Jonas, 2001)
		7.5			excluded	
FI	8	6	5	as AT	excluded	(Monni, 2004b)
NL	8	4.4		all**	included	(Amstel, 2000)
NO	-1	21		all**	excluded	(Rypdal, 2000)
PL	6	6	3.8	as AT	included	(Gawin, 2002)
RU	0	17		CO_2		(Nilsson, 2000)
UK	8	19		all**	excluded	(Charles, 1998)

* – Stands for Land Use Change and Forestry.

** – All gases as mentioned in Annex I to the Kyoto Protocol: CO_2, CH_4, N_2O, HFCs, PFCs, SF_6

Source: (Jonas, 2004a), modified.

By δ we denote the fraction of a party emission that is to be reduced in the commitment year(s) according to the obligation. The value of δ may be negative for parties, which were alloted limitation of the emission increase. Let us also denote by t_b the basic year and by t_c the commitment year. To simplify formulae we introduce the short notation $x_b = x(t_b)$ and $x_c = x(t_c)$. Now, the following inequality should be satisfied to prove the compliance

$$x_c - (1 - \delta)x_b \leq 0 \tag{1}$$

The problem arises because neither x_c nor x_b are known precisely enough. Instead, only the difference of estimates can be calculated

$$\hat{x}_c - (1-\delta)\hat{x}_b \tag{2}$$

where both $\hat{x}_c$ and $\hat{x}_b$ are known with intolerable low accuracy. Examples of uncertainty values are shown in Table I.

3. Compliance proving

The intuition behind the method developed in this section is that given the GHG inventory we only know that the true emission is within some interval around it, due to uncertainty. Thus, to be more secure that the requested limit has been actually reached, the reported emission has to be appropriately smaller, in dependence on an imposed confidence. This is called *undershooting*. The value α corresponding to the one-tailed significance level in the stochastic framework is called here *a risk*. As the Kyoto Protocol concentrates on the difference of emissions in the basic and commitment years, we adopt the above reasoning to the difference (2). Moreover, we consider not only the stochastic type of uncertainties but also the interval one where knowledge of the real emission is limited to a symmetric interval around the reported value containing the real value.

3.1. Interval type uncertainty

Assuming that the uncertainty intervals at the basic and the commitment years are $\pm\Delta_b$ and $\pm\Delta_c$, respectively, we have

$$x_b \in [\hat{x}_b - \Delta_b, \hat{x}_b + \Delta_b], \qquad x_c \in [\hat{x}_c - \Delta_c, \hat{x}_c + \Delta_c]$$

from where, using the interval calculus rules, we get

$$x_c - (1-\delta)x_b \in [D\hat{x} - \Delta_{bc}, D\hat{x} + \Delta_{bc}] \tag{3}$$

where

$$D\hat{x} = \hat{x}_c - (1-\delta)\hat{x}_b \tag{4}$$

and

$$\Delta_{bc} = \Delta_c + (1-\delta)\Delta_b \tag{5}$$

However, big part of uncertainty is related to the method of calculation itself. This refers particularly to the coefficients in formulae, which are often known with rather low accuracy, for example from expert judgment. This kind of uncertainty is present in inventories prepared in both basic and commitment years, due to the same calculation method. It causes that the uncertainties of both inventories are dependent and the uncertainty of the difference is actually smaller than that obtained from (5). This is why, besides the so called total or level uncertainties Δ_b and Δ_c, sometimes also the trend uncertainty Δ_{bc} is computed independently (Rypdal, 2001).

This kind of dependence between variables has not been considered, up to the present authors knowledge, in the interval calculus theory. Correlation of variables has been discussed within the fuzzy set theory. Although the fuzzy sets inherit interval calculus rules, most of the correlation coefficient notions formulated there reduce to the trivial 0 or 1 values in the interval case. An interesting definition of the correlation coefficient, with meaningful interpretation in the interval case, is given in (Hung, 2001). However, it (as others) lacks development of the relevant calculus for the correlated variables.

We propose to model the dependence between x_b and x_c by subtracting from (5) an interval Δ

$$\Delta_{bc} = \Delta_c + (1-\delta)\Delta_b - \Delta \tag{6}$$

The interval Δ may be structured by imagining that it contains a part of uncertainty included in Δ_b and Δ_c, i. e. we assume that it is of the form $\Delta = \xi\Delta_c + \zeta(1-\delta)\Delta_b$, giving

$$\Delta_{bc} = (1-\xi)\Delta_c + (1-\zeta)(1-\delta)\Delta_b \tag{7}$$

It can be difficult to identify both parameters ξ and ζ in (7). It may be then useful to assume $\xi = \zeta$ to obtain

$$\Delta_{bc} = (1-\zeta)[\Delta_c + (1-\delta)\Delta_b] \tag{8}$$

Calculation for few values from the data presented in the literature ((Gawin, 2002; Winiwarter, 2001)) gives $\zeta \sim 0.65 \div 0.7$. So, the dependence of inventories is quite high. It is perhaps worth to mention that 5% trend uncertainty is suggested as a frequent value in (Monni, 2004b). This claim is, however, also based only on few calculated cases.

To be fully credible, that is to be sure that (1) is satisfied even in the worst case, the party should prove $D\hat{x} + \Delta_{bc} \leq 0$, see Fig. 1. Our proposition is to admit for some agreed chance of not satisfying the obligations. In other wording, we want to take risk not greater then α ($0 \leq \alpha \leq 0.5$) that the reduction in the commitment year t_c is not fulfilled. We then say that the party *proves the compliance with risk* α if $D\hat{x} + \Delta_{bc} \leq 2\alpha\Delta_{bc}$, see Fig. 1 for the geometrical interpretation. The lower bound $\alpha = 0$ corresponds to the inclusion of half of the uncertainty interval (full credibility). The value $\alpha = 0.5$ corresponds to ignoring completely the uncertainty. The parameter α is to be set beforehand, common for all market participants. After simple algebraic manipulations we obtain from the above definition the condition

$$\hat{x}_c \leq (1-\delta)\hat{x}_b - (1-2\alpha)\Delta_{bc} \tag{9}$$

Thus, to prove the compliance with risk α the party has to undershoot its obligation with the value $(1-2\alpha)\Delta_{bc}$, dependent on the uncertainty measure Δ_{bc}.

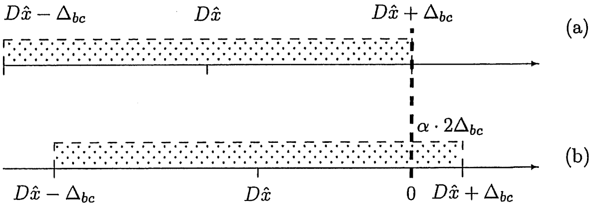

Figure 1. Full compliance (a) and the compliance with risk α (b) in the interval uncertainty approach.

Alternatively, the condition (9) can be written as $\hat{x}_c + (1-2\alpha)\Delta_{bc} \leq (1-\delta)\hat{x}_b$, with interpretation of correcting upward the emission estimate $\hat{x}_c$, as adopted e. g. in (Gillenwater, 2005).

The condition (9) can be also rewritten as

$$\hat{x}_c \leq [1-\delta-(1-2\alpha)R_{bc}]\hat{x}_b \tag{10}$$

where

$$R_{bc} = \frac{\Delta_{bc}}{\hat{x}_b}$$

is the half relative uncertainty interval with respect to the reported emission in the basic year $\hat{x}_b$. It is seen from (10) that the compliance with risk α induces redefinition of the reduction fraction

$$\delta \quad \rightarrow \quad \delta_{Ui} = \delta + (1-2\alpha)R_{bc} \tag{11}$$

Analogously to the definition of R_{bc} we define

$$R_b = \frac{\Delta_b}{\hat{x}_b} \qquad R_c = \frac{\Delta_c}{\hat{x}_c}$$

3.2. Stochastic type uncertainty

Let us assume now that $\hat{x}(t)$ is normally distributed with the mean $E[\hat{x}(t)] = x(t)$ and variance $var[\hat{x}(t)] = \sigma^2$, with obvious notations σ_b^2 and σ_c^2 in the years $t = t_b$ and $t = t_c$, respectively. Wider class of distributions can be considered but it is out of scope of this paper. The variable $\hat{x}_c - (1-\delta)\hat{x}_b$ is then normal with the mean $x_c - (1-\delta)x_b$ and the variance

$$\sigma_{bc}^2 = (1-\delta)^2\sigma_b^2 - 2(1-\delta)\rho_{bc}\sigma_b\sigma_c + \sigma_c^2 \tag{12}$$

where ρ_{bc} is the correlation coefficient of $\hat{x}_b$ and $\hat{x}_c$. Calculation for few cases provides the value $\rho_{bc} \sim 0.8$. Again it is seen that the correlation is high.

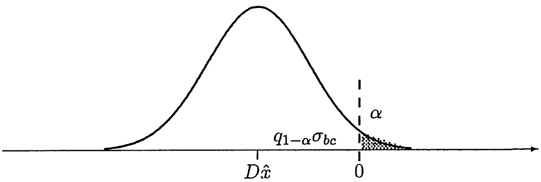

Figure 2. Compliance with risk α in the stochastic approach.

We require that the probability of noncompliance is not higher than α

$$\mathcal{P}\{\frac{(1-\delta)\hat{x}_b - \hat{x}_c - (1-\delta)x_b + x_c}{\sigma_{bc}} \geq q_{1-\alpha}\} \leq \alpha$$

where $q_{1-\alpha}$ is the $(1-\alpha)$th quantile of the standard normal distribution. This provides the condition

$$\hat{x}_c \leq (1-\delta)\hat{x}_b - (1-\delta)x_b + x_c - q_{1-\alpha}\sigma_{bc} \tag{13}$$

If $x_c > (1-\delta)x_b$, then (13) follows from

$$\hat{x}_c \leq (1-\delta)\hat{x}_b - q_{1-\alpha}\sigma_{bc} \tag{14}$$

If $x_c < (1-\delta)x_b$, then the committed obligation is fulfilled anyway. Thus, we conclude that fulfillment of (14) is sufficient for proving compliance with risk α in the stochastic approach. A sketch in Fig. 2 shows analogy of the stochastic and the interval approaches.

Condition (14) can be also written as

$$\hat{x}_c \leq [1 - \delta - q_{1-\alpha}R_{bc}]\hat{x}_b$$

Table II. Recalculated reduction commitments δ_{Ui}, in [%].

Country	δ	R_b	R_{bc}	δ_{Ui} interval $\alpha = 0.1$	δ_{Ui} interval $\alpha = 0.3$	δ_{Ui} stochastic $\alpha = 0.1$	δ_{Ui} stochastic $\alpha = 0.3$
AT	8	12	7.5	14.0	11.0	12.8	10.0
		9.8	5.1	12.1	10.0	11.3	9.3
		15	10*	16.0	12.0	14.4	10.6
		7.5	4.7*	11.8	9.9	11.0	9.2
FI	8	6	5	12.0	10.0	11.2	9.3
NL	8	4.4	3*	10.4	9.2	9.9	8.8
NO	-1	21	14.7*	10.8	4.9	8.5	2.9
PL	6	6	3.8	9.0	7.5	8.4	7.0
RU	0	17	11.9*	9.5	4.8	7.6	3.1
UK	8	19	12.8*	18.2	13.1	16.2	11.4

* – estimated using $\zeta = 0.7$

where

$$R_{bc} = \frac{\sigma_{bc}}{\hat{x}_b}$$

This case induces redefinition of the reduction fraction according to the following scheme

$$\delta \quad \longrightarrow \quad \delta_{Us} = \delta + q_{1-\alpha} R_{bc} \tag{15}$$

Similarly we also define

$$R_b = \frac{\sigma_b}{\hat{x}_b} \qquad R_c = \frac{\sigma_c}{\hat{x}_c}$$

Comparison of few recalculated values of reduction commitments for the interval and stochastic case and for two values of αs are presented in Table II. For those countries where R_{bc}s were not available, estimates with $\zeta = 0.7$ have been used. For $\alpha = 0.3$ and the stochastic case the shifts are not so big, even less than 1% for the smallest uncertainty and around 4% for the biggest one. For $\alpha = 0.1$ and the interval case they are much bigger, reaching almost 12% in the worst case.

4. Adjustment of the basic committed level

A critique of the undershooting concept may be connected with increase of the required reduction of the reported emission caused by additional expression dependent on uncertainty. This way more than the agreed 5.2% estimated reduction would occur. This excess reduction can be corrected by appropriately shifting the reference reduction level. The idea presented here is to compare the uncertainty distributions with a reference one, which satisfies the original committed obligation and has a chosen uncertainty measure. More specifically, we require that both the reference distribution and the distribution of a party considered have the same upper $(1-\alpha)$th limits of their uncertainty intervals, see Figs. 3 and 4. Having established this interdependencies, the reduction fraction δ_{Ui} for all countries are adjusted (decreased) with the reference reduction fraction. The adjustment leaves the differences in commitment levels obtained from the undershooting but shifts them close to the original Kyoto values. In particular, it may be required that 5.2% total reported reduction is preserved.

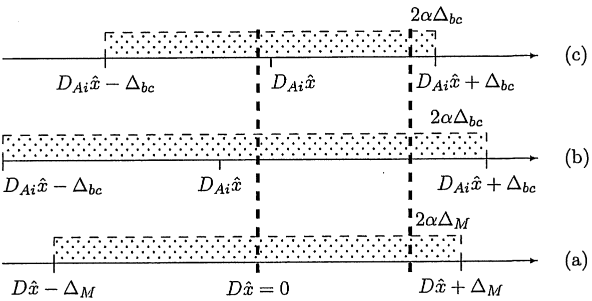

Figure 3. Adjustment of the committed level in the interval uncertainty approach, (a) reference model, (b) $\Delta_{bc} > \Delta_M$, (c) $\Delta_{bc} < \Delta_M$. $D_{Ai}\hat{x} = \hat{x}_c - (1-\delta_{Ai})\hat{x}_b$.

4.1. Interval type uncertainty

We assume that the reference distribution satisfies exactly the committed reduction level and therefore its reduction fraction is δ. At its upper limit of the $(1-\alpha)$th uncertainty interval it holds $\hat{x}_c = (1-\delta)\hat{x}_b + (1-2\alpha)\Delta_M$, where Δ_M is a chosen half reference interval. Similarly, for the same upper limit of the party with the adjusted committed fraction δ_{Ai} we have $\hat{x}_c = (1-\delta_{Ai})\hat{x}_b + (1-2\alpha)\Delta_{bc}$. As both these upper limits have to be equal we get the equation, see also Fig. 3

$$(1-\delta_{Ai})\hat{x}_b + (1-2\alpha)\Delta_{bc} = (1-\delta)\hat{x}_b + (1-2\alpha)\Delta_M \tag{16}$$

This can be also written as

$$[1-\delta_{Ai} + (1-2\alpha)R_{bc}]\hat{x}_b = [1-\delta + (1-2\alpha)(R_M - R_{bc}) + (1-2\alpha)R_{bc}]\hat{x}_b$$

where $R_M = \Delta_M/\hat{x}_b$. This yields the following relationship for the redefinition of the reduction fraction

$$\delta \quad \rightarrow \quad \delta_{Ai} = \delta - (1-2\alpha)(R_M - R_{bc}) \tag{17}$$

The reduction fraction δ_{Ai} is smaller than δ_{Ui}, as the difference is

$$\delta_{Ui} - \delta_{Ai} = (1-2\alpha)R_M$$

4.2. Stochastic type uncertainty

Likewise, for the stochastic approach we get, see Fig. 4

$$(1-\delta_{As})\hat{x}_b + q_{1-\alpha}\sigma_{bc} = (1-\delta)\hat{x}_b + q_{1-\alpha}\sigma_M$$

where σ_M is a chosen reference standard deviation. Finally

$$\delta \quad \rightarrow \quad \delta_{As} = \delta - q_{1-\alpha}(R_M - R_{bc}) \tag{18}$$

where $R_M = \sigma_M/\hat{x}_b$.

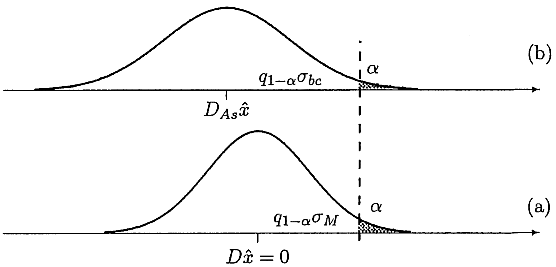

Figure 4. Adjustment of the committed level in the stochastic approach: (a) reference model, (b) $\sigma_{bc} > \sigma_M$. $D_{As}\hat{x} = \hat{x}_c - (1-\delta_{As})\hat{x}_b$.

4.3. CHOICE OF R_M

An obvious choice of R_M is to keep possibly unchanged the reduction level of the Kyoto compliance. At least two interpretations are, however, possible, even if only the interval uncertainty is considered, as it is actually done in the sequel. Let us assume that N parties, $n = 1, \ldots, N$, take part in the Kyoto reduction project. We can require that mean committed reduction fractions before and after adjustment are equal

$$\frac{1}{N}\Sigma_{n=1}^{N}\delta_A^{(n)} = \frac{1}{N}\Sigma_{n=1}^{N}\delta^{(n)}$$

After inserting for $\delta_A^{(n)}$ from (17) or (18) this induces the condition

$$R_M^{av} = \frac{1}{N}\Sigma_{n=1}^{N}R_{bc}^{(n)} \tag{19}$$

which is the average value of all reduction fractions.

Alternatively, we can require that mean committed reduction quota is constant

$$\frac{1}{N}\Sigma_{n=1}^{N}\delta_A^{(n)}\hat{x}_b^{(n)} = \frac{1}{N}\Sigma_{n=1}^{N}\delta^{(n)}\hat{x}_b^{(n)}$$

The resulting condition is a weighted average

$$R_M^{wav} = \frac{\Sigma_{n=1}^{N}\Delta_{bc}^{(n)}}{\Sigma_{n=1}^{N}\hat{x}_b^{(n)}} = \Sigma_{n=1}^{N}w_b^{(n)}R_{bc}^{(n)} \tag{20}$$

where

$$w_b^{(n)} = \frac{\hat{x}_b^{(n)}}{\Sigma_{n=1}^{N}\hat{x}_b^{(n)}}$$

is the share of the reported emission of the party n in the total reported emission in the basic year.

5. Uncertainties in emission trading

Admitting the above compliance proving policy it is possible to develop rules that include uncertainty in emission trading and this way to solve the problem of different quality of this good among trading partners. The main line of reasoning in deriving the final formula is as follows. Assume that during the trade the uncertainty related to the trading quota of reported emission is transferred from the seller to the buyer. The transferred uncertainty increases the buyer's uncertainty and therefore the purchased emission is less worthy for the buyer within the proposed

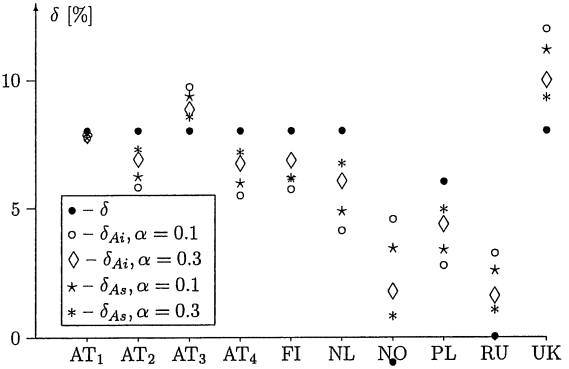

Figure 5. Comparison of different δ_As for R_M^{av} and $\alpha = 0.1, 0.3$.

earlier compliance proving mechanism. The diminished value is called an effective traded emission. It is then expressed in effective traded permits. This way conversion ratios of the reported emission to the effective permits is established. In comparison with the trading ratios among two trading partners the effective permits form a common basis for comparison of reported emissions for all trading parties when their uncertainties are also taken into account.

5.1. Interval type uncertainty

Let us consider a selling party, recognized by the superscript S in variables. The trend uncertainty used in proving compliance of the selling party is $\Delta_{bc}^S = (1-\zeta)[\Delta_c^S + (1-\delta^S)\Delta_b^S]$ or $R_{bc}^S = \Delta_{bc}^S/\hat{x}_b^S$. It seems then reasonable to assign to the sold emission this part of the uncertainty Δ_{bc}^S, which is connected with the commitment year t_c, i. e. $(1-\zeta^S)\Delta_c^S$ or $(1-\zeta^S)R_c^S = (1-\zeta^S)\Delta_c^S/\hat{x}_c^S$. Thus, the unit $\hat{E}^S$ of the sold reported emission brings with it the uncertainty

$$(1-\zeta^S)\hat{E}^S R_c^S = \frac{\hat{E}^S}{\hat{x}_c^S}(1-\zeta^S)\Delta_c^S = (1-\zeta^S)\hat{e}^S\Delta_c^S$$

where $\hat{e}^S = \hat{E}^S/\hat{x}_c$ is the share of the sold emission in the seller's total emission.

If the buying party, recognized by the superscript B, purchases n units $\hat{E}^S$, then its emission balance becomes

$$\hat{x}_c^B - n\hat{E}^S \tag{21}$$

As countries prepare their inventories independently, it is reasonable to assume that there is *no dependence* of these estimates. Thus, we calculate the uncertainty of the buying country, after inclusion of freshly bought one, as

$$\Delta_{bc}^B + n(1-\zeta^S)\hat{e}^S\Delta_c^S \tag{22}$$

The case with dependence is discussed later on.

Before the trade the following compliance-proving-with-risk-α inequality had to be satisfied

$$\hat{x}_c^B + (1-2\alpha)\Delta_{bc}^B \leq (1-\delta^B)\hat{x}_b^B \tag{23}$$

After the trade it changes to

$$\hat{x}_c^B - n\hat{E}^S + (1-2\alpha)[\Delta_{bc}^B + n(1-\zeta^S)\hat{e}^S\Delta_c^S] \leq (1-\delta^B)\hat{x}_b^B \tag{24}$$

Comparing (23) and (24) it is seen that they differ in the following component, which will be called *the effective traded emission*

$$nE_{eff} = n\hat{E}^S - n(1-2\alpha)(1-\zeta^S)\hat{e}^S\Delta_c^S = n[1-(1-2\alpha)(1-\zeta^S)R_c^S]\hat{E}^S$$

The effective reduction in the buyer's balance from one purchased unit $\hat{E}^S$ is

$$E_{eff} = [1-(1-2\alpha)(1-\zeta^S)R_c^S]\hat{E}^S \tag{25}$$

Thus, the bigger the seller's uncertainty is, the less the purchased unit counts for the buyer.

Note that for proving compliance the efficient emission is directly subtracted from the buyer's emission inventory, without any uncertainty considerations.

In the economic literature it is common to express the effects on trading by the trading ratios, see (Gillenwater, 2005). This can be easily calculated using the effective emissions. Let $\hat{x}^B$ be the buyer's and $\hat{x}^S$ the seller's reported emissions, both equivalent to the same effective emission x_{eff}. Thus, we have

$$\hat{x}^B[1-(1-2\alpha)(1-\zeta^B)R^B] = x_{eff} = \hat{x}^S[1-(1-2\alpha)(1-\zeta^S)R^S]$$

Then the trading ratio r_t among these two parties is

$$r_t = \frac{\hat{x}^B}{\hat{x}^S} = \frac{1-(1-2\alpha)(1-\zeta^S)R^S}{1-(1-2\alpha)(1-\zeta^S)R^S}$$

At the present situation uncertainties of the prospective buyer's inventories are typically smaller than those of the prospective seller, i. e. $R^B < R^S$. In this case $r_t < 1$. This means that in order to allow increasing the buyer's reported emission by one unit the seller has to reduce $1/r_t > 1$ units of his reported emission. This way the total number of reported emission units is reduced. Moreover, reduction of more inaccurate reported emission decreases the final total relative uncertainty.

Let us discuss now, what would be the impact of existence of *dependence* of the buyer's and seller's inventories on the result. Then, according to the simplified version of (7), (22) is changed to

$$(1-\zeta^{BS})(\Delta_{bc}^{B}+n(1-\zeta^{S})\hat{e}^{S}\Delta_{c}^{S})$$

where ζ^{BS} is the dependence parameter. Then (24) becomes

$$\hat{x}_c^B - n\hat{E}^S + (1-2\alpha)(1-\zeta^{BS})[\Delta_{bc}^{B}+n(1-\zeta^{S})\hat{e}^{S}\Delta_{c}^{S}] \leq (1-\delta^B)\hat{x}_b^B$$

causing the effective reduction of the buyer's balance

$$E_{eff} = [1-(1-2\alpha)(1-\zeta^{BS}v)(1-\zeta^{S})R_c^S]\hat{E}^S \tag{26}$$

with

$$v = \frac{1}{\eta p}\frac{R_{bc}^B}{R_c^S} \tag{27}$$

where $\eta = \frac{\hat{x}_c^B}{\hat{x}_b^B}$ is the estimated buyer's emission reduction and $p = \frac{n\hat{E}^S}{\hat{x}_c^S}$ is the buyer's ratio of the purchased emission to its total emission in the commitment time. In (27) both η and the ratio $\frac{R_{bc}^B}{R_c^S}$ are close to 1, while p is of the order of few hundredths. Thus, v is big, say $50 \div 100$, and then $1-\zeta^{BS}v$ is positive only when ζ^{BS} is small enough. Existence of positive dependence of the trading countries can then lead to troubles in definition of the effective reduction.

5.2. Stochastic type uncertainty

In the stochastic case it is difficult to extract from σ_{bc}^S the part connected only with t_c. That is why we consider here only *uncorrelated* inventories, with $\rho_{bc}^S = 0$. It will be obvious in the sequel that it is not only one difficulty connected with the stochastic case. Thus, we admit that the unit $\hat{E}^S$ of the sold reported emission brings with it the following uncertainty

$$\hat{E}^S R_c^S = \frac{\hat{E}^S}{\hat{x}_c^S}\sigma_c^S = \hat{e}^S\sigma_c^S$$

Having purchased n units, the emission balance of the buying party becomes

$$\hat{x}_c^B - n\hat{E}^S$$

and its uncertainty is calculated from the expression

$$\sqrt{(\sigma_{bc}^B)^2 + (n\hat{e}^S\sigma_c^S)^2 - 2n\hat{e}^S\rho^{BS}\sigma_{bc}^B\sigma_c^S}$$

where it is assumed that the correlation between the trading countries inventories exist and then ρ^{BS} is the correlation coefficient of the variables $\hat{x}_c^B - (1-\delta^B)\hat{x}_b^B$ and $\hat{x}_c^S$. To fulfill the obligations, the original emission of the buying country should satisfy the following condition

$$\hat{x}_c^B - (1-\delta^B)\hat{x}_b^B + q_{1-\alpha}\sigma_{bc}^B \leq 0 \tag{28}$$

where σ_{bc} is given by (12). After purchasing $n\hat{E}^S$ units from the selling country, the new condition is

$$\hat{x}_c^B - n\hat{E}^S + q_{1-\alpha}\sqrt{(\sigma_{bc}^B)^2 + (n\hat{e}^S\sigma_c^S)^2 - 2n\hat{e}^S\rho^{BS}\sigma_{bc}^B\sigma_c^S} \leq (1-\delta^B)\hat{x}_b^B$$

This can be written in the form

$$\hat{x}_c^B - n\hat{E}^S + q_{1-\alpha}\sigma_{bc}^B + q_{1-\alpha}\Big(\sqrt{(\sigma_{bc}^B)^2 + (n\hat{e}^S\sigma_c^S)^2 - 2n\hat{e}^S\rho^{BS}\sigma_{bc}^B\sigma_c^S} - \sigma_{bc}^B\Big)$$

$$\leq (1-\delta^B)\hat{x}_b^B \tag{29}$$

Subtracting (29) and (28), and then dividing by n, we get

$$E_{eff} = \Big[1 - q_{1-\alpha}R_c^S\Big(\sqrt{(\frac{\sigma_{bc}^B}{n\hat{E}^S R_c^S})^2 + 1 - 2\rho^{BS}\frac{\sigma_{bc}^B}{n\hat{E}^S R_c^S}} - \frac{\sigma_{bc}^B}{n\hat{E}^S R_c^S}\Big)\Big]\hat{E}^S \tag{30}$$

The expression on the right hand side is nonlinear in R_c^S, even if $\rho^{BS} \neq 0$, and can not be reduced to a linear form similar to (25). Let us try, however, to estimate the value in the parenthesis.

Denoting the component in the parenthesis by P, it can be transformed as follows

$$P = \frac{1 - 2\rho^{BS}v}{\sqrt{v^2 + 1 - 2\rho^{BS}v} + v}$$

where

$$v = \frac{\sigma_{bc}^B}{n\hat{E}^S R_c^S} = \frac{1}{\eta p}\frac{R_{bc}^B}{R_c^S}$$

with the same definitions as in the interval uncertainty case

$$\eta = \frac{\hat{x}_c^B}{\hat{x}_b^B} \qquad p = \frac{n\hat{E}^S}{\hat{x}_c^B}$$

As before, υ is big, then under the square root $1-2\rho^{BS}\upsilon$ can be ignored in comparison with υ^2, which provides the approximate formula for E_{eff}

$$E_{eff} = \Big(1 - q_{1-\alpha} R_c^S \frac{1-2\rho^{BS}\upsilon}{2\upsilon}\Big)\hat{E}^S$$

Similar to the interval uncertainty case the value of $1-2\rho^{BS}\upsilon$ is positive only for very small correlation coefficient ρ^{BS}. Thus, we assume $\rho^{BS}=0$ to get finally

$$E_{eff} = \Big(1 - q_{1-\alpha} R_c^S \frac{\eta p}{2} \frac{R_c^S}{R_{bc}^B}\Big)\hat{E}^S \tag{31}$$

This formula depends not only on R_c^S, but also on the ratio $\frac{R_c^S}{R_{bc}^B}$, as well as on η and p. Particularly the multiplication by p causes that the stochastic approach gives much smaller deviations from the exact observation solutions than in the interval one.

6. Tradable permits under uncertainty

Usual instruments applied for limitation of a pollutant emission are tradable emission permits. The theory of the tradable permits has been elaborated for exactly known emissions (Montgomery, 1972). With big uncertainties, like in the GHG case, our proposition is to use for permits the efficient emissions introduced in the previous section. The derivations here are conducted only for the *interval type uncertainty*.

The effective tradable permit E_{eff} corresponding to one unit of the reported emission $\hat{E}$ is then defined as

$$E_{eff} = \hat{E}[1 - (1-2\alpha)(1-\zeta)R] \tag{32}$$

where R is the relative uncertainty of $\hat{x}$. Other way round, the reported emission $\hat{x}$ is equivalent to $\hat{x}[1-(1-2\alpha)(1-\zeta)R]$ units of the effective tradable permits. The formula directly reflects the following rule: higher the uncertainty – less units of effective emission permits a party is allocated with.

6.1. Compliance with undershooting

Let us consider a party taking part in the Kyoto Protocol project. According to condition (9), in the commitment year the party has permission to emit $\hat{x}_c$ units of GHG satisfying

$$\hat{x}_c \le (1-\delta)\hat{x}_b - (1-2\alpha)\Delta_{bc} =$$

$$= (1-\delta)[1-(1-2\alpha)(1-\zeta)R_b]\hat{x}_b - (1-2\alpha)(1-\zeta)\Delta_c$$

Adding to both sides $(1-2\alpha)(1-\zeta)\Delta_c$ and denoting, according to (32),

$$l_c = [1-(1-2\alpha)(1-\zeta)R_c]\hat{x}_c \qquad l_b = [1-(1-2\alpha)(1-\zeta)R_b]\hat{x}_b \quad (33)$$

i. e. the numbers of the effective permits equivalent to the emission $\hat{x}_c$ and $\hat{x}_b$, respectively, yields

$$\frac{1+(1-2\alpha)(1-\zeta)R_c}{1-(1-2\alpha)(1-\zeta)R_c} l_c \leq (1-\delta) l_b$$

As typically relative uncertainty R_c for a party may be of the order $0.1 \div 0.2$, then approximately

$$(1-\delta)\frac{1-(1-2\alpha)(1-\zeta)R_c}{1+(1-2\alpha)(1-\zeta)R_c} \approx 1-\delta-2(1-2\alpha)(1-\zeta)R_c$$

and therefore we can use the approximation

$$l_c \leq [1-\delta-2(1-2\alpha)(1-\zeta)R_c] l_b \qquad (34)$$

Relation (34) expresses commitment condition in the effective tradable permits. It has the same form as the original commitment condition for the reported emissions (9). But now, the following redefinition of the reduction fraction applies

$$\delta \qquad \rightarrow \qquad \delta_{pUi} = \delta + 2(1-2\alpha)(1-\zeta)R_c \qquad (35)$$

6.2. Compliance with adjustment of the commitment level

To introduce adjustment of the basic committed level of Sec. 4 let us consider again the basic equation (16) with the new adjustment fraction δ_{pAi}

$$(1-\delta_{pAi})\hat{x}_b + (1-2\alpha)(1-\zeta)[\Delta_c + (1-\delta_{pAi})\Delta_b] = (1-\delta)\hat{x}_b + (1-2\alpha)\Delta_M$$

It can be written as

$$(1-\delta_{pAi})[1+(1-2\alpha)(1-\zeta)R_b]\hat{x}_b = (1-\delta)[1-(1-2\alpha)(1-\zeta)R_b]\hat{x}_b +$$

$$+(1-2\alpha)(1-\zeta)[(1-\delta)R_b\hat{x}_b - R_c\hat{x}_c] + (1-2\alpha)R_M\hat{x}_b$$

or using definition of l_b (33)

$$(1-\delta_{pAi})\frac{1+(1-2\alpha)(1-\zeta)R_b}{1-(1-2\alpha)(1-\zeta)R_b} l_b =$$

$$= (1-\delta)l_b + (1-2\alpha)R_M\hat{x}_b + (1-2\alpha)(1-\zeta)[(1-\delta)R_b - R_c\frac{\hat{x}_c}{\hat{x}_b}]\hat{x}_b$$

Now, after similar approximate reasoning as in the undershooting case, the above equality can be transformed as follows

$$(1-\delta_{pAi})l_b \approx [1-\delta-2(1-2\alpha)(1-\zeta)R_b]l_b +$$

$$+\frac{1-(1-2\alpha)R_M}{1+(1-2\alpha)(1-\zeta)R_b}l_b + \frac{(1-2\alpha)(1-\zeta)[(1-\delta)R_b - R_c\frac{\hat{x}_c}{\hat{x}_b}]}{1+(1-2\alpha)(1-\zeta)R_b}l_b$$

or

$$(1-\delta_{pAi})l_b \approx [1-\delta-2(1-2\alpha)(1-\zeta)R_b]l_b +$$

$$+\frac{1-(1-2\alpha)R_M}{1+(1-2\alpha)(1-\zeta)R_b}\left(1+\frac{(1-2\alpha)(1-\zeta)[(1-\delta)R_b - R_c\frac{\hat{x}_c}{\hat{x}_b}]}{1-(1-2\alpha)R_M}\right)l_b$$

As the following approximations can be used

$$\frac{(1-2\alpha)R_M}{1+(1-2\alpha)(1-\zeta)R_b} \approx (1-2\alpha)(1-\zeta)R_M$$

and

$$\frac{(1-2\alpha)(1-\zeta)[(1-\delta)R_b - R_c\frac{\hat{x}_c}{\hat{x}_b}]}{1-(1-2\alpha)R_M} \ll 1$$

then, finally, approximately we get

$$(1-\delta_{pAi})l_b \approx \Big(1-\delta+(1-2\alpha)[R_M - 2(1-\zeta)R_b]\Big)l_b$$

This provides the reduction fraction for permits with adjustment

$$\delta \quad \longrightarrow \quad \delta_{pAi} = \delta - (1-2\alpha)[R_M - 2(1-\zeta)R_b] \qquad (36)$$

Above, (19) or (20) can be substituted for R_M, with R_{bc} given by (8).

Calculating, as before, the difference

$$\delta_{pUi} - \delta_{pAi} = (1-2\alpha)R_M + 2(1-2\alpha)(1-\zeta)(R_c - R_b)$$

we see that it is close to $\delta_{Ui} - \delta_{Ai}$, and even equal to it when $R_c = R_b$, and therefore almost surely positive.

6.3. Compliance proving and trading mechanism

Thus, the compliance proving and trading mechanism with the uncertain observations and adjustment of the basic committed level requires the following steps.

(i) At a (successive) basic year the allotted reported emissions are converted to the effective permits according to the expression

$$l_b = \hat{x}_b[1 - (1 - 2\alpha)(1 - \zeta)R_b] \tag{37}$$

(ii) The committed obligations, in effective permits, in the commitment year are calculated from the condition

$$l_c \leq (1 - \delta_{pAi})l_b = \Big(1 - \delta + (1 - 2\alpha)[R_M - 2(1 - \zeta)R_b]\Big)l_b \tag{38}$$

which is equivalent to the reported emission

$$\hat{x}_c = \frac{l_c}{1 - (1 - 2\alpha)(1 - \zeta)R_c} \approx l_c[1 + (1 - 2\alpha)(1 - \zeta)R_c] \tag{39}$$

(iii) The effective permits l_c can be traded and directly added to the effective permits of any party taking part in the project.

Let us notice that if $R_M = R_{bc}$, i.e. uncertainty of the party equals the reference one, and $R_{bc} = 2(1-\zeta)R_b$, then $R_M = 2R_b$ and therefore $\delta_{pAi} = \delta$. In this case (38) reduces to the condition $l_c \leq (1-\delta)l_b$, where the reduction fraction is equal to the original one.

The above scheme reduces the trade in the uncertain case to the classical tradable permits problem. Once the reported emission are recalculated to the effective permits, they are traded and counted for compliance proving without further consideration of the uncertainties in the emission inventories.

7. Simulation of a carbon market with effective permits

The aim of this section is to employ the introduced earlier ideas into a market optimization problem, i.e. to simulate trading with effective permits within both the undershooting and adjustment framework. In constructing the market model the basic decision of each participating country is considered. Is it cheaper to abate the emission or to buy the permits on the market? The answer depends on the permit price, which is settled on the market as a result of optimisation of the total cost of all participants.

7.1. Data base

In order to perform carbon market simulation one needs to know cost functions of GHG abatement for market participants. Availability of data forced us to consider the original Kyoto protocol participants aggregated into five groups: United States (US), OECD Europe (OECDE), Japan, Canada/Australia/New Zealand (CANZ) and finally Eastern Europe/Former Soviet Union (EEFSU), instead of continuing calculations for the countries mentioned earlier in the paper. Data for regional abatement cost functions come from (Godal, 2003b)[1]. Data on uncertainty level were derived from (Godal, 2003a; Rypdal, 2001) and partly assumed (for Japan). The results here and particularly in the sequel should be regarded as illustrative and not the ultimate solution due to partly estimated data. Table III depicts the situation of the groups before any exchange of permits has been made, and according to the present regulations, i. e. without undershooting.

Table III. Base year emissions, committed changes in emissions, inventory uncertainty, total and marginal costs of compliance without trade.

	Base year emissions	Kyoto target	Inventory uncertainty	Total costs	Marginal costs
Units	MtC/year	%	%	MUS$	$/tC
US	1 345	7.0	13	89 343	-313.7
OECDE	934	7.9	10	28 652	-322.7
Japan	274	6.0	15	21 077	-453.8
CANZ	217	0.7	20	10 477	-216.5
EEFSU	1337	1.7	30	0	0.0
Total	4 107			149 549	

One can immediately spot from Table III a substantial gap between the Kyoto targets and the magnitude of inventory uncertainties. Although some objections can be rosed toward accuracy of the uncertainty levels accepted here, the situation generally follows earlier observations, e.g. (Rypdal, 2001), revealing potential troubles with verification of the Kyoto protocol compliance.

7.2. No uncertainty market

The following notation will be used:

$n = 1, 2, \ldots, N$ – the index of a party of the Kyoto Protocol;

[1] Kind provision of data from Odd Godal is gratefully acknowledged.

$x_c^{(n)}$ – emission level of the party n in the commitment year;

$c^{(n)}(x_c^{(n)})$ – cost of reducing emission to the level $x_c^{(n)}$;

$\delta^{(n)}$ – fraction of the party n base year emission that is to be reduced according to the Kyoto obligation;

$x_b^{(n)}$ – base year emission of the party n.

The task is to meet the target of the Kyoto protocol and not to allow the costs to become higher than it is necessary (Baumol, 1998; Tietenberg, 1985):

$$\begin{gathered}\min_{x_c^{(n)}} \sum_n c^{(n)}(x_c^{(n)}) \\ \text{s.t. } \sum_n (x_c^{(n)} - (1-\delta^{(n)})x_b^{(n)}) = 0\end{gathered} \tag{40}$$

The border condition takes the form of equation, as we assume parties never overcomply. Constructing the Lagrangian we obtain the condition for the static market equilibrium:

$$\lambda = -\frac{\partial c^{(n)}(x_c^{(n)})}{\partial x_c^{(n)}}$$

where λ is the Lagrange multiplier being interpreted as the market shadow (equilibrium) price.

7.3. Market with uncertainties

7.3.1. *Effective emission permits*

Based on the formula (32) the relationship between the reported emission level $x^{(n)}$ and the effective emission permits $l^{(n)}$ is

$$l^{(n)} = [1-(1-2\alpha)(1-\zeta)R^{(n)}]x^{(n)} \tag{41}$$

where $R^{(n)}$ is the relative uncertainty of the inventory [2].

Since an effective permit will be the standard permit used in our setting, the cost of emission abatement is expressed in terms of the effective permit units

$$c^{(n)}(x_c^{(n)}) = c^{(n)}\Big(\frac{l^{(n)}}{1-(1-2\alpha)(1-\zeta)R^{(n)}}\Big) \tag{42}$$

[2] Here, uncertainties for the base year and for the commitment year are assumed to be equal, therefore the subscripts b and c are dropped. Consideration of uncertainty reduction would require to include also the cost of such action in the optimization problem (40) (compare (Godal, 2003a; Obersteiner, 2000)).

This way the argument of the abatement cost function is shifted depending on party's uncertainty level $R^{(n)}$, dependence of the commitment and base years uncertainty parameter ζ and the assumed risk level α. Market decisions will be made on the basis of the cost function (42).

7.3.2. *Market with undershooting*
Having expressed abatement costs in terms of the effective permits, the next step is to apply the undershooting rule so that parties might become, respectively, awarded or penalized for their uncertainty level. Inserting from (33) for l_b in (34) the commitment condition is now expressed as follows

$$l^{(n)} \leq [1-\delta^{(n)}-2(1-2\alpha)(1-\zeta)R^{(n)}]x_b^{(n)}[1-(1-2\alpha)(1-\zeta)R^{(n)}] \quad (43)$$

It differs from the standard border condition in (40) since the Kyoto original emission obligation is decreased according to the undershooting rule depending on the inventory uncertainty $R^{(n)}$ and considered risk level α. The last two terms on the right hand side of inequality (43) correspond to effective permits in the base year.

The cost-effective fulfillment of the protocol commitments expressed in terms of effective permits is now as follows

$$\min_{l^{(n)}} \sum_n c^{(n)}\Big(\frac{l^{(n)}}{1-(1-2\alpha)(1-\zeta)R^{(n)}}\Big) \quad (44)$$

subject to

$$\sum_n (l^{(n)}-[1-\delta^{(n)}-2(1-2\alpha)(1-\zeta)R^{(n)}]x_b^{(n)}[1-(1-2\alpha)(1-\zeta)R^{(n)}]) = 0$$

Constructing the Lagrangian yields the condition

$$\lambda = -\frac{\partial c^{(n)}\Big(\frac{l^{(n)}}{1-(1-2\alpha)(1-\zeta)R^{(n)}}\Big)}{\partial l^{(n)}} \quad (45)$$

7.3.3. *Market with adjustments*
Since undershooting decreases Kyoto emission liabilities and results in increase of abatement costs, we consider also market with adjustment of the commitment level. The adjustment turns the border condition in our optimization model into the following one

$$\min_{l^{(n)}} \sum_n c^{(n)}\Big(\frac{l^{(n)}}{1-(1-2\alpha)(1-\zeta)R^{(n)}}\Big) \quad (46)$$

subject to

$$\sum_n (l^{(n)} - [1 - \delta^{(n)} + (1-2\alpha)(R_M - 2(1-\zeta)R^{(n)})]\times$$
$$\times x_b^{(n)}[1 - (1-2\alpha)(1-\zeta)R^{(n)}]) = 0$$

Results for both cases of reference uncertainty R_M (e.i. R_M^{av} and R_M^{wav}) will be analyzed.

7.4. Simulation results

Below we present results of market optimization problem as formulated in (44) and (46).

7.4.1. *Trading with effective permits under undershooting*

Table IV shows the results of trading with effective permits under undershooting, for few values of the parameter α and for an assumed value of dependence coefficient $\zeta = 0.7$ common for all the parties [3]. The table starts with $\alpha = 0.5$, which corresponds to neglecting uncertainty. Obviously, effective permits and reported emission are equal in this case for any party and we obtain the standard solution with the market shadow price 142.5 \$/tC and the total abatement cost for all parties 37 150 MUS\$, very much diminished from the situation of no trade – 149 549 MUS\$. EEFSU is the only net seller of permits.

Setting $\alpha = 0.3$ we accept the risk of 30% that a party's actual emission is above the Kyoto target. This is reflected in different levels of effective permits and reported emissions. Market price (marginal cost) settled on the market of effective permits, $\frac{\partial c^{(n)}(l^{(n)})}{\partial l^{(n)}}$, has increased and equals 198.4 \$/tC. However, it is worth to note that marginal costs of reported emissions for each party at the equilibrium points, $\frac{\partial c^{(n)}(x^{(n)}(t_c))}{\partial x^{(n)}(t_c)}$, differ, ranging from 191 \$/tC (EEFSU) to 196 \$/tC (OECDE). This reflects different levels of inventory uncertainty (Table III). Total abatement cost has also increased considerably achieving the sum of almost 70 000 MUS\$.

The situation evolves in the same direction when the parameter α is decreased further. Generally, smaller is the risk α accepted, smaller is the excess saved emission for sale. For example, when α is small, then EEFSU group can sell less of effective permits due to its high inventory uncertainty. At the same time OECDE, with a lower inventory

[3] Obviously, the simulation results will heavily depend on the parameter ζ. However, we do not consider sensitivity of results on ζ since this parameter depend mainly on the method of inventory calculation and one can hardly imagine tuning this parameter in practice.

Table IV. Trading with effective permits under undershooting for different levels of risk α ($\zeta = 0.7$) - results at the equilibrium points; A - marginal cost of reported emission; B - marginal cost of effective permit.

	Effective emission permits	Reported emissions	Effective permits traded	A	B	Total costs
Units	MtC/y	MtC/y	MtC/y	\$/tC	\$/tC	MUS\$
Variable	$l^{(n)}$	$x_c^{(n)}$		$\frac{\partial c^{(n)}(x_c^{(n)})}{\partial x_c^{(n)}}$	$\frac{\partial c^{(n)}(l^{(n)})}{\partial l^{(n)}}$	$c^{(n)}(l^{(n)})$
	$\alpha = 0.5$					
US	1 561.6	1 561.6	310.8	-142.5	-142.5	18 433
OECDE	959.4	959.4	99.1	-142.5	-142.5	5 602
Japan	321.1	321.1	63.5	-142.5	-142.5	2 059
CANZ	248.4	248.4	32.9	-142.5	-142.5	4 583
EEFSU	807.8	807.8	-506.3	-142.5	-142.5	6 473
Total	3 898.3	3 898.3	0			37 150
	$\alpha = 0.3$					
US	1 442.9	1 465.8	252.9	-195.3	-198.4	34 618
OECDE	918.7	929.8	90.9	-196.0	-198.4	10 598
Japan	304.9	310.5	61.7	-194.8	-198.4	3 848
CANZ	219.9	225.3	19.8	-193.6	-198.4	8 461
EEFSU	748.8	776.7	-425.3	-191.2	-198.4	11 658
Total	3 635.2	3 708.1	0			69 183
	$\alpha = 0.1$					
US	1 327.5	1 370.3	197.0	-247.9	-255.9	55 790
OECDE	878.6	900.2	82.8	-249.8	-255.9	17 208
Japan	289.2	299.9	59.9	-246.7	-255.9	6 169
CANZ	183.0	202.8	7.7	-243.6	-255.9	13 394
EEFSU	693.5	747.3	-347.4	-237.5	-255.9	17 976
Total	3 371.8	3 520.5	0			110 537
	$\alpha = 0$					
US	1 271.1	1 322.7	169.8	-274.1	-285.3	68 222
OECDE	858.7	885.3	78.7	-276.7	-285.3	21 124
Japan	281.5	294.7	59.0	-272.4	-285.3	7 525
CANZ	180.2	191.7	2.1	-268.2	-285.3	16 229
EEFSU	667.2	733.2	-309.6	-259.6	-285.3	21 482
Total	3 258.7	3 427.6	0			134 582

uncertainty, buys less permits. Finally, requiring undershooting of the full uncertainty belt $\Delta_{bc}^{(n)}$, as defined in equation (8), we would have to accept the effective permit shadow price of 285.3 \$/tC and the sum of total abatement costs of 134 582 MUS\$ (compare Fig. 6). This was the reason to examine also adjusted Kyoto obligations according to (46). Let us note that the total abatement costs in this case are still smaller than that with no trade from Table III.

Table V. Trading with effective permits according to the adjusted Kyoto obligation for R_M^{av} and R_M^{wav} ($\alpha = 0$, $\zeta = 0.7$) - results at the equilibrium points; A - marginal cost of reported emission; B - marginal cost of the effective permit; a - $\frac{\partial c^{(n)}(x^{(n)}(t_c))}{\partial x^{(n)}(t_c)}$; b - $\frac{\partial c^{(n)}(l^{(n)})}{\partial l^{(n)}}$.

	Effective emission permits	Reported emissions	Effective permits traded	A	B	Total costs
Units	MtC/y	MtC/y	MtC/y	\$/tC	\$/tC	MUS\$
Variable	$l^{(n)}$	$x^{(n)}(t_c)$		a	b	$c^{(n)}(l^{(n)})$
	$R_M^{av} = 0.10365$ ($\alpha = 0$)					
US	1 475.0	1 534.8	239.7	-157.2	-163.6	22 448
OECDE	921.9	950.4	47.9	-158.7	-163.6	6 950
Japan	303.9	318.3	54.4	-156.3	-163.6	2 476
CANZ	228.7	243.3	29.4	-153.8	-163.6	5 340
EEFSU	731.4	803.7	-371.4	-148.9	-163.6	7 068
Total	3 660.9	3 850.5	0			44 282
	$R_M^{wav} = 0.108$ ($\alpha = 0$)					
US	1 483.5	1 543.7	242.7	-152.3	-158.5	21 069
OECDE	924.5	953.2	46.7	-153.8	-158.5	6 523
Japan	304.9	319.3	54.2	-151.4	-158.5	2 324
CANZ	230.7	245.4	30.6	-149.0	-158.5	5 012
EEFSU	734.1	806.7	-374.2	-144.2	-158.5	6 634
Total	3 677.7	3 868.3	0			41 562

7.4.2. *Trading with effective permits under adjustment*

Adjustment of each party commitment obligation using a reference uncertainty distribution has proved to be a practical solution. Results of trade under adjustment for both R_M^{av} and R_M^{wav} are presented for

$\alpha = 0$ in Table V. As $R_M^{wav} = 0.108$ is higher than $R_M^{av} = 0.10365$, the adjusted reduction target δ_i^{Ap} is higher in the case of *average* R_M^{av} and participants have to reduce more. The total reported emission equals 3 850.5 MtC/year, as compared with 3 868.3 MtC/year under the *weighted* R_M^{wav}. Permit price on the effective permit market settles at 163.6 and 158.5 \$/tC, respectively. Total abatement costs differ by 2 720 MUS\$. Changing parameter α the influence of uncertainty can be partially relaxed (see Fig. 6) making both the marginal price λ and the cost decreasing.

Summing up, inclusion of uncertainty in the trading scheme bears some additional cost (total abatement cost in the equilibrium point) as compared to the standard system, even with the adjusted target level. It is inevitable under assumptions taken, as the abatement cost function is increasing and convex. However, this additional cost seems to be reasonable. The increase is from 37 150 MUS\$ to 41 562 MUS\$ (in the case of R_M^{wav}) when full uncertainty is considered ($\alpha = 0$).

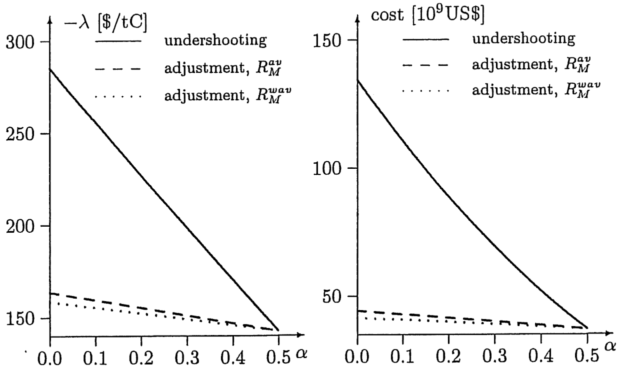

Figure 6. Dependence of the marginal cost λ (left) and the cost (right) on α. $\zeta = 0.7$.

8. Conclusions

The presented approach of including uncertainty in the reported emissions can be used to solve the problem of different qualities of emission inventories encountered in compliance proving and emission trading and due to high and non homogeneous errors corresponding to different greenhouse gases. The advantage of the presented approach is in complete treatment of the uncertainty problem and its reduction to known rules for exact observations. In particular, introduction of the effective permits reduces the permit trade under uncertain inventory to the well known permit trade rules with no uncertainty. To apply the approach, the knowledge of uncertainty estimates of inventories for all parties involved is needed. Then, this would also stimulate research in documentation and decreasing the national inventory uncertainty estimates.

Application of this approach requires different agreements between parties participating in the emission reduction project than the present state of the Kyoto Protocol law. The most difficult points in negotiations might be changes of committed reductions. Proposed adjustment method make the changes much smaller. Moreover, some free parameters may help to find the most convenient solution.

The above reasoning was centered on national emission inventories but it can be extended to the case when uncertainties of different emitted gases are considered in trading, provided the uncertainties are not too high and justify the approximations made. Then, the uncertainty measures R_c connected with each activity could be used for determining the number of the effective tradable permits. The idea can also be applied to other flexible mechanisms, provided the respective uncertainty measures are known for them.

While adequate conditions for the undershooting and adjustment has been presented, the definition of effective permits in a stochastic case still remains unsolved due to encountered nonlinearities. Yet, the stochastic case is important, as it reflects better the reality. Moreover, for the same risk α the confidence intervals for the stochastic case are smaller than in an interval case, particularly when algebraic transformations of variables are involved. This is due to the effect of concentration of probability around the mean value.

An intermediate solution can be obtained using the fuzzy uncertainty model. The calculus applied there inherits rules from the interval model, but the uncertainty may be more concentrated around the average value. An idea of this approach has been mentioned in (Nahorski, 2005b) and a more mature presentation was submitted for publication (Nahorski, 2005a). This way a generalized formula for the effective permits was obtained.

A closer attention deserves the problem of inaccuracy of the uncertainty measure, viz. the standard deviation σ or the uncertainty half interval Δ. Due to the way of estimating the variance this problem seems to have no ready solution even for the stochastic type of uncertainty. The authors are neither aware of any solution for the interval type. The inaccuracy of the uncertainty measure is also important from the implementation point of view. This is discussed in detail in (Gillenwater, 2005) and will not be repeated here. It is perhaps only worth to add that an interim solution may be to use the uncertainty classes as proposed in (Jonas, 2005).

Another related problem is whether errors in inventories have an additive or a multiplicative character, see (Nahorski, 2005c). The theory in the present paper assumes implicitly that the errors are additive. However, it seems to be relatively easy to adopt the results to the multiplicative errors using logarithms of the accounted data, like in (Nahorski, 2005c).

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