

Temperature and rate effects in metal plasticity

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THE PHYSICAL basis of plastic flow in metals is reviewed and the most important controlling processes are discussed. Theoretical treatments of some of the relevant micro-mechanisms are outlined and the resulting predictions of temperature and rate effects are indicated. In particular, the thermal-activation rate theory for the motion of dislocations is considered in some detail, and the significance of activation parameters is discussed. Attention is given to the existence of history effects and the conditions under which the correlation of temperature and rate sensitivities is possible. Experimental results obtained in dynamic tests on various materials are summarized and discussed in terms of the theories of micro-mechanisms already reviewed. An account is given of some recent work at very high strain rates and under combined stresses, and the paper concludes with a discussion of the problem of formulating constitutive equations for application to important practical problems.

Dokonano przeglądu fizykalnych podstaw plastycznego płynięcia w metalach oraz przedyskutowano najważniejsze procesy kontrolujące. Podano zarys teoretycznej analizy kilku istotnych mikromechanizmów i na ich podstawie określono przewidywane wartości temperatury i prędkości. W szczególności rozważono szczegółowo prędkościową teorię termicznych aktywacji i przedyskutowano znaczenie parametrów aktywacji. Zwrócono uwagę na istnienie efektów historii oraz warunków, przy których możliwa jest korelacja wrażliwości materiału na temperaturę i prędkość. Otrzymane wyniki eksperymentalne w badaniach dynamicznych nad różnymi materiałami zostały podsumowane i przedyskutowane w ramach wcześniej omówionych teorii mikromechanizmów. Wzięto pod uwagę kilka ostatnich prac dotyczących bardzo dużych prędkości odkształcenia i złożonego stanu naprężenia. W zakończeniu przeprowadzono dyskusję problemu formułowania równań konstytutywnych celem zastosowania ich do ważnych zagadnień praktycznych.

Проведено обозрение физических основ пластического течения в металлах и обсуждены важнейшие проверочные процессы. Дается очерк теоретического анализа нескольких подходящих макромеханизмов и на их основе определены предвиденные значения температуры и скорости. В частности подробно рассмотрена скоростная теория термических активаций и обсуждено значение параметров активации. Обращено внимание на существование эффектов истории, а также условий, при которых возможна корреляция чувствительности материала на температуру и скорость. Полученные экспериментальные результаты в динамических исследованиях разных материалов подытожены и обсуждены в рамках ранее рассмотренных теорий микромеханизмов. Принято в рассмотрение несколько последних работ, касающихся очень больших скоростей деформации и сложного напряженного состояния. В заключение проведено обсуждение задачи формулировки определяющих уравнений с целью применения их для важных практических вопросов.

1. Introduction

THE CLASSICAL papers of OROWAN, POLANYI and TAYLOR, all published in 1934, provided the answer to the long-standing question concerning the large difference between the theoretical and experimental values for the flow stresses of metals. It became clear that plastic flow occurs by the movement of line defects, or dislocations, through the crystal lattice. It was, however, another two decades before it was realized that in general this movement must be considered as a dynamical process, so that a satisfactory theory of

plastic flow must include the strain rate as a basic parameter. This fact was also implicit in the development of macroscopic plasticity theory, which relates plastic strain increments, or rates, to the applied stresses; in this context, however, time is eliminated by the assumption of ideal or rate-independent plasticity.

The basic kinematic equation which is required in relating the macroscopic flow to the microscopic rate-controlling mechanism was first stated by OROWAN [1] in 1940. It is an expression of the fact that the strain rate is proportional to the magnitude of the Burgers vector b and to the area swept out by the dislocations in unit time. OROWAN considered this area to be the product of the total length of dislocations and the average velocity with which they move. If these quantities can be related to the stress, strain, temperature or other macroscopic parameters, Orowan's equation provides the link between the microscopic and macroscopic. This approach has been extensively used by JOHNSTON and GILMAN, HAHN and others [2-5] to calculate stress-strain curves based on various empirical formulae for the dislocation density and mean dislocation velocity in terms of macroscopic quantities. These studies have shown that it is quite possible to generate qualitatively correct stress-strain curves in this way. There is, however, a basic difficulty in determining experimentally the underlying relationships, since the instantaneous mobile dislocation density and velocity cannot be measured. Indeed, it can be argued that it is in principle impossible to measure these quantities separately except under special conditions where dislocation density is so low that the motion of individual dislocations can be observed; such conditions are of course very different from those obtaining in ordinary polycrystalline metals and alloys.

It is well known that at low strain rates or moderately high temperatures many metals show small temperature and rate dependences, and this has led to the concept of an "athermal" stress which is governed by relatively large-scale internal stresses caused by grain boundaries, precipitate particles or other obstacles to dislocation motion. It is usually assumed, following SEEGER [6], that in general the flow stress is the sum of this athermal component and a thermal component. The latter is required to enable dislocations to move past short-range barriers such as solute atoms or intersecting dislocations; in this process, thermal energy is also available, so that the strain rate is governed by a relation of the Arrhenius type. This approach has been successful in explaining the observed rate dependence of various metals and alloys, though there are some difficulties in its general application.

Accepting the validity of the assumption that a total activation energy ΔH_0 is required to overcome a short-range barrier, part of which may be supplied by thermal motion, it is clear that if the applied stress is large enough the whole of the energy ΔH_0 will be supplied by mechanical work, so that thermal activation will no longer be required. The critical value of applied stress must correspond with the flow stress at absolute zero of temperature. For stresses greater than this the flow ceases to be controlled by the short-range barriers, and must depend on dissipative processes which occur when a dislocation moves through the lattice. A review of several such processes has been given by NABARRO [7]. In many cases, the dissipation is of a linearly viscous nature, so that a linear dependence of flow stress on strain rate would be expected.

At extremely high strain rates, relativistic effects may become dominant, since it can

be shown that the energy associated with a moving dislocation becomes indefinitely large as its speed approaches the sonic wave speed. Thus there is a limiting strain rate which corresponds to the motion of all the dislocations at this same speed. However, this rate is, for ordinary metals, so high that it seems doubtful whether it is reached except possibly under explosive loading.

2. Thermally-activated plastic flow

From absolute rate theory the dislocation velocity is given by

$$(2.1) \quad v = 2v_0 \exp\left(-\frac{\Delta H_0}{kT}\right) \operatorname{sh}\left(\frac{b}{kT} \int_0^{\tau^*} A d\tau^*\right),$$

where v_0 is the limiting velocity attained when the total activation energy ΔH_0 is supplied by external work, b — the Burgers vector, A — the area swept out during activation (the activation area), τ^* — the thermal component of the shear stress, k — Boltzmann's constant, and T the absolute temperature.

For small values of τ^* this gives a linear dependence of v on τ^* , but for large τ^* we may write

$$(2.2) \quad v = v_0 \exp\left(-\frac{\Delta H_0 - \int_0^{\tau^*} V d\tau^*}{kT}\right),$$

where $V = Ab$ is the activation volume.

It should be noted that a different definition of activation volume is sometimes used (mean activation volume):

$$V' = \frac{b}{\tau^*} \int_0^{\tau^*} A d\tau^* = \frac{1}{\tau^*} \int_0^{\tau^*} V d\tau^*,$$

in which case (2.2) becomes

$$(2.3) \quad v = v_0 \exp\left(-\frac{\Delta H_0 - V'\tau^*}{kT}\right).$$

The two definitions coincide if A does not depend on τ^* ; but in general both V and V' depend on τ^* .

Combining (2.3) with Orowan's kinematic relation,

$$(2.4) \quad \dot{\gamma}^p = b\rho v,$$

where $\dot{\gamma}^p$ is the plastic shear strain rate and ρ is the mobile dislocation density, we obtain

$$\tau^* = \frac{\Delta H_0}{V'} + \frac{kT}{V'} \ln\left(\frac{\dot{\gamma}^p}{\dot{\gamma}_0}\right),$$

where $\dot{\gamma}_0 = b\rho v_0$.

Thus

$$(2.5) \quad \tau = \tau_A + \frac{\Delta H_0}{V'} - \frac{kT}{V'} \ln\left(\frac{\dot{\gamma}_0}{\dot{\gamma}^p}\right),$$

where τ_A is the athermal component of stress. Thus τ is a function of the "velocity-modified temperature" $T_{\text{mod}} = T \ln(\dot{\gamma}_0/\dot{\gamma}^p)$. If V and V' are independent of τ^* , the stress varies linearly with the logarithm of the plastic strain rate and reaches the limiting value $\dot{\gamma}_0$ at a stress

$$(2.6) \quad \tau_0 = \tau_A + \frac{\Delta H_0}{V'}$$

If, however, V' varies with τ^* , the relation between τ^* and $\ln \dot{\gamma}^p$ is non-linear although there is still a limiting stress corresponding to $\dot{\gamma}^p = \dot{\gamma}_0$. In general we may write

$$V = kT \left(\frac{\partial \ln \dot{\gamma}^p}{\partial \tau^*} \right)_T \approx kT \left(\frac{\partial \ln \dot{\gamma}^p}{\partial \tau} \right)_T,$$

provided that $\dot{\gamma}_0$ does not vary significantly with τ^* . Thus if V is also independent of τ^* , plots of thermal stress against the logarithm of the plastic strain rate should be straight lines of slope proportional to absolute temperature; if V depends on τ^* but not on T , these plots are curved, though at a given stress τ^* the slopes are still proportional to T .

Theoretical non-dimensional plots have been calculated by DAVIDSON and LINDHOLM [8] for various assumed relations between V and τ^* , which correspond to different models for the short-range barriers. They conclude that for f.c.c. metals, in particular aluminium, Seeger's assumption that V is independent of τ^* gives an adequate representation of experimental data over wide ranges of strain rate and temperature, under constant-rate conditions. This behaviour is illustrated in Fig. 1.

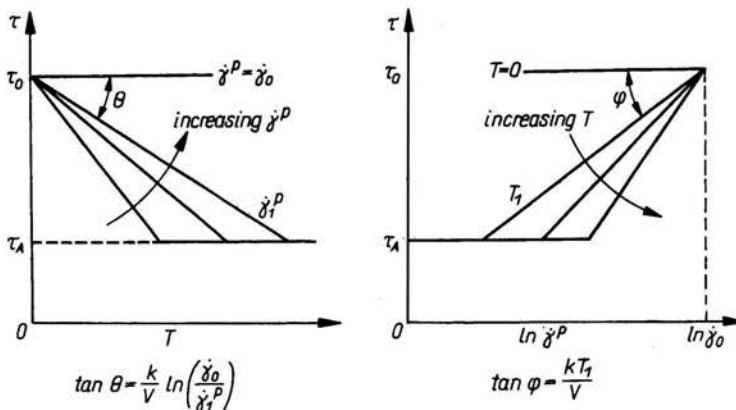


FIG. 1. Temperature and rate sensitivity of flow stress for a metal with constant activation volume.

For b.c.c. metals, on the other hand, it seems that the behaviour cannot be fully described by this model unless V varies with both τ^* and T . This conclusion also follows from measurements of dislocation velocity as a function of stress and temperature, made by TURNER and VREELAND [9] on iron single crystals.

It is possible that the measured strain rate is controlled by two or more processes corresponding to different thermally-activated mechanisms. If these mechanisms operate on different segments of dislocation which do not interact, we may expect the plastic strain rate to be the sum of two rates, each related to the stress by an equation such as

(2.5). Then at a given stress the high-rate mechanism will dominate, so that at low temperatures the mechanism with smaller activation energy will be controlling, and at high temperatures that with larger activation energy will be controlling. If, on the other hand, each dislocation segment is subjected to two types of barrier, the plastic strain rate will be governed by the low-rate mechanism, so that the contrary will apply. The available data for b.c.c. metals are consistent with the first of these two possibilities.

A further indication that the flow stress of b.c.c. metals may be dependent on more than one thermally-activated mechanism is given by strain-rate change tests on molybdenum and niobium [10]. These show that the rate sensitivity of the flow stress depends on the previous strain history in a complex manner, the behaviour showing a significant change as the initial rate is changed from a quasi-static rate to a rate of order 10^{-1} s^{-1} .

In spite of these difficulties in applying (2.5) to b.c.c. metals, it has been shown by ARMSTRONG and CAMPBELL [11] that an equation of this type nevertheless holds as a first approximation for constant rate conditions with V inversely proportional to τ^* . This relation implies that ΔH_0 and $\int_0^{\tau^*} V d\tau^*$ are both infinite, though their difference — the activation energy ΔH — remains finite; it is given by

$$(2.7) \quad \Delta H = \int_{\tau^*}^{\tau_0^*} V d\tau^* = C \ln \left(\frac{\tau_0^*}{\tau^*} \right),$$

where $C = V\tau^*$ is a constant. The Eq. (2.2) then becomes

$$(2.8) \quad v = v_0 \left(\frac{\tau^*}{\tau_0^*} \right)^{C/kT}.$$

An empirical equation of the form (2.7) was first suggested by YOKOBORI [12] on the basis of calculations made by COTTRELL and BILBY [13]. The power law (2.8) was used to describe experimental observations of dislocation motion made by JOHNSTON and GILMAN [2] for lithium fluoride, and by STEIN and LOW [14] for silicon-iron.

From (2.4) and (2.8) we obtain:

$$(2.9) \quad \tau^* = \tau_0^* \left(\frac{\dot{\gamma}_0}{\dot{\gamma}^p} \right)^{-kT/C}.$$

According to this approach, therefore, the flow stress is given by

$$(2.10) \quad \text{or} \quad \begin{aligned} \tau &= \tau_A + \tau_0^* (\dot{\gamma}_0 / \dot{\gamma}^p)^{-\beta_0 T}, \\ \tau &= \tau_A + \tau_0^* \exp(-\beta T), \end{aligned}$$

where τ_A is the athermal component of stress and $\beta_0 = k/C$, $\beta = \beta_0 \ln \left(\frac{\dot{\gamma}_0}{\dot{\gamma}^p} \right)$.

The Eqs. (2.9) and (2.10) are illustrated in Fig. 2.

In general, it appears that much of the constant-rate data for f.c.c. metals can be reasonably represented by (2.5), while (2.10) may be used to describe results for b.c.c. metals, though here the choice of τ_A is crucial and some discrepancies are evident. Data for h.c.p. metals are less extensive than for f.c.c. and b.c.c. metals. Since in general slip must occur

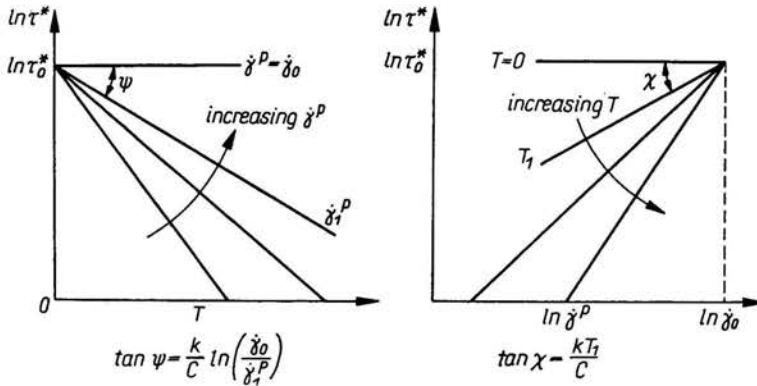


FIG. 2. Temperature and rate sensitivity of flow stress for a metal with activation volume inversely proportional to thermal component of stress.

on both prismatic and basal planes and may therefore be governed by more than one mechanism, the behaviour is more complex than for f.c.c. metals.

The basis of the treatment discussed above is that the stress may be considered as the sum of an athermal and a thermal component. This assumption has been examined by LI [15] who pointed out that a dislocation will in general move in a fluctuating stress field; taking the fluctuation to be sinusoidal, LI calculated the mean velocity in terms of the total stress and the amplitude of the stress fluctuations, assuming that the instantaneous velocity is a power-function of the local stress. Hence he calculated an effective internal stress which if subtracted from the applied stress, would give a (constant) velocity equal to the mean velocity. LI concluded that in most cases the effective internal stress is nearly equal to the amplitude of the stress fluctuations.

It should be noted, however, that such a conclusion is not easily applied to a real material, since the internal stress fluctuations will be of a random nature rather than sinusoidal. A statistical treatment is thus required, and it is not clear that the usual practice of subtracting a constant internal stress can be justified.

LI also calculated apparent activation parameters, and showed that they are larger than those without an internal stress field.

Another uncertainty involved in the thermal-activation-rate analysis is the possibility of a stress-dependent entropy of activation Δs . This is related to the activation volume by the Maxwell relation

$$(2.11) \quad \left(\frac{\partial \Delta s}{\partial \tau^*} \right)_T = \left(\frac{\partial V}{\partial T} \right)_{\tau^*}$$

Experimental results for b.c.c. metals indicate that these terms are not zero, or alternatively that more than one mechanism is operative.

Finally, the role of the pre-exponential factor $\dot{\gamma}_0$ is not yet clear. It is commonly assumed to depend only on strain, but a significant dependence on stress cannot be ruled out.

For all these reasons, it is evident that the theory is not sufficiently well established to be used with confidence to predict the rate and temperature dependence of stress for

real materials. Its chief use is in giving a qualitatively correct description of behaviour and in suggesting forms for empirical relationships which can describe the observed behaviour in certain ranges of the variables.

3. Quasi-viscous flow

At sufficiently high stresses, the local obstacles to dislocation motion become ineffective in controlling the strain rate, since thermal activation is not required. The stress at which this first occurs is

$$(3.1) \quad \tau_0 = \tau_A + \tau_0^*$$

or, for a stress-independent activation volume V ,

$$(3.2) \quad \tau_0 = \tau_A + \frac{\Delta H_0}{V}.$$

At this stress, the plastic strain rate becomes equal to the pre-exponential factor $\dot{\gamma}_0$, at any temperature; the stress τ_0 is also the limiting flow stress at absolute zero temperature, for any strain rate up to $\dot{\gamma}_0$.

At stresses above τ_0 or strain rates above $\dot{\gamma}_0$, flow must be controlled by dissipative processes within the crystal lattice, as dislocations move through it. A review of theories of a number of such processes has been given by NABARRO [7], who concluded that several of them cause sufficient resistance to restrict the dislocation speed under normal conditions to values considerably less than the sonic speed. Several mechanisms involve linearly viscous dissipation, and of these phonon viscosity appears to make the largest contribution. MASON's formula [16] for the shear stress required to move a dislocation at a speed v against this resistance is

$$(3.3) \quad \tau_v = \eta b v / 8 \pi r_0^2,$$

where η is the (microscopic) viscosity and r_0 is the effective core radius of the dislocation. MASON obtained agreement by taking r_0 to be of the order of b , but NABARRO [7] has argued that a considerably larger value should be used.

In general, for viscous damping we write

$$(3.4) \quad \tau_v = B v / b,$$

where B is the resistive force acting on unit length of dislocation line moving at unit velocity.

Combining (3.4) with (2.4) we obtain

$$(3.5) \quad \tau_v = \alpha \dot{\gamma}^p,$$

where $\alpha = B/\rho b^2$ is the macroscopic viscosity.

If it is assumed that for $\tau > \tau_0$ the stress must be sufficient to overcome the long-range stresses, the short-range barriers and the viscous resistance, the total stress is given by

$$(3.6) \quad \tau = \tau_A + \tau_0^* + \alpha \dot{\gamma}^p.$$

Experiments on single-crystal and polycrystalline metals and alloys have given results consistent with (3.6), with the following values of α , in units of kNsm^{-2} (10^4 Poise), at room temperature [17]:

	Al	Cu	Zn	Brass	Mild steel
Single Crystal	1.2	10.8	0.5	—	—
Polycrystalline	1.4, 2.1	3.6	—	5.5	2.1, 2.8

For aluminium single crystals, α was found to decrease slightly as the temperature increased from 20 to 500°K, while for zinc in basal shear it was found to be independent of temperature in the range 300 to 633°K; for both materials no temperature dependence was observed for the value of the intercept corresponding to $\dot{\gamma}^p = 0$, i.e. τ_0 . For mild steel, however, this intercept decreased considerably as the temperature increased from 293 to 713°K; from (2.6), this implies that the activation volume increases with increasing temperature. This could be caused by an increase in the core size of the dislocation with increasing temperature, or by a change in the rate-controlling mechanism with change of temperature.

4. Effect of strain rate history

It has been mentioned already that there is experimental evidence that in general the flow stress of a metal depends on the strain history as well as the instantaneous values of strain and strain rate. Such a dependence is to be expected from the known fact that the dislocation structure is in general history-dependent. This is indicated by the observations that slip-bands are finer in mild steel after rapid deformation than after slow deformation to the same strain, and that the static strength is lower.

Detailed studies of rate-history effects have been made by LINDHOLM [18], KLEPACZKO [19], and FRANTZ and DUFFY [20] for aluminium, by NICHOLAS and WHITMIRE [21] for aluminium, mild steel and titanium, and by CAMPBELL and BRIGGS [10] for molybdenum and niobium. It appears that for aluminium and titanium the dynamic flow stress is always reduced if pre-straining occurs at a low rate, whereas for mild steel, molybdenum and niobium it may be increased. KLEPACZKO has interpreted the results for aluminium in terms of a recovery process which operates during the pre-straining period. The results for b.c.c. metals on the other hand appear to be related to changes in dislocation distribution.

Recently, an investigation has been made of the behaviour of copper subjected to very large strain rate increases (10^{-3} to 10^3 s $^{-1}$) in pure shear, at various initial temperatures [22]. The results show a positive rate sensitivity and positive strain hardening. The initial response to the rate increase is elastic, but a considerable incremental strain occurs before the stress reaches that value obtained in constant rate tests at the higher rate. These results, which are similar to those obtained by KLEPACZKO for aluminium, show that the material behaviour is not consistent with the existence of a mechanical equation of state relating stress, strain, strain rate and temperature.

5. Behaviour under combined stresses

Most of the available data on rate and temperature effects relate to the behaviour under simple states of stress. However, many applications concern materials subjected

to combined stresses, so that it is necessary to develop more general constitutive relations. The different approaches which have been made to this problem have been reviewed by PERZYNA [23, 24] and CRISTESCU [25]. PERZYNA has formulated a theory for elastic-viscoplastic materials; in its simplest form, this is a generalization of the Prandtl-Reuss flow rule of ideal plasticity, and may be written

$$(5.1) \quad \dot{\epsilon}_{ij}^p = 2\lambda \langle \Phi(F) \rangle \frac{\partial f}{\partial \sigma_{ij}},$$

where $F = f/k - 1$, $f(\sigma_{ij})$ being the yield function and k the "static" yield stress. The symbol $\langle \rangle$ is defined so that

$$\langle \Phi(F) \rangle = \begin{cases} 0 & \text{for } F \leq 0, \\ \Phi(F) & \text{for } F > 0. \end{cases}$$

Assuming the Huber-Mises yield function

$$(5.2) \quad f = \left(\frac{1}{2} S_{ij} S_{ij} \right)^{1/2},$$

where S_{ij} is the deviatoric stress, (5.1) becomes

$$(5.3) \quad \dot{\epsilon}_{ij}^p = \lambda \langle \Phi(F) \rangle S_{ij} / f.$$

In general, the strain rate parameter λ , the "static" yield stress k and the function Φ depend on the temperature T . The Eq. (5.3) is a generalization of the relations introduced by SOKOLOVSKY [26] and MALVERN [27] for uniaxial stress. PERZYNA has used various forms of the function Φ in fitting (5.3) to experimental data for several metals; he has also generalized the equation to take account of work-hardening and history effects [24]. LINDHOLM [18] has used a relation of the form (5.3) in interpreting the results of his tension-torsion tests on aluminium; he assumed a function Φ corresponding to the thermal-activation Eq. (2.3), taking the activation volume to be independent of stress, and obtained a reasonable agreement over a range of temperatures and constant strain rates.

BODNER [28] has pointed out that it may not be possible to define a "static" yield stress, and has postulated a functional relationship between the second invariants of the deviators of the plastic deformation rate and the stress. The function is a continuous one, which can be applied at all stages of the deformation, so that no distinction is made between elastic and visco-plastic conditions, nor between loading and unloading. The validity of this approach has not yet been established for high rates of straining.

Little information is available for non-radial straining paths. LINDHOLM [18] found in medium rate tension-torsion tests on aluminium that a flow rule of the form (5.3) was obeyed approximately, though there was some indication that during a sudden change in the orientation of the strain rate tensor, deviations from the rule occurred. Similar effects have been reported for copper tested at low and medium strain rates [29, 30]. It is therefore not yet clear whether this type of constitutive law is valid in situations such as those which occur in metal-forming and crack propagation, when very rapid changes in the magnitude and orientation of the plastic strain rate tensor take place. To obtain experimental data relevant to such conditions and to determine the most appropriate forms of theory to describe the observed behaviour are perhaps the most important tasks in macro-plasticity at the present time.

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