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Risk and utility of sustainable development

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Risk and Utility of Sustainable Development

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Abstract. The paper deals with evaluation of sustainable development by employing the concept of two factors utility of sustainable development (USD). The first factor represents the long-term expected profit of capital investment. The second factor represents the worse case profit necessary for survival of crises. Using the USD concept the support of cooperation of individuals and institutions in the era of globalization is analysed. A theorem, on fair division of benefits, enables one to derive the best cooperation strategy and individual partner's benefits in an explicite form. The methodology presented can be implemented in the form of a support system stimulating the matching of prospective partners (in exploitation of innovations and new technologies), which contributes to the economic growth and competiveness of national economy.

Key words: sustainable development, utility, decision support, cooperation, join ventures, negotiation, globalization, benefits, fair division, risk management.

I. Introduction

Development is usually defined as a welfare improving change in the set of opportunities (challenges) open to the society. Welfare is regarded in multidimensional context as the level of:

a. natural capital (land, forest etc),
b. built (engineered systems) capital,
c. social (human, intellectual, cultural) capital,
d. institutional (institutions that a society has at disposal) capital.

According to the Rio de Janerio (1992) Declaration "the right to development must be fulfilled so as to equitable meet development and environmental needs of present and future generations". Sustainable development requires that individuals and organizations exist for a long time interval and survive the short time crises.

In order to evaluate and choose the best, from the sustainable development point of view, alternatives the concept of two factors utility of sustainable development (introduced by R. Kulikowski [1] in 1998) can be used.

In the present paper the application of that concept to the cooperation of individuals and institutions in the era of globalization will be analysed.

II. Utility of Sustainable Development (U.S.D.)

The USD function (F), which is a measure of welfare increments resulting from the investment (I), being a part $x = I / P$ of the investors liquid capital P, is assumed in the form

$$U(x) = F[Zx, Y]$$

where

a. Z is the long term (strategic) expected profit equal PR, $R=(P_m - I):I$ is the expected rate of return (i.e. $R=E\{\widetilde{R}\}$, where the variance of random variable $\widetilde{R}: v\{\widetilde{R}\}=\sigma^2$);

b. $Y = Z - \kappa\sigma P$ is the short term "worse case profit", where κ is a subjective parameter characterizing the fear of failure. The value $VAR = \kappa\sigma P$ is called "Value at Risk".

Since Z, Y, are expressed in monetary terms and U should not change when one changes monetary units, it is assumed that F is "constant return to scale" and can be approximated by Cobb-Douglas function, i.e.

$$U(x) = F[Zx, Zs] \cong PRS^{1-\beta}x^{\beta}, \quad \beta \in [0,1], \tag{1}$$

where

$S = 1-\kappa\dfrac{\sigma}{R}$ is called the safety index, $0 < S \le 1$,

$\beta \cong \dfrac{\Delta U}{U}:\dfrac{\Delta x}{x}$ is a subjective parameter characterizing the investor's entrepreneurship (when $\beta = 1$ the value of $S^{1-\beta}=1$ so the investor ignores the risk and for $\Delta x / x = const$ his $\Delta U / U$ attains maximum value).

In order to evaluate R and S one can use the simple success-failure model, where the maximum return R_u (reward) is attained with probability p and the failure $(\widetilde{R} = R_d = 0)$ with probability $1-p$. Using that model one gets $R = pR_u$ and

$$S = 1-\kappa\frac{\sqrt{p(1-p)}}{p} = 1-\kappa\sqrt{\frac{1-p}{p}}. \tag{2}$$

The subjective parameter κ can be evaluated using the following scaling procedure.

a. Find the lower admissible value of success probability $\overline{p}$ $(p \ge \overline{p})$ such that during the worse interval the expected return $PR_u\overline{p}$ will cover the expected, minimal liabilities (costs) minus the level of liquid capital reserves (working capital) A, i.e.

$$PR_u\overline{p} = L_m - A, \text{ or}$$

$$\overline{p} = \lambda / R_u, \quad \lambda = \frac{L_m - A}{P}.$$

b. Find the lower bound of S and $U(x)$ for the worse interval when $U(x)$ drops to the minimal value $U_0(x) = PR_u\overline{p}S_0^{1-\beta}x^{\beta}$.

Assuming that the lowest utility is also attained for risk-free investment (in government bonds) with utility $U_F(x) = PR_F x^{\beta}$ one can write $U_0(x) = U_F(x)$ and find

$$S_0 = (R_F / \lambda)^{\gamma}, \quad \gamma = \frac{1}{1-\beta}. \tag{4}$$

Since, on the other hand, $S_0 = 1-\kappa\sqrt{\dfrac{1-\overline{p}}{\overline{p}}}$, one gets

$$\kappa(\lambda) = (1-S_0)\sqrt{\frac{\overline{p}}{1-\overline{p}}} = \left[1-(R_F/\lambda)^{\gamma}\right]\sqrt{\frac{\lambda}{R_u - \lambda}} \tag{5}$$

and

$$S = 1 - \left[1 - (R_F / \lambda)^{f}\right] \sqrt{\frac{\lambda}{R_u - \lambda} \cdot \frac{1-p}{p}} \,. \tag{6}$$

The USD concept can be also interpreted in terms of statistical estimation theory. Using e.g. Tchebyshev inequality one can construct the confidence interval for the expected value R employing the spot estimator $R_n = \frac{1}{n}\sum_{t=1}^{n} \tilde{R}_t$:

$$P_r(R_n - \varepsilon < R < R_n + \varepsilon) \geq 1 - \frac{\sigma^2}{n\varepsilon^2}, \tag{7}$$

where $E\{\hat{R}\} = R$, $V\{\tilde{R}\} = \sigma^2$. The bounds $(R_n - \varepsilon, R_n + \varepsilon)$ are random variables such that the probability of covering of the unknown R by the interval $(R_n - \varepsilon, R_n + \varepsilon)$ can be assumed equal $\alpha = 1 - \frac{\sigma^2}{n\varepsilon^2}$ (called the confidence level).

Since, according to (3) the lower bound of R is $\bar{p}R_u = \lambda$, $(\bar{p} \leq p)$ so assuming $R_n - \varepsilon = \lambda$ one can write (7) in an equivalent form

$$P_r(\lambda < R < 2R_n - \lambda) \geq 1 - \frac{1}{n}\left(\frac{\sigma}{R_n - \lambda}\right)^2 \stackrel{\Delta}{=} \alpha(\lambda), \tag{8}$$

where $\alpha(\lambda)$ can be called the confidence level of survival.

It is possible to observe that when the historic data sample n and the confidence interval $2\varepsilon = 2(R_n - \lambda)$ decrease (as a result of growing λ) the confidence level of survival $\alpha(\lambda)$ as well as S (according to (6)) decline.

Expressing USD (1) in the equivalent form

$$U(x) = s(p)PR_u x^{\beta}, \tag{9}$$

where $s(p) = pS^{1-\beta}$ can be called the subjective success probability, one can see that $s(p) < p$ (where p is the objective probability of success). As shown in [3] for small λ (but large R_u and small p) $s(p) > p$. In such a situation the gamblers participation in the lotteries is rational despite the fact that the expected value of the lottery is negative.

In order to analyse the impact of the long and short term strategies on the USD consider the binominal success-failure model with k successes in n trials. Each trial (such as a service of one client or production of one unit) requires the given time interval ΔT. When the production or service capacity is π the number of trials per 1 year is $n = \pi\Delta T$. If, for example, one is analysing the production project with $\pi = 3$ unit/day and 1 year planning horizon (with 12 x 26 = 312 working days) $n = 3 \cdot 312 = 936$.

Suppose that the estimated (by historical data) probability of success is $p = k/n$. The planned rate of return $R_u = P_m / p - 1$ where P_m - market price of the unit of production, P - production cost per unit. When k units are sold (i.e. k elementary successes achieved) the expected rate of return becomes $R_u p$.

Assume the binominal probability distribution function of successes in the form

$$P_r\{x=k\}=\binom{n}{k}p^k(1-p)^{n-k}, \quad k=1,2,...,n. \tag{10}$$

Since $E\{x\}=np$, $V\{x\}=np(1-p)$ one can find

$$\sigma/R=\frac{\sqrt{np(1-p)}}{np}=\sqrt{\frac{1-p}{np}}$$

and

$$S=1-\kappa(\lambda)\sqrt{\frac{1-p}{np}}. \tag{11}$$

The $\kappa(\lambda)$ can be derived using the scaling procedure for 1 month worse-case subinterval, characterized by $\bar{n}=n/12$.

Then

$$\kappa(\lambda)=(1-S_0)\sqrt{\frac{\bar{n}\bar{p}}{1-\bar{p}}}, \quad S=1-(1-S_0)\sqrt{\frac{\bar{n}\bar{p}}{1-\bar{p}}\cdot\frac{1-p}{np}} \tag{12}$$

Using the maximum likelihood estimators for $p=k/n$ and $\bar{p}=\bar{k}/\bar{n}$ one gets

$$S=1-[1-(R_F/\lambda)^2]\sqrt{\frac{1/k-1/n}{1/\bar{k}-1/\bar{n}}}, \tag{13}$$

$$VaR=P[1-(R_F/\lambda)^2]\sqrt{\frac{1/k-1/n}{1/\bar{k}-1/\bar{n}}}. \tag{14}$$

It is possible to observe that the growing $\bar{n}/n$ ratio increases VaR and decreases S and USD.

Numerical example. The car dealer estimates USD for the coming 1 year horizon and 1 month worse time survival interval. His clients servicing capacity is 3 cars/day so n = 12 x 26 x 3 = 936. The expected selling (estimated by historical data) is $k=624$ cars/year at the price $P_m=\$$ 15.000 and costs $P=\$$ 13.043 so $R_u=0.15$ and success probability $p=\frac{624}{936}=2/3$.

In order to survive the worse month $(n=26\cdot3=78)$ the dealer has to sell at least $\bar{k}=47$ cars. Then $\bar{p}=47/78=0.603$.

Assuming $R_F=0.05$, $\beta=0.5$ he gets $S_0=\left(\frac{0.05}{0.603\cdot0.15}\right)^2=0.306$.

Then $S=1-0.694\sqrt{\frac{624^{-1}-936^{-1}}{47^{-1}-78^{-1}}}=0.826$ and USD value (per 1 trial i.e. $n=1$) becomes

$$U=PR_u p\sqrt{S}=13043\cdot0.15\cdot2/3\sqrt{0.826}=\$\ 1187.$$

As shown in [2,3,4,5] the USD methodology can be used effectively to support decisions in concrete problems connected with allocation of resources, risk and knowledge management, education etc.

III. Support of Cooperation in the Era of Globalization

In the era of globalization, when the firms, institutions, regions and countries are trying to extend the cooperation links, the problem of choosing the best cooperating partners and negotiate with them the fair division of common benefits gains the ultimate importance. Using the USD concept, to describe the goals of individual partners, and the Nash principle of fair division of benefits it is possible to derive the best cooperation strategy which can be used as a methodological support in the process of negotiation among the cooperative parties.

Suppose that n partners (research institutes, producers, investors etc) consider the cooperation aimed at exploitation of an innovation or a new technology which will result in the expected market profit P_m. Denote the profit division strategy by y_i, $i = 1,2,\ldots,n$, $\sum_{i=1}^{n} y_i = 1$. In the case when cooperation creates a company the y_i strategy can be used to divide the shares among the partners (share holders). The i-th partner, who engages I_i / P_i part of his capital P_i in the joint venture gets $P_m y_i$ of reward. The USD of i-th partner becomes

$$U_i(y_i) = \bar{I}_i R_i(y_i) S_i^{1-\beta_i}, \quad \bar{I}_i = P_i x_i^{\beta_i} = P_i \left(\frac{I_i}{P_i} \right)^{\beta_i}, \tag{15}$$

where

$$R_i(y_i) = P_m y_i / I_i - 1, \quad \forall i.$$

Denote also, by U_{iT} the utility of traditional activity (status quo) of i-th partner

$$U_{iT} = \bar{I}_i R_{iT} S_{iT}^{1-\beta_i}, \qquad \forall i \tag{16}$$

It is assumed that cooperationm will hold when the utility increments, resulting from switching from the traditional to the innovative technology are positive i.e.

$$\Delta U_i(y_i) = U_i(y_i) - U_{iT} = M_i (y_i - E_i) > 0, \qquad \forall i \tag{17}$$

where $M_i = P_m (P_i S_i / I_i)^{1-\beta_i}$ - cooperation motivation factor,

$$E_i = \frac{I_i}{P_m} \left[1 + R_{iT} (S_{iT} / S_i)^{1-\beta_i} \right]$$ - cooperation expense (I_i), factor.

Using Nash principle the fair division of common benefits strategy $y_i = \hat{y}_i$, $\forall i$ can be derived by solving the problem: $\max_{y \in \Omega} \phi(y) = \phi(\hat{y})$, where

$$\phi(y) = \prod_{i=1}^{n} \Delta U_i(y_i), \quad \Omega = \left\{ y_i \mid \sum_{i=1}^{n} y_i = 1, \quad y_i > 0, \quad \forall i \right\} \tag{18}$$

The optimum strategy $y_i = \hat{y}_i$, $\forall i$, can be derived in an explicite form using the following theorem (on fair division of benefits).

Theorem. There exists an unique fair (in the Nash sense) strategy $y_i = \hat{y}_i$, $i = 1,2,\ldots,n$, of benefits division $(y_i P_m)$ among the n cooperating parties, characterized by positive USD increments $\Delta U_i(y_i) = M_i (y_i - E_i) > 0$, such that $\Delta U_i(\hat{y}_i) = M_i \delta$, $\forall i$, where

$$\hat{y}_i = \frac{1}{n}\left[1+(n-1)E_i - \sum_{k\neq i}^{n} E_k\right], \quad \forall i \tag{19}$$

$$\delta = \frac{1}{n}\left[1-\sum_{k=i}^{n} E_k\right]. \tag{20}$$

The δ parameter, called cooperation benefit indicator (CBI) expresses the cummulative benefit resulting from cooperation. The E_i coefficients characterize the individual expense level (I_i / P_m), an increase of safety ratio $\left[\left(\frac{S_{iT}}{S_i}\right)^{1-\beta_i}\right]$ and the profits foregone (R_{iT}), resulting from swithing (from traditional status quo) to the innovative technology.

Consider, for example, cooperation of two partners:

$$\hat{y}_1 = \frac{1}{2}\left\{1+\frac{I_1}{P_m}\left[1+R_{1T}\left(\frac{S_{1T}}{S_1}\right)^{1-\beta_1}\right]-\frac{I_2}{P_m}\left[1+R_{2T}\left(\frac{S_{2T}}{S_2}\right)^{1-\beta_2}\right]\right\}, \; \hat{y}_2 = 1-\hat{y}_1 \tag{21}$$

The optimum strategy rewards more the first partner (i.e. $\hat{y}_1 > \hat{y}_2$) when he: a. spends more $(I_1 > I_2)$, b. undertakes bigger risk $\left[(S_{1T}/S_1)^{1-\beta_1} > (S_{2T}/S_2)^{1-\beta_2}\right]$, c. loses more by switching from traditional to innovative technology $(R_{1T} > R_{2T})$. When, for example, the dealer is negotiating a contract with the car producer the optimum strategy (21) may help them to establish the fair factory price, for the new car models sold by the dealer at the market price P_m.

Proof

Since $\phi(y)$ is strictly concave in Ω the necessary and sufficient conditions for the strategy y_i, $i = 1,2,\dots,n$, to be optimum become

$$\phi'_{y_k} = \left[\prod_{i=1}^{n-1}(y_i - E_i)\left(1-\sum_{i=1}^{n-1} y_i - E_n\right)\right]'_{y_k} = \left[\prod_{i=1}^{n-1}(y_i - E_i)\right]'_{y_k}\left(1-\sum_{i=1}^{n-1} y_i - E_n\right) - \prod_{i=1}^{n-1}(y_i - E_i) = 0,$$

$k = 1,2,\dots,n-1$

Since $\left[\prod_{i=1}^{n-1}(y_i - E_i)\right]'_{y_k} = \prod_{i\neq k}^{n-1}(y_i - E_i)$, $k = 1,2,\dots,n-1$, then

$$\phi'_{y_k} = \prod_{i=1}^{n-1}(y_i - E_i)\left[\frac{1-\sum_{i=1}^{n-1} y_i - E_n}{y_k - E_k} - 1\right] = 0, \quad k = 1,2,\dots,n-1 \tag{21}$$

Since $\prod_{i=1}^{n-1}(y_i - E_i) > 0$ (by assumption) the necessary optimization conditions boil down to

$$1-\sum_{i=1}^{n-1} y_i - E_n = y_k - E_k, \quad k = 1,2,\dots,n-1.$$

Taking into account that $1-\sum_{i=1}^{n-1} y_i = y_n$, one gets

$$y_k - E_k = y_n - E_n \overset{\Delta}{=} \delta , \quad k = 1,2,\ldots,n-1 .$$

Then

$$\hat{y}_i = E_i + \delta , \quad i = 1,2,\ldots,n . \qquad (22)$$

Since $\sum_{i=1}^{n} \hat{y}_i = \sum_{k=1}^{n} E_k + n\delta = 1$, then

$$\delta = \frac{1}{n}\left[1 - \sum_{k=1}^{n} E_k\right]$$

and by (22) one gets:

$$\hat{y}_i = E_i + \frac{1}{n}\left[1 - \sum_{k=1}^{n} E_k\right] = \frac{1}{n}\left[1 - (n-1)E_i - \sum_{k \neq 1}^{n} E_k\right], \quad i = 1,2,\ldots,n .$$

The utility increments for optimization strategy $y_i = \hat{y}_i$, $i = 1,\ldots,n$ by (17) become

$$\Delta U_i(\hat{y}) = M_i(\hat{y}_i - E_i) = M_i\delta , \quad \forall i$$

It should be also noted that in the case when $S_{iT} / S_i = 1$, $\forall i$, the necessary condition for creation of beneficial partnership $(\delta > 0)$ requires that the expected market profit P_m (connected with exploitation of an innovation) exceeds the aggregate partner's traditional profit $\sum_{i=1}^{n} I_i(1+R_{iT})$.

IV. Implementation of USD Methodology

The success of welfare increasing sustainable development requires an exploitation of innovations and new technologies, developed in research institutes and implemented by cooperation with producers and investors. The cooperation is effective when the cooperation yields common benefit $(\delta > 0)$ and the benefits division among the cooperation partners is fair.

The proposed methodology can be used for searching, choosing and matching of prospective partners, who are characterized by small E_i (expenses) producing large δ and ΔU_i and, are able to exploit the innovations and new technologies. It can be also used for supporting the negotiation processes concerning the benefits division among the partners who have accepted the participation in the joint ventures.

The processes of searching, choosing and matching of prospective partners can be implemented by broking and consulting organizations employing exchange of information by internet. The wide dissemination of USD methodology can contribute to the acceleration of economic growth and competitiveness of national economy. It can also help the procedures, who have already exploited the traditional technology, and are loosing the cooperating clients, to avoid bankruptcy by switching to the new (innovative) technologies.

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