241/2002

Raport Badawczy

Research Report

RB/69/2002

Management of Intellectual Capital

T. Banek, R. Kulikowski

Instytut Badań Systemowych
Polska Akademia Nauk

Systems Research Institute
Polish Academy of Sciences

POLSKA AKADEMIA NAUK

Instytut Badań Systemowych

ul. Newelska 6

01-447 Warszawa

tel.: (+48) (22) 8373578

fax: (+48) (22) 8372772

Kierownik Pracowni zgłaszający pracę:
Prof. dr inż. Roman Kulikowski

Warszawa 2002

Management of Intellectual Capital

Tadeusz Banek[1+2], Roman Kulikowski[2]
(1) PL Lublin, (2) IBS PAN

December 12, 2002

Abstract

Two different types of firm's activity are considered as portfolio selection problems. First activity is an ordinary one. Innovative and extraordinary is the second. To be profitable it must have enough of an "intellectual capital" (IC). Optimization problems for the firm are solved and the notion of IC is defined. Two different methodologies are discussed.

1 Introduction

Let $r > 0$ be a rate of return from the firm's activity (production, services) carried out so far in stable economical conditions and established and implemented technologies. We assume that r is a known (deterministic) value. The firm carries out research (scientific, technical, technological, intelligence activities, etc.) on new solutions (technical: constructional, technological, commercial: new points of sale, new types of promotion and advertisement, etc.), which can bring a success in the form of a greater rate of return.

Formally, let $R(\omega)$ means a rate of return from expenses on **research and implementation**, where

$$R(\omega) = \begin{cases} R_u & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases} \tag{1}$$

If the firm spends on research and implementation some part (fraction) of its capital, then the increase $\Delta\xi$ of its wealth ξ (for a single period) is

$$\Delta\xi = (1-x)\, r\xi + xR(\omega)\,\xi \tag{2}$$

where $x \in [0,1]$ is the fraction mentioned above.

In result, the resultant rate of return from both types of activities (a routine and an innovative one) is

$$\rho(x,\omega) = \frac{\Delta\xi}{\xi} = (1-x)\, r + xR(\omega) \tag{3}$$

Let us remark that the fraction $(1-x)\, r\xi$ represents this part of the increase of the firm's wealth on which its material resources mainly work, whereas the fraction $xR(\omega)\xi$ represents the part of the increase of the firm's wealth on which its immaterial (intellectual) resources mainly work.

Problem 1 *Find*

$$\max_{x\in[0,1]} EU\left((1-x)\,r\xi + xR(\omega)\,\xi\right) \tag{4}$$

where $U(\cdot)$ *denotes a utility function of the firm,* E *- expected value operator.*

Case I

The world of decision and the world of chance are disjoint. The probability of success does not depend on decisions concerning e.g. the level of financing a research project.

Case II

Decision affects probabilities. The probability of success depends on the level of financing a research project, i.e.

$$p = p(x\xi)\,,\ \text{or } p = p(x)$$

for example

$$p(x\xi) = 1 - e^{-\lambda x\xi},\ \text{or } p(x) = 1 - e^{-\lambda x}$$

In this case, an alternative formulation of the problem is possible.

Let $Q(\omega)$ be a rate of return (with the probability distribution like in (1)) **from an activity carried out in the new technology**, obtained thanks to the research. The firm spends the fraction x_1 on an activity carried out in the old technology, x_2 spends on research and plans to spend x_3 on an activity in the new technology - when the research succeeds. The increase $\Delta\xi$ of the wealth ξ is

$$\Delta\xi = x_1 r\xi + x_3 Q(\omega)\,\xi$$

where

$$Q(\omega) = \begin{cases} R_u & \text{with probability } p(x_2) \\ 0 & \text{with probability } 1 - p(x_2) \end{cases}$$

The equivalent of **Problem 1** will be now

Problem 2 *Find*

$$\max_{x_1+x_2+x_3=1} EU\left(x_1 r\xi + x_3 Q(\omega)\,\xi\right)$$

2 Solutions

To Problem 1

Because

$$\begin{aligned} & EU\left(\left(1-x\right)r\xi+xR\left(\omega\right)\xi\right) \\ = & pU\left(\left(1-x\right)r\xi+xR_u\xi\right)+\left(1-p\right)U\left(\left(1-x\right)r\xi\right) \end{aligned} \tag{5}$$

so

$$\max_{x\in[0,1]} EU\left(\left(1-x\right)r\xi+xR\left(\omega\right)\xi\right)= \tag{6}$$

Case I

$$=\max_{x\in[0,1]}\left\{pU\left(\left(1-x\right)r\xi+xR_u\xi\right)+\left(1-p\right)U\left(\left(1-x\right)r\xi\right)\right\} \tag{7}$$

and the (Fermat) condition of disappearing of the first derivative gives

$$\frac{U'\left(\left(1-x\right)r\xi+xR_u\xi\right)}{U'\left(\left(1-x\right)r\xi\right)}=\frac{\left(1-p\right)r}{p\left(R_u-r\right)} \tag{8}$$

Example 3 *Let*

$$U\left(x\right)=\ln x$$

then

$$\frac{\left(1-x\right)r}{\left(1-x\right)r+xR_u}=\frac{\left(1-p\right)r}{p\left(R_u-r\right)}$$

and, finally

$$x_{opt}=\frac{pR_u-r}{R_u-r},\ when\ \frac{pR_u-r}{R_u-r}\in[0,1]$$

Example 4 *Let*

$$U\left(x\right)=x^\beta,\ \beta\in\left(0,1\right)$$

then

$$\frac{\left(1-x\right)r+xR_u}{\left(1-x\right)r}=\left(\frac{p\left(R_u-r\right)}{\left(1-p\right)r}\right)^{1/1-\beta}\triangleq\alpha$$

and, finally

$$x_{opt}=\frac{\alpha-r}{R_u+r\left(\alpha-1\right)},\ when\ \frac{\alpha-r}{R_u+r\left(\alpha-1\right)}\in[0,1]$$

Case II

$$= \max_{x\in[0,1]} \{p(x)\,U((1-x)\,r\xi + xR_u\xi) + (1-p(x))\,U((1-x)\,r\xi)\} \tag{9}$$

or

$$\max_{x\in[0,1]} \{p(x)\,U_1(x) + (1-p(x))\,U_2(x)\} \tag{10}$$

where

$$U_1(x) = U((1-x)\,r\xi + xR_u\xi)\,,\; U_2(x) = U((1-x)\,r\xi) \tag{11}$$

and the (Fermat) condition of disappearing of the first derivative gives

$$\begin{aligned} & p(x)\,U_1'(x)\,\xi\,(R_u - r) - (1-p(x))\,U_2'(x)\,r\xi \\ & +p'(x)\,[U_1(x) - U_2(x)] \\ = \; & 0 \end{aligned} \tag{12}$$

Example 5 *Let*

$$U(x) = \ln x$$

then

$$\begin{aligned} & \frac{R_u - r}{(1-x)\,r + xR_u} - \frac{1-p(x)}{(1-x)\,p(x)} \\ & +\frac{p'(x)}{p(x)} \ln \frac{(1-x)\,r\xi + xR_u\xi}{(1-x)\,r\xi} \\ = \; & 0 \end{aligned}$$

or

$$\begin{aligned} & \exp\left[\frac{1-p(x)}{(1-x)\,p(x)} - \frac{R_u - r}{(1-x)\,r + xR_u}\right] \\ = \; & \left[\frac{(1-x)\,r\xi + xR_u\xi}{(1-x)\,r\xi}\right]^{p'(x)/p(x)} \end{aligned}$$

Example 6 *Let*

$$U(x) = x^\beta$$

then

$$\begin{aligned} & p(x)\,\beta\,((1-x)\,r\xi + xR_u\xi)^{\beta-1}\,\xi\,(R_u - r) - (1-p(x))\,\beta\,((1-x)\,r\xi)^{\beta-1}\,r\xi \\ & +p'(x)\left[((1-x)\,r\xi + xR_u\xi)^\beta - ((1-x)\,r\xi)^\beta\right] \\ = \; & 0 \end{aligned}$$

To Problem 2
Because

$$\begin{aligned} & EU\left(x_1 r\xi + x_3 Q\left(\omega\right)\xi\right) \\ = \ & p\left(x_2\right) U\left(x_1 r\xi + x_3 R_u \xi\right) + \left(1 - p\left(x_2\right)\right) U\left(x_1 r\xi\right) \end{aligned}$$

so

$$\begin{aligned} & \max_{x_1+x_2+x_3=1} EU\left(x_1 r\xi + x_3 Q\left(\omega\right)\xi\right) \\ = \ & \max_{\substack{0\leq x_1, x_3\leq 1 \\ x_1+x_3\leq 1}} \left\{p\left(1-x_1-x_3\right) U\left(x_1 r\xi + x_3 R_u \xi\right) + \left(1 - p\left(1-x_1-x_3\right)\right) U\left(x_1 r\xi\right)\right\} \end{aligned}$$

3 Application of the R. Kulikowski methodology

To Problem 1

$$\max_{0\leq x\leq 1} \left\{u_1\left(x\right) + u_2\left(x\right)\right\}$$

where (see [1]}

$$u_1\left(x\right) = \left(1-x\right)^{\beta} r\xi$$

is the utility of the activity up to date (with no risk), where as the utility of the research activity is

$$\begin{aligned} u_2\left(x\right) &= s\left(p\right) x^{\beta} R_u \xi \\ s\left(p\right) &= p\left[1 - \left(1-S_0\right)\sqrt{\overline{q}/q}\right]^{1-\beta} \end{aligned}$$

By the Hőlder inequality we have

$$x_{opt} = \frac{a}{a+b} \tag{13}$$

where

$$\begin{aligned} a &= \left(r\xi\right)^{1/1-\beta} \\ b &= \left(R_u\xi\right)^{1/1-\beta}\left[1 - \left(1-S_0\right)\sqrt{\overline{q}/q}\right]^{1-\beta} \end{aligned}$$

To Problem 2

$$\max_{x_1+x_2+x_3=1} \left\{u_1\left(x_1\right) + u_2\left(x_2, x_3\right)\right\}$$

in **Problem II** (the "classical" methodology)

$$V_2(\xi) = \max_{x_1+x_2+x_3=1} EU\left(x_1 r\xi + x_3 Q(\omega)\xi\right)$$

the R. Kulikowski methodology

$$V_2^K(\xi) = \max_{x_1+x_2+x_3=1} \left\{u_1(x_1) + u_2(x_2, x_3)\right\}$$

denote maximal values of the utility function where using intellectual capital, where as

$$\begin{aligned} v_1(\xi) &= v_2(\xi) = U(r\xi) \\ v_1^K(\xi) &= v_2^K(\xi) = u_1(x) \end{aligned}$$

denote corresponding values of the utility function without using intellectual capital. One can then define intellectual capital as
a difference (the "classical" methodology)

$$\begin{aligned} K_i(\xi) &= V_i(\xi) - v_i(\xi) \\ i &= 1, 2 \end{aligned}$$

the R. Kulikowski methodology

$$\begin{aligned} K_i^K(\xi) &= V_i^K(\xi) - v_i^K(\xi) \\ i &= 1, 2 \end{aligned}$$

Another method of estimating intellectual capital is also possible, based e.g. on the notion of the value of information (see e.g. [3],[4],[5])

Bibliography

[1] R.Kulikowski, Problemy zarządzania ryzykiem, RB/28/2001 IBS PAN (lepiej „On [illegible] Theory")

[2] M.Romanowska, Kształtowanie wartości firmy w oparciu o kapitał intelektualny, System Informacji Strategicznej, DIFIN, W-wa 2001

[3] T. Banek, Rachunek Ryzyka, WSAiZ w Zamościu, Lublin 2000, Seria Monografie, Nr 2

[4] T. Banek, Wartość informacji, Konferencja KBO UŚ, Ustroń 2001

[5] T. Banek, R. Kulikowski, Value of Information, Control and Cybernetics