Raport Badawczy
Research Report

RB/34/2001

Decision support in safety oriented transport systems

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Warszawa 2001

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Abstract

The paper deals with decision support in transport systems. Two decision models are introduced. The first called "cost-risk allocation" enables one to allocate the given budget among the different risk areas in such a way that the resulting risk is minimum. The second model is based on the utility, return and safety, derived for the concrete decision maker. The optimum strategy maximizes the utility and enables evaluation, ranking and acceptance of the concrete transport investment projects. Numerical examples are included.

1. Introduction

The problem of transport-risk management is generally multidimensional. Each individual, as well as an institution or firm, is exposed to different threats or menaces, such as road accidents, damages or lost property, bankruptcy etc. At the same time individual, as well as the institutions, can spend a part of its budget on the risk prevention devices or diagnostic and prophylactic activities. Since the prevention effectiveness (i.e. prevention cost growth corresponding to the growth of safety, or decline of risk) changes much for different risk areas the problem of the best allocation of given budget among the set of risk areas can be formulated. For that purpose one should construct a formal model of cost-risk relationship for different sources of transport risks.

It became customary to assign transport risks to the three main sources: the road, the vehicle and the human element (driver, passenger, pedestrian) [1]. Each main source consists, in turn, on a number of factors; e.g. the driver element can be subdivide into; speeding, safety belts, alcohol, young drivers etc.

It is also possible to identify and assign a part of the total risk figure, (such as the probability of being killed in the road accident) to the corresponding factors. According to

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Crowley [1], "in nine out of ten accidents, at least one road user factor is present; in at least one in four a road factor is present; and in one in eight a vehicle factor (OECD, 1975)".

The next important step in the risk analysis is to identify the individuals (or authorities), who control allocation of budget for prevention etc activities, within the set of specified factors. As the main controllers (decision makers) within the decentralized, hierarchical control structure one should mention:

- State (transport ministry)
- Regions
- Firms and corporations
- Individuals.

In the opinion of many people the transport risk controlling system in many countries does not work efficiently. The maine causes of poor performance of many existing systems are:

a. Poor risk management knowledge and the lack of suitable control methodology
b. Poor risk assessment and lack of suitable risk information and statistical data
c. Poor correlation of risk involving decisions taken by the separate controllers.

The present paper main objective is to improve transport risk-management efficiency by a methodological support of the decision processes.

For that purpose two concrete optimization models are proposed.

The first, called cost-risk allocation model enables the decision maker to allocate the given budget for different risk factors in such a way that the resulting risk is minimum. The second, called utility maximization model enables one to choose the best risk prevention strategy, which maximizes the utility function of the decision maker. In that type of model the risk prevention budget is not fixed beforehand. The risk prevention cost is deducted from the financial return on a given investment. The model can be used therefore for situations when the utility depends on the safety and return within a long planning interval. The optimum strategy can be derived here by using the so called URS methodology which was described in the papers [3÷7].

2. Cost-risk allocation model

It is obvious that problems of people's safety arise in different areas such as, health-care, environment quality, transport system, criminal activity etc. In each area there is a probability (p_i) of peoples death or – survival $1-p_i$. Assuming that the survival events are statistically independent, the cumulative probability of survival $(1-p)$ of all (n) threats becomes

$$1-p=\prod_{i=1}^{n}(1-p_i)^i$$

and the cumulative death probability

$$p=1-\prod_{i=1}^{n}(1-p_i)^i$$

Since p_i are generally small numbers

$$p \cong \sum_{i=1}^{n} p_i . \qquad (1)$$

The p_i probabilities depend generally on local conditions, such as environment protection, organization of health and transport system etc., as well as – on the regional budget (i.e. the activities in health, education, police, transport etc.) which prevents regional population from disasters. The main problem, when the regional budget is planned can be formulated as: How to allocate the budget among different activities in order to get minimum p ?. To solve such a problem it is necessary to construct a model describing relation between probabilities p_i and the cost of prevention $C_i(p_i)$, $\forall i$.

In the model studied one assumes that the relative detriments $-\Delta p_i / p_i |_{p_i=\overline{p}_i}$ (at the fixed level $p_i = \overline{p}_i$) require the proportional cost increments $\Delta C_i / C_i$, i.e.

$$\frac{\Delta C_i}{C_i} = -e_i \frac{\Delta p_i}{p_i |_{p_i=\overline{p}_i}}, \quad \forall i \tag{2}$$

Using such a model one can easily compare the future (planned) state (p_i, C_i) with the existing state $(\overline{p}_i, \overline{C}_i)$.

The relation (2) for small increments $(\Delta \stackrel{\Delta}{=} d)$:

$$\frac{dC_i}{C_i} = -e_i \frac{dp_i}{p_i |_{p_i=\overline{p}}} = -e_i \frac{dx_i}{x_i |_{x_i=1}}, \tag{3}$$

is equivalent to the cost function

$$C_i(p_i) = C_i(\overline{p}_i x_i) = \overline{C}_i e^{e_i(1-x_i)}, \quad \forall i \tag{4}$$

where $x_i = p_i / \overline{p}_i$; $x_i > 0$, $\forall i$ are called the control variables.

The parameters

$$e_i \equiv -\frac{\Delta C_i}{\overline{C}_i} : \left(\frac{\Delta p_i}{\overline{p}_i} \right) = -\frac{\overline{p}_i}{\overline{C}_i} \cdot \frac{\Delta C_i}{\Delta p_i}$$

can be regarded as the so called elasticities (percentage increments of costs due to the risk reduction $-\Delta p_i / p_i$). If, for example in a given area the death rate reduction by 20% can be achieved by 10% increase of prevention costs (e.g. an early oncological diagnostic, screening, vaccinations etc. in the health-care system) one gets $e_i = 0.1 : 0.2 = 0.5$. When in another area the elasticity $e_j < 0.5$; one can say that in j-th area the prevention costs are acting more effectively as compared to the i-th area.

One can formulate now the problem of optimum budget allocation among the n areas, characterized by given $\overline{p}_i$, e_i, $\forall i$, parameters. Assume that in last planning interval the costs: $C_i(\overline{p}_i) = \overline{C}_i$ and the budget $\sum_i \overline{C}_i = \overline{C}$ are given.

The past risk, characterized by $\overline{p}_i$, i.e.

$$\overline{p} = \sum_{i=1}^{n} \overline{p}_i ,$$

is also known.

In the new planning interval, when the strategy $\{x_i\}_1^n$, is used the total risk becomes

$$p = \sum_{i=1}^{n} p_i = \sum_{i=1}^{n} \overline{p}_i x_i . \tag{5}$$

To achieve such a risk the following (new) budget

$$C=\sum_{i=1}^{n} C_i(\overline{p}_i x_i)=\sum_{i=1}^{n} \overline{C}_i e^{e_i(1-x_i)}, \tag{6}$$

is required.

The optimum strategy of risk prevention $\hat{x}=\{\hat{x}_1,\hat{x}_2,...,\hat{x}_n\}$, subject to the limited budget, can be derived by solving the following convex optimization problem:

$$\min_{x_i} = \sum_{i=1}^{n} \overline{p}_i x_i , \tag{7}$$

subject to

$$\sum_{i=1}^{n} \overline{C}_i e^{e_i(1-x_i)} \le C, \tag{8}$$

$$0 \le x_i \le 1/\overline{p}_i, \quad \forall i. \tag{9}$$

The solution for the simplified situation (neglecting (9) and inequality constraint in (8)) can be easily obtained by the classical Lagrange – multiplier technique.

Introducing the lagrangean:

$$\phi(x,L)=\sum_{i=1}^{n} \overline{p}_i x_i + L\left[\sum_{i=1}^{n} C_i(\overline{p}_i x_i)-C\right],$$

where $L=$ Lagrange multiplier, one can write down the necessary conditions of optimality

$$\phi'_{x_i}=\overline{p}_i - Le_i\overline{p}_i C_i(\overline{L}_i x_i)=0, \quad \forall i \tag{10}$$

$$\phi'_L=\sum_{i=1}^{n} C_i(\overline{p}_i x_i)-C=0. \tag{11}$$

Since by (10)

$$C_i(\overline{p}_i x_i)=\frac{1}{Le_i}, \quad \forall i$$

and by (11)

$$\sum_{j=1}^{n} \frac{1}{Le_i}=C;$$

one gets

$$L=\frac{1}{C}\sum_{j=1}^{n}\frac{1}{e_i},$$

and

$$C_i(\overline{p}_i\hat{x}_i)=\frac{1/e_i}{\sum_j 1/e_j}C, \quad \forall i. \tag{12}$$

According to (12) the optimum costs are proportional to $1/e_i$ and the budget shares for risk prevention in i-th area:

$$l_i=\frac{1}{C}C_i(\overline{p}_i\hat{x}_i)=\frac{1/e_i}{\sum_j 1/e_j}, \quad \forall i \tag{13}$$

where

$$\sum_{i=1}^{n} l_i = 1.$$

It should be observed that the linearity of the objective function (5) and strict convexity of constraint (8) are sufficient for the existance and uniqueness of the solution (i.e. the minimum of objective function). The optimum strategy by (4) and (12):

$$\hat{x}_i = 1 - \frac{1}{e_i} \ln C_i / \overline{C}_i, \quad \forall i \tag{14}$$

where

$$C_i \stackrel{\Delta}{=} C_i(\overline{p}_i \hat{x}_i) = \frac{1/e_i}{\sum_{j=1}^{n} 1/e_j} C,$$

$$\overline{C}_i \stackrel{\Delta}{=} C_i(\overline{p}_i).$$

One can observe that in order to satisfy (9) the conditions

$$e^{(1-1/\overline{p}_i)e_i} \le C_i / \overline{C}_i \le e^{e_i}, \quad \forall i$$

should hold, i.e. the "new/old" budget ratio $C_i / \overline{C}_i$, should not change much. In such a case, which usually holds in practical applications, the constraints (9) are not active so they can be dropped before the optimization is carried out. One can also observe that due to the convexity of constraints the optimum solution $\hat{x}$ is located on the border of constraints domain. Then the equality sign in (8) holds so one can apply the Lagrange multiplier technique (instead the more complicated Kuhn-Tucker approach).

The formulae (13) (14) express the cost allocation principle in the risk prevention problems. According to that principle the risk (5) is minimum when costs are allocated proportionally to $1/e_i$ (the inverse of cost elasticities).

In order to illustrate the application of proposed methodology consider a numerical example.

Numerical example 1

Suppose the regional planning office finds that the number of death casualties in transport has increased to the level $\overline{p}_1 = 0.01$, which is equal the rest of regional casualties $\overline{p}_2 = 0.01$. The existing budget for prevention of casualties in transport was $\overline{l}_1 = \overline{C}_1 / \overline{C} = 0.15$ and in the remaining areas $\overline{l}_2 = \overline{C}_2 / \overline{C} = 0.85$. The experts believe that elasticities of prevention costs are $e_1 = 2.5$, $e_2 = 1$. It is assumed that an increase of total budget $C / \overline{C} = 1.3$; in the planned period will take place.

The office wants to find the best new allocation (l_1, l_2) of budget between transport and rest of activities.

According to (13) one finds the new budget allocation:

$$l_1 = \frac{1/2.5}{1 + 1/2.5} = 0.286; \qquad l_2 = \frac{1}{1 + 1/2.5} = 0.714.$$

Then

$$C_1 / \overline{C}_1 = \frac{0.286C}{0.15\overline{C}} = 2.479; \quad C_2 / \overline{C}_2 = \frac{0.714C}{0.85\overline{C}} = 1.092.$$

By formulae (14) one gets the optimum strategy:

$$\hat{x}_1 = 1 - \frac{1}{2.5} \ln 2.479 = 0.637$$

$\hat{x}_2 = 1 - \ln 1.092 = 0.912$.

The optimum strategy enables the reduction of resulting risk, i.e. $p_1 = 0.01 \cdot 0.637 = 0.00637$; $p_2 = 0.01 \cdot 0.912 = 0.00912$ so $p = p_1 + p_2 = 0.015$, which is by $\frac{\overline{p} - p}{\overline{p}} \cdot 100\% = \frac{0.02 - 0.015}{0.02} = \frac{0.02 - 0.015}{0.02} \cdot 100\% = 25\%$ less the initial risk $\overline{p} = 0.02$.

It should observed that if we want to improve risk management in general and death prevention in particular, according to the proposed methodology, it is important to analyse the total risk situation, with transport risk included. It is important to keep data and evaluate accidents rate $(\overline{p}_i)$, as well as – the elasticities of cost of preventions (e_i). Since the risks and prevention budgets are decentralized, to get the most effective system of social safety (i.e. risk prevention) it is important to coordinate the policies and strategies taken by different decision units, including regional and state authorities, private business and individuals.

3. Utility maximization model

In the risk-cost allocation model the decision maker is allocating the given budget among a number of risk preventing activities in such a way that the resulting risk probability is minimum. Such a model is very useful when the risk prevention is resulting immediately (i.e. within the planed period), as in the case of allocation of maintenance costs, connected e.g. with number of road policemen, or prevention the road surface from skidding (especially in winter time) etc. Risk-allocation model can help also to choose transportation speed (one can reduce transportation risk at the expense of cargo delivery delay cost).

There is however a large number of transportation problems, where the risk prevention activities result mainly in the future, i.e. when one invests in the future safety. Usually investment is supposed to make a profit (return) at the admissible level of safety. A typical problem of that category is the evaluation of a highway investment project or investment in the new transport system or vehicle by an investor (firm or corporation). The individuals solve, as well, problems of that category when they decide how much to spend on health, education or vehicle to get a safe return in the future.

In order to construct the safety/return transportation model one has to start with the appropriate methodology.

3.1. URS methodology

URS methodology is a tool for supporting present decisions, which result (in future), in uncertain consequences.

The methodology deals with utility U based on the expected rate of return R and safety S, i.e. a notion, which is opposite to the risk.

In the present section main concepts (described in details in Ref. []) of URS methodology will be given.

For that purpose assume that one invests at $t = 0$ the capital $P(0) = P_0$, expecting to get at $t = 1$ the capital $P_1 = P(1) > P_0$. The return $\tilde{R} = \frac{P_1 - P_0}{P_0}$ is a random normally distributed variable, with given expected value $R = E\{\tilde{R}\}$ and the variance σ^2. The decision-maker (i.e. an investor) is interested in two monetary values:

a) expected monetary return $Z = P_0 R$

b) worse cases or-net monetary return $Y = P_0[R - \kappa\sigma]$, where κ can be called the "price of fear" of the worse case consequences.

Introducing the popular recently notion of "value at risk", i.e.

$$VaR = P_0 \kappa \sigma \tag{15}$$

and the complementary notion of "value at safety":

$$VaS = P_0 RS, \tag{16}$$

$$S = 1 - \kappa \frac{\sigma}{R}. \tag{17}$$

One gets also

$$VaR + VaS = VaE, \tag{18}$$

where VaE is the expected value $VaE = P_0 R$.

In the case when T and R are determined ex post (e.g. from historical data) for $t = T \neq 1$, one should replace (15) by $VaR = P_0 \kappa \sigma \sqrt{T}$ and $VaS = 1 - \kappa \frac{\sigma\sqrt{T}}{R}$, respectively.

The parameter κ can be also interpreted as a quantile of the normal probability distribution, shown in Fig. 1. The value $RS = Y / P_0$ splits the range of $\tilde{R}$ on two subsets with the probabilities.

$$Pr\{\tilde{R} - R \le \kappa_p \sigma\} = p$$

$$Pr\{|\tilde{R} - R| < |\kappa|_{1-2p}\, \sigma\} = 1 - 2p$$

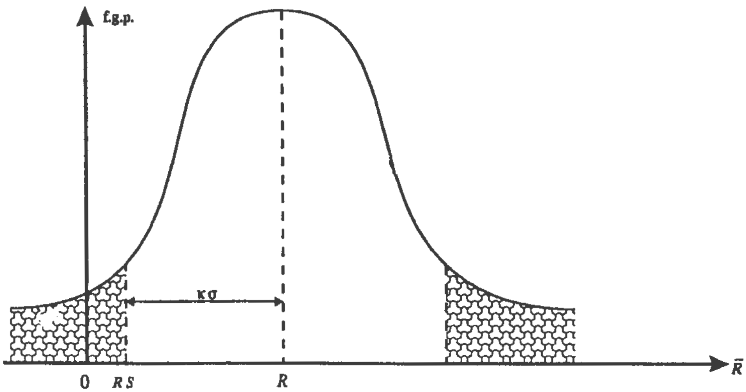

Fig. 1. Interpretation of κ as a quantile

The numerical values of $\kappa_p = |k|_{1-2p}$ for a given p can be easily derived using tables of normal p.d.f. E.g. $\kappa_{0.025} = |\kappa|_{0,95} \approx 1{,}96$, $\kappa_{1/6} = |\kappa| \cong 1$, etc.

One should observe that the index S, called in [5÷7] the index of safety or assurance, has a practical meaning for the investments, which are acceptable by the investor. Namely it should offer him a premium for bearing the risk, i.e. $R \geq R_F + \kappa\sigma$, where R_F is a return on risk-free investments (e.g. government bonds). The S is a positive number $S \in (0,1]$. One can see also, by (18), that an increase (decrease) of VaR requires a decrease (increase) of VaS.

It should be also noted that by introducing the notion of VaR or VaS one is able to deal with subjectively perceived notion of risk and safety in an explicit, numerical form.

In order to evaluate the utility of an investment the decision-maker has to introduce a suitable utility function. The concept of single factor utility $(F(z))$ was introduced in economic sciences long ago. That function was axiomatically justified in the well-known paper by von Neumann and Morgenstern (1949). It was, however, criticised by some economists (I. Fisher, M. Allais), who argued that people make decisions using the expected values as well as the variances σ^2. For these risons instead of single factor utility in the papers [3÷7] the concept of two factors-utility function, was introduced.

The first factor is $VaE = P_0RN$, where N is the number of items (e.g. shares of stock), with the price P_0 and expected rate of return R.

The second

$$VaS = P_0R\left(1 - \kappa\frac{\sigma}{R}\right) = P_0RS.$$

Then the utility function becomes

$$U = F[VaE, VaS] = F[P_0RN, P_0RS].$$

U is increasing function of both factors (VaE, VaS) and it reduces to the classical single factor utility $F(z)$ when one deals with deterministic cases, i.e. when $S = 1$.

Since both factors are expressed in monetary values the function F should be "constant return to scale". Otherwise it would change, when one changes monetary units (e.g. US \$ to 100 cents). The simplest function of required type is the Cobb-Douglas function, i.e.

$$U = (P_0RN)^{\beta}(P_0RS)^{1-\beta}, \quad S \in (0,1], \quad \beta \in [0,1]. \tag{19}$$

The function (19) increases along with R and S, as well as, number N. One can observe that utility (19) is expressed in monetary terms. It attains the maximum value for $S = 1$. When one invests in a stock with expected return P_0R [US \$] the utility at $S = 0.8$ and $\beta = 0.5$ is only 0.89 P_0R [US \$]. Then the factor $S^{1-\beta}$ discounts the expected value (P_0R) in the presence of a risk or threat. An increase of U along with N is negatively accelerated as in the case of classical single factor utility.

Using (19) it is possible to plot the constant utility curves on the $R(S)$ plane. These curves are described by the function

$$R = \overline{U} / S^{1-\beta}, \quad \text{where} \quad U / P_0N^{\beta} = const. \tag{20}$$

It should be noted that when a decision maker wants to apply URS methodology he should start with identification of his β and κ parameters. For that purpose he can use the "indifference to exchange" or – "certainty equivalent" concept. According to that concept an investor, having utility (19) is indifferent to exchange a risk free investment (e.g. the government bonds with utility $U_F = P_0 R_F$) for the risky investment (with utility $\overline{U} = P_0 R^* (\overline{S})^{1-\beta}$) or - vice versa. Since $\overline{U} = U_F$ and $\overline{S} = 1 - \overline{\kappa}\frac{\overline{\sigma}}{R^*}$, one finds easily

$$\overline{\kappa} = \left[1 - \left(\frac{R_F}{R^*}\right)^{\frac{1}{1-\beta}}\right]\frac{R^*}{\overline{\sigma}}. \tag{21}$$

Observe that $R^* > R_F$ so $\overline{\kappa}$ is a positive number increasing along with R^* when $\overline{\sigma} = const$. Using the indifference to exchange concept investors can choose (within the set of shares, with different R_i, and $\sigma_i = \overline{\sigma}$, $\forall i$) a share having the R^* return, which makes $\overline{U}$ equal U_F. Then, setting R^* in (21) one finds $\overline{\kappa}$.

When $\overline{\kappa}$ is identified investors can evaluate the safety (S_i) of any other investment by the formula

$$S_i = 1 - \overline{\kappa}\frac{\sigma_i}{R}, \quad \forall i. \tag{22}$$

In other words the identification of $\overline{\kappa}$ enables one the scaling of S as well as ranking and comparing different shares or risky investments. Obviously $\overline{\kappa}$ is a subjective parameter and it can vary in time for any individual. For that reason the rationally thinking decision maker, who wants to be consistent, should check $\overline{\kappa}$ before a new decision is taken. Consistency requires application of the scaling process, i.e. a periodic analysis of past decisions in order to detect (ex post) $\overline{\kappa}$ and correct it, if necessary, when the safety level $(\overline{\sigma})$ and consequently R^* changes.

In many risk management problems the return R and safety S depend parametrically on a control parameter y. Then the problem of choosing optimum strategy $y = \hat{y}$ can be formulated. An example of such a procedure, will be given in the next section.

It is assumed that $R(S)$ is strictly concave, decreasing function of y. Then the strategy y exists and is unique. One can also assume that $\{R,S\} > \beta$. Then one can also derive y formally by the condition

$$\delta U = \delta R + (1-\beta)\delta S = 0, \tag{23}$$

where $\delta U = \dot{U}/U$; $\delta R = \dot{R}/R$; $\delta S = \dot{S}/S$ are small increments. The condition (23) can be written also in the form

$$\frac{S(y)}{R(y)} = \frac{1-\beta}{\omega(y)}, \tag{24}$$

where $\omega(y) = -\frac{dR}{dy} : \frac{dS}{dy}$, called the optimum S/R principle.

Using (24) one can also construct an interactive algorithm for deriving $\hat{y}$ strategy [4].

It should be observed that in order to use the URS-methodology one should get as well the numerical values for the subjective parameter β. For determining β assume that R and S are constant, while P_0 and N get the small increments $\delta P_0 = \dot{P}_0 / P_0$; $\delta N = \dot{N}/N$. Then one get by $\delta U = \delta P_0 + \beta \delta N = 0$ or $\beta = -\delta P_0 / \delta N$. One can imagine that an offer of the stock seller is made to sell $N(1+\delta N)$ stocks at a decreased price $P_0(1-\delta P_0)$ each. If the buyer accepts the transition one can assume that his utility is characterised by

$\beta = -\delta P_0 / \delta N$. If e.g. the decrease of price by 0.5% is achieved by the increase of number of stocks bought by 1%, one gets $\beta = 0.5$.

The URS-methodology enables one also to set a simple rule of acceptance of projects characterised by given set $\{R_i, S_i\}$, $i = 1,2,...$. Indeed, the investor will accept a project only in the case, when it offers him utility $U(R_i, S_i) = P_i R_i S_i^{1-\beta}$ at least equal the utility of risk-free investment $U(R_F, 1) = P_i R_F$. Then the accepted project return should satisfy the following condition:

$$R_i \geq \frac{R_F}{S_i^{1-\beta}}, \quad i = 1,2,... . \tag{25}$$

3.2. Binominal success – failure model

Consider an investor who is studying the prospects of an investment. He applies for that purpose the binominal model with two possible states:

1. The success, when with probability $(1 - \overline{p}/y)$ the large enough return $R^u(y)$ will be achieved.
2. The failure, when with probability $\overline{p}/y$ the close to zero return $R^d(y)$ will be achieved.

The failure is the synonym of firm bankruptcy, i.e. the invested capital does not generate cash flow of return to meet future obligations. For an individual it means that the return on his investments does not generate the future earnings to support his existence or survival. The state of zero earnings (i.e. zero consumption) the individual can regard as equivalent to the lost life resulting e.g. from traffic accident.

In the present model y is a positive number, called safety control. When y increases $\overline{p}/y$ declines so the safety of the investor is growing. Usually the increased safety is achieved at the expense of declining return $R^u(y)$ so the main problem facing the investor is to choose the best compromise between safety and return. In other words, he would like to know what is the optimum value of $\overline{y} \overset{\Delta}{=} \hat{y}$.

Since

$$R^u(y) = \frac{P_1(y)\rho}{P_0(y)} - 1, \tag{26}$$

where

$P_0(y)$ = investment value,

$P_1(y)\rho$ = the present value of return "cash flow" ($P_1(y)$ each year),

$\rho = \sum_{t=1}^{T_e} (1+r)^{-t}$, T_e = planning interval, r = discount rate.

Observe that for large T_e, $\rho \cong 1/r$ and $P_0(y)$ is an increasing function of y. It is therefore assumed that there exists a positive elasticity $\mu_0 = P_0'(y)/P_0(y)$ so $P_0(y)$ can be approximated by the exponential function

$$P_0(y) = P_0(1) e^{\mu_0 (y-1)}.$$

In the similar way one can define $\mu_1 = P_1'(y)/P_1(y)$ and elasticity of yield $V(y) \stackrel{\Delta}{=} P_1(y)/P_0(y)$:

$$\mu = -\dot{V}(y)/V(y) = \mu_0 - \mu_1. \tag{27}$$

To find the numerical values of the elasticities introduced one can use the relations:

$$\mu_0 \cong \frac{\Delta P_0}{P_0} : \frac{\Delta y}{y}\bigg|_{y=1} \tag{28}$$

$$\mu_1 \cong \frac{\Delta P_1}{P_1} : \frac{\Delta y}{y}\bigg|_{y=1}. \tag{29}$$

The relations (28) (29) have a simple interpretation. Namely μ_0 can be regarded as the percentage of investment cost increase $(\frac{\Delta P_0}{P_0} 100\%)$ necessary to get the fixed percentage of control increase $(\frac{\Delta y}{y} 100\%)$. Since y growth requires additional costs μ_0 is positive. A similar situation is, generally, with μ, i.e. $\mu > 0$, because usually $\mu_1 < \mu_0$.

It is now possible to approximate $R^u(y)$ by the function

$$R^u(y) = V(1)e^{-\mu(y-1)} - 1,$$

and find the expected return,

$$R(y) = R^u(y)(1 - \overline{p}/y) + R^d \overline{p}/y$$

and the variance

$$[\sigma(y)]^2 = (1 - \overline{p}/y)[R(y) - R^u(y)]^2 + \overline{p}/y[R(y) - R^d(y)]^2 =$$
$$= (1 - \overline{p}/y)\overline{p}/y[R^u(y) - R^d(y)]^2$$

Then one can derive the safety index S by (17):

$$S(y) = 1 - \overline{\kappa}\frac{\sigma(y)}{R(y)},$$

where, for the sake of brevity, one assumes $R^d(y) = 0$, so

$$S(y) = 1 - \overline{\kappa}\frac{\sqrt{(1 - \overline{p}/y)\overline{p}/y}\; R^u(y)}{(1 - \overline{p}/y)R^u(y)} = 1 - \overline{\kappa}\sqrt{\frac{\overline{p}}{y - \overline{p}}}. \tag{30}$$

The value of $\overline{\kappa}$, derived by (21) for $y = 1$, becomes:

$$\overline{\kappa} = \left[1 - \left(\frac{R_F}{R^*}\right)^{\frac{1}{1-\beta}}\right]\frac{R^*(1 - \overline{p})}{\sqrt{\overline{p}(1 - \overline{p})}R^*} = a\sqrt{\frac{1 - \overline{p}}{\overline{p}}}, \quad a = 1 - \left(\frac{R_F}{R^*}\right)^{\frac{1}{1-\beta}}.$$

Then (30) becomes

$$S(y) = 1 - a\sqrt{\frac{1 - \overline{p}}{y - \overline{p}}},$$

which for small $\overline{p}$, can be written

$$S(y) \cong 1 - \frac{a}{\sqrt{y}}. \tag{31}$$

Since $\overline{p}/y$ is small $(y \geq 1)$ one can also approximate $R(y)$ by $R^u(y)$:

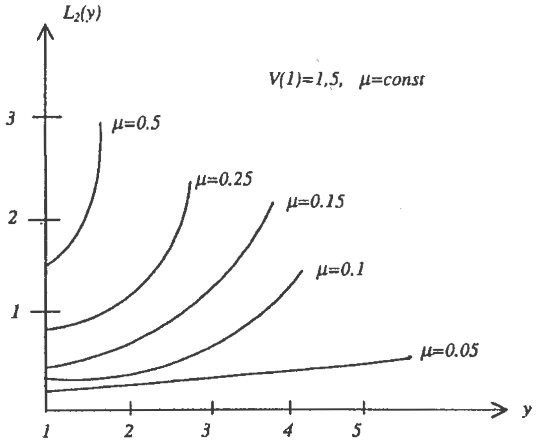

Fig. 2b. Plots of loss of return lines

The plots of $L_1(y)$ for $\beta = 0.5$ and $a = const.$, as well as the plots of $L_2(y)$ for $V(1) = 1.5$ and $\mu = const.$, are given in Fig. 2a, b. In Fig. 2a the intersection of two plots (for $\mu = 0.05$ and $a = 0.9$) which determines $\hat{y}$ is also shown.

It can be observed that when the elasticity of yield (μ) increases (declines) while risk aversion (a) is constant the safety factor $\hat{y}$ declines (increases). When $\mu = const.$ and a declines (increases) the safety factor $\hat{y}$ declines (increases). These observations are in agreement with a behavioral model of an investor. When his aversion to risk (a) increases he will choose bigger value of $\hat{y}$ and he will decrease $\hat{y}$ when the yield elasticity of prevention cost (μ) is growing.

To illustrate the application of the proposed methodology for a concrete investment problem consider the following example.

Numerical example 2

A taxi driver considers buying a car equiped optionally with a number of risk reducing devices, such as ABS, air bugs, winter tyres, chains etc., which are characterized by the corresponding y-factors. He beliefs that $P_1(1)/P_0(1) = 0.225$ while ρ (for $r = 0.15$, $T_e \to \infty$) is equal 6.67 so $V(1) = 0.225 \cdot 6.67 = 1.5$. He calculates also yield elasticities and gets $\mu_0 = 0.1$; $\mu_1 = 0.05$ so $\mu = \mu_0 - \mu_1 = 0.05$. Then the driver calculates his risk aversion a. He estimates $R_F / R^* = 0.32$, i.e. his indifferent return R^* is 3.12 times the risk free return R_F (which could be obtained e.g. from investments in government bonds). Then, assuming $\beta = 0.5$ the driver finds

$$a = 1 - (0.32)^2 = 0.9, \quad \alpha = 0.225.$$

Using the plots, shown in Fig. 2a, he finds his best risk prevention strategy $\hat{y} = 2.25$. For that strategy the expected return

$$R(y) \cong R''(y) = V(1)e^{-\mu(y-1)} - 1. \tag{32}$$

It should be noted that approximations used in (31) (32) are justified in the problems studied. Analyzing e.g. the bankruptcy of US firms within 1925÷1990 one finds [2] that the failure rate was changing from 0.0004 (in 1945) up to 0.0154 (in 1932) with the average in recent years (1970÷1990) around $\overline{p} = 0.005$. As far as the mortality in road accidents is concerned the data collected by Main Statistical Office (GUS) identicate that an average mortality rate in Poland (number of casualties per registred vehicles) was in 1998 arround $\overline{p} = 0.0006$.

Now it is possible to derive the optimum control strategy $y \stackrel{\Delta}{=} \hat{y}$, using the optimum S/R principle (23). For that purpose one has to compute:

$$\delta S = \frac{dS}{dy} : S = \frac{a}{2} \frac{1/y}{\sqrt{y-a}}, \tag{33}$$

$$\delta R = \frac{dR}{dy} : R = -\mu \left[1 - \frac{1}{V(1)} e^{\mu(y-1)}\right]^{-1} \tag{34}$$

and find the intersection of two plots of functions:

$$(1-\beta)\delta S \stackrel{\Delta}{=} L_1(y) = \alpha \frac{y^{-1}}{\sqrt{y-a}}, \quad \text{where} \quad \alpha = \frac{a(1-\beta)}{2}, \tag{35}$$

$$\delta R \stackrel{\Delta}{=} L_2(y) = \mu \left[1 - \frac{1}{V(1)} e^{\mu(y-1)}\right]^{-1}. \tag{35}$$

The parameter α can be called the risk aversion factor, while $L_1(y)$ for $a = const.$, is called the "risk line" and $L_2(y)$ for $\mu = const.$, $V(1) = const.$ is called "loss of return line".

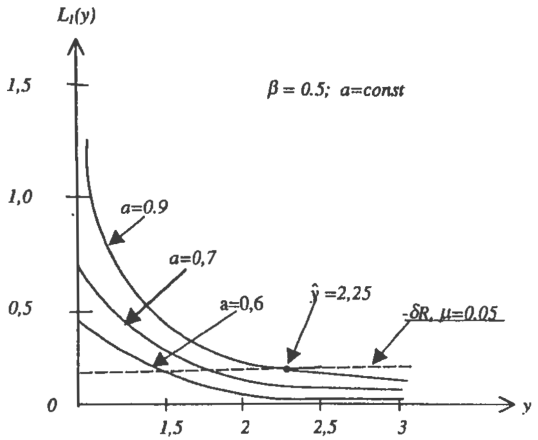

Fig. 2a. Plots of risk lines

$$R(2.25) = 1.5\, e^{-0.1 \cdot 1.25} - 1 = 0.324$$

and safety

$$S(2.25) = 1 - \frac{0.9}{\sqrt{2.25}} = 0.400\,.$$

The initial driver's utility

$$U(1) = R(1)\sqrt{S(1)}\, P_0(1) = 0.5\sqrt{0.1}\, P_0(1) = 0.158\, P_0(1)$$

and for $\hat{y} = 2.25$ the utility becomes

$$U(2.25) = R(2.25)\sqrt{S(2.25)}\, P_0(2.25) = 0.324\sqrt{0.4}\, P_0(2.25) = 0.205\,.$$

Then, due to the optimum strategy, the utility U / P_0 has increased from 0.158 up to 0.205. The increase of investment cost due to the risk prevention becomes

$$\Delta P = P_0(1)[e^{0.1 \cdot 1.25} - 1] \approx 0.13\, P_0(1), \text{ i.e. } 13\% \text{ of } P_0(1).$$

3.3. Applications of URS methodology to support decision in transport problems

The methodology based on investor's utility, return and safety (i.e. the URS methodology) is a general tool to support risk involving decisions in the numerical (computerized) form. Using such a tool one can avoid ambiguity due to the subjective perception of risk and safety by individual decision makers. After the scalling procedure (i.e. self identification of β and $\bar{\kappa}$ parameters) the investor can express his risk, safety and utility in an explicite numerical or monetary form. He can also derive the best strategy for his (personal) model of utility. The strategy suggested by the model depends on the individually evaluated risk aversion (a) as well as yield etasticity (μ), which can be derived for each concrete risk management or risk prevention problem. Such a personalized approach by URS methodology helps to avoid arbitrary decisions or decisions influenced by public pressure or noncompetent emotions. The methodology helps investors to be consistent in planning and management of risky project.

The methodology is also helphull in the evaluation of complex risk involving projects, such as the project of a new highway. The highway can be regarded as detrimental to the regional population as it produces air pollution, noise etc., which increase death probability, and destruct the ecology. In order to protect people the local authorities require the highway parameters to adhere to costly strandards (for example the A2-highway project is required to pass Ursynów district in Warsaw by an underground tunnel). The costly standards increase the elasticity of investment cost (μ_0 and μ).

On the other hand there is a positive effect of highway project on the regional economic growth and regional welfare so the authorities can contribute to an increase of μ_1. The resulting $\mu = \mu_0 - \mu_1$ is the net effect of both tendencies, i.e. the desire of investors to get large return and the desire of regional authorities to have increasing welfare and safety. In order to find a consensus between these controversional tendencies one can use the Nash approach, which requires that the product of utilities of both parties concerned is maximum. Using such an approach the negotiation supporting methodology was developed in Systems Research Institute, which for brevity reason, one is not pursueing in the present paper.

Observe also that when the optimum $\hat{y}$ strategy is derived, by URS methodology, one can easily find the expected return $R(\hat{y})$ and safety $S(\hat{y})$ of the highway project. Then the investor is able to check is the project profitable using the acceptance condition (25), i.e.

$$R(\hat{y}) \geq \frac{R_F}{S(\hat{y})^{1-\beta}}, \quad R_F = \text{risk free return} \tag{36}$$

When (36) does not hold there is no chance the investor will engage in the project. Using relation (36) one can also evaluate and rank different transport projects. Suppose e.g. the city should decide which projects, dealing with extension of road system or underground, to choose within the limited investment budget. Using the URS methodology one can compute $R_i(\hat{y}_i)$, $S_i(\hat{y}_i)$, $\forall i$ and using (36) choose a subset of projects having biggest utilities.

Using URS methodology one can also support decisions regarding speed limits for the city traffic. The decision to introduce a low speed limit is justified by the desire to have low death casualties. On the other hand the low speed limit results in an increased transportation time and decline of economic returns. It is possible to formulate the problem in terms of $\overline{p}/y$, where y is a decreasing function of speed limit, and find such limit which maximizes the drivers utility function.

References

[1] Crowley F. Impact of new technologies on efficiency and safety. The European Conference of Ministers of Transport, ECMT, Paris, 402-447, 1993.

[2] Levy H., Sarnat M. Capital investment & financial decisions, Prentice Hall, New York 1994.

[3] Kulikowski R. Porfolio optimization: Two factors-utility approach, Control and Cybernetics, No 3, 1998.

[4] Kulikowski R. Optimum safety/return principle and applications, Bulletin of Polish Academy of Sciences, Technical Sciences, 48(2), 2000.

[5] Kulikowski R. Optimization of survival strategy by application of safety dependent utility model. Control and Cynernetics, 29(1), 2000.

[6] Kulikowski R. Wspomaganie decyzji inwestora w warunkach ryzyka. Rynek Terminowy No 2, 2000.

[7] Kulikowski R., Libura M., Słomiński L. Wspomaganie decyzji inwestycyjnych. IBS PAN, Warszawa 1998.

[8] Reynolds S.F. The cost of road accidents. Journal of the Royal Statistical Society, 119, 393-708, 1956.

[9] Smeed R.I. The usefulness of formulae in traffic engineering and road safety. Acc. Analysis & Prevention, 4(4), 1972.

[10] Van Ouwerkerk. Quality of life social costs. The European Conference of Ministers of Transport, ECMT, Paris, 391-424, 1989.