Archives of Mechanics • Archiwum Mechaniki Stosowanej • 27, 5-6, pp. 693-699, Warszawa 1975

On the dynamic stability of thermomechanical processes in viscoplastic bodies (*)

A. BALTOV (SOFIA)

AN energetic stability criterion for visco-plastic dynamically loaded bodies is proposed, taking into account the available temperature field. A thermodynamic method and the Liapunov's stability definition are applied. The criteria for the stability of motion, of shape, and the thermal and the internal structural instability are also obtained.

Zaproponowano energetyczne kryterium stateczności dla dynamicznie obciążonych ciał lepkoplastycznych przy uwzględnieniu danego pola temperatury. Zastosowano metodę termodynamiczną oraz definicję stateczności Lapunowa. Otrzymano również kryteria stateczności ruchu i kształtu oraz niestateczności termicznej i struktury wewnętrznej.

Предложен энергетический критерий устойчивости для динамически нагруженных вязкопластических тел, при учете данного поля температуры. Применен термодинамический метод и определение устойчивости по Ляпунову. Получены тоже критерия устойчивости движения и формы, а также термической неустойчивости и внутренней структуры.

1. Statement of the problem

INVESTIGATION on the thermomechanical processes in viscoplastic bodies is very important in technology. A lot of materials under dynamic mechanical and thermal loading possess viscoplastic properties. For that reason the problems of describing such processes and establishing their stability condition, have an essential meaning in solid mechanics. In many cases, like metal forming with plastic deformation, thermal and mechanical loading resistance of machine elements etc. the instability of the process is undesired. In other cases it could be a desired state e.g. metal cutting forming a thin zone [1]. Stability problems are especially complicated in dynamical cases. Instability occurs in all dynamic experiments with viscoplastic materials — necking at dynamic tensile experiments, forming of stripes at dynamic torsion of thin-walled tubes [2] etc.

An energetic stability criterion in dynamically loaded bodies under temperature field is proposed in this paper. A thermodynamic method of investigation as well as the Liapunov's stability definition is used. The thermodynamic method is often used in the cases of equilibrium stability problems of elastic bodies [3, 4, 5 etc.]. The consideration of the dynamic character of the process involves essential changes in the statement of the stability problem.

(*) The paper has been presented at the *EUROMECH* 53 *COLLOQUIUM* on "*THERMOPLASTICITY*", Jabłonna, September 16–19, 1974.

Let us consider the body $\mathscr{B}$ occupying the region Ω_0 of the three-dimensional Euclidean space E_3 at a certain moment t_0. The volume of the body at that moment is V and it is bounded by a closed regular surface S. At the moment t the body occupies the region $\Omega_t \subset E_3$. We assume Ω_0 to be the reference configuration with the coordinate system $\{\mathbf{X}\}$; Ω_t is the actual configuration with the coordinate system $\{\mathbf{x}\}$; the law of motion is $\mathbf{x} = \mathbf{x}(\mathbf{X}, t)$. The body is loaded on the part S_P of the boundary surface with dynamically applied traction $\hat{\mathbf{T}}\mathbf{N}$, varying according to a given programme in the time interval (t_0, t_1). On the remaining part S_v of the surface ($S_v \cup S_P = S$) the velocities $\mathbf{v}$ are prescribed. The surrounding is under a constant temperature $\hat{\theta}$.

The boundary conditions of the process are:

$$
\begin{aligned}
&\mathbf{TN} = \hat{\mathbf{T}}(\mathbf{X}, t)\mathbf{N}, \quad \mathbf{X} \in S_P, \quad t \in (t_0, t_1),\\
(1.1)\qquad &\mathbf{v} = \mathbf{v}(\mathbf{X}, t), \quad \mathbf{X} \in S_v, \quad t \in (t_0, t_1),\\
&\mathbf{QN} = -(\theta - \hat{\theta})h_T, \quad \mathbf{X} \in S, \quad t \in (t_0, t_1).
\end{aligned}
$$

$\mathbf{T}$ is the first Piola-Kirchhoff stress tensor; $\mathbf{N}$ is the outward unit normal to the surface S in Ω_0; $\mathbf{v} = \dot{\mathbf{x}}(\mathbf{X}, t)$ is the velocity in terms of the material variables $(\mathbf{X}, t)$; $\mathbf{Q}$ is the heat flux vector in Ω_0; $\theta = \theta(\mathbf{X}, t)$ is the absolute temperature; h_T is the heat transfer coefficient.

A thermomechanical process occurs in the body under these conditions. A model of a body with internal state variables is assumed in order to describe this process.

The body is considered as a closed thermodynamic system, consisting of an infinite number of local thermodynamic systems — the infinitesimal neighbourhoods of the particles $\mathbf{X}$, having mass dm. The full thermodynamic state of any local thermodynamic system $(\mathbf{X}, dm)$ is characterized by means of:

a) $\mathbf{x}(\mathbf{X}, t)$ and the mass density $\varrho(\mathbf{X}, t)$, determining the position of the system;

b) the strain tensor $\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$ and the Piola-Kirchhoff second stress tensor $\tilde{\mathbf{T}}(\mathbf{X}, t)$, describing the internal mechanical state of the system. $\mathbf{F} = \partial_{\mathbf{X}}\mathbf{x}$ is the deformation gradient;

c) the specific entropy $\eta(\mathbf{X}, t)$ and the absolute temperature $\theta(\mathbf{X}, t)$, describing the internal thermal state of the system;

d) the internal state variables $\boldsymbol{\chi}^{(\alpha)}_{\mathbf{d}}(\mathbf{X}, t)$, $(\alpha = 1, 2, \ldots, n)$ and the conjugated dissipative forces $\mathbf{P}^{(\alpha)}(\mathbf{X}, t)$, describing the internal structural state of the system. The parameters $\boldsymbol{\chi}^{(\alpha)}$ could be tensors of different orders and may have different physical meanings. The tensors representing in macroscale the microdamage density (dislocations, microcracks, etc.) are often chosen to describe the viscoplastic material properties. The viscoplastic strain tensor $\mathbf{E}^a_{\mathbf{d}}$ itself could be chosen as an internal state variable [7];

e) the specific internal energy $u = u(\mathbf{X}, t)$ describing the internal energy state of the system;

f) the temperature gradient $\mathfrak{S} = \partial_{\mathbf{X}}\theta$ and the entropy displacement $\mathbf{h}(\mathbf{X}, t)$, describing the heat transfer, $\dot{\mathbf{h}} = \mathbf{Q}/\theta$.

Let us assume that the first group of constitutive equations is known [6],

$$
\begin{aligned}
(1.2)\qquad &u = u(\mathbf{E}, \eta, \boldsymbol{\chi}^{(\alpha)}),\\
&\tilde{\mathbf{T}} = \varrho_0\,\partial_{\mathbf{E}}u, \quad \theta = \partial_\eta u, \quad \mathbf{P}^{(\alpha)} = -\varrho_0\,\partial_{\boldsymbol{\chi}^{(\alpha)}}u, \quad (\alpha = 1, 2, \ldots, n),
\end{aligned}
$$

Taking into account the law of mass conservation

(1.3) $$\varrho = \frac{\varrho_0}{\det\{\mathbf{F}\}},$$

and the law of heat transfer

(1.4) $$\mathbf{Q} = \mathscr{F}(\mathbf{E}, \theta, \boldsymbol{\chi}^{(\alpha)})\mathfrak{S},$$

the state of the local thermodynamic system could be described by means of the following system of functions $M_{\mathbf{X}}(t) \equiv \{\mathbf{v}, \mathbf{F}\,(\text{or } \mathbf{E}), \eta, \boldsymbol{\chi}^{(\alpha)}\}(\mathbf{X}, t)$, $\mathbf{X} \in \Omega_0$, $t \in (t_0, t_1)$. The local thermodynamic process in the elementary system is described by the change of these functions $\dot{M}_{\mathbf{X}}(t) \equiv \{\dot{\mathbf{v}}, \dot{\mathbf{F}}\,(\text{or } \dot{\mathbf{E}}), \dot{\eta}, \dot{\boldsymbol{\chi}}^{(\alpha)}\}(\mathbf{X}, t)$. This change is determined by the following differential equations system

$\varrho_0 \dot{\mathbf{v}} = \operatorname{Div}\mathbf{T}$ (the equations of motion, where the body forces are neglected)

$\dot{\mathbf{F}} = \partial_{\mathbf{X}}\mathbf{v}, \left(\text{or } \dot{\mathbf{E}} = \frac{1}{2}(\dot{\mathbf{F}}^T\mathbf{F} + \mathbf{F}^T\dot{\mathbf{F}})\right)$ (the geometrical relations)

(1.5) $\varrho_0\dot{\eta} = \operatorname{Div}\left(\frac{1}{\theta}\mathbf{Q}\right) + \frac{1}{\theta}\varrho_0\delta$ (the balance entropy equation where the heat supply is neglected),

$\dot{\boldsymbol{\chi}}^{(\alpha)} = \boldsymbol{\Phi}^{(\alpha)}(\mathbf{E}, \eta, \boldsymbol{\chi}^{(\alpha)})$, $(\alpha = 1, 2, \ldots, n)$ (the equations of evolution for the internal state variables),

with

(1.6) $\varrho_0\delta = \mathfrak{S}\cdot\dot{\mathbf{h}} + \sum_{\alpha=1}^{n} \operatorname{tr}\{\mathbf{P}^{(\alpha)}\dot{\boldsymbol{\chi}}^{(\alpha)}\} \geqslant 0$ (the Clausius-Duhem dissipation energy inequality).

The global thermodynamic process, taking place in the body $\mathscr{B}$ is determined by means of the multitude $M \equiv \{M_{\mathbf{X}}(t)\}$, at all $\mathbf{X} \in \Omega_0$, $t \in (t_0, t_1)$ and obeys the first and the second thermodynamic principles, applied to the global thermodynamic system of the body

(1.7)
$$\frac{d}{dt}\int_V \left(\varrho_0 u + \frac{1}{2}\varrho_0\mathbf{v}\cdot\mathbf{v}\right)dV = \int_S \mathbf{v}\cdot\mathbf{T}\mathbf{N}dS + \int_S \mathbf{Q}\cdot\mathbf{N}dS,$$
$$\frac{d}{dt}\int_V \varrho_0\eta dV - \int_S \frac{1}{\theta}\mathbf{Q}\cdot\mathbf{N}dS = \int_V \frac{1}{\theta}\varrho_0\delta dV \geqslant 0.$$

The Eqs. (1.5) and (1.6), after taking into account (1.2), (1.3), (1.4) and (1.7), describe the considered thermomechanical process in the body $\mathscr{B}$. The loading programme is given with boundary conditions (1.1); the initial state of the body is assumed to be known e.g. it is given $M_0 = M(t_0)$.

The form of the constitutive equations must be determined in each particular case for the kind of material. As an example of the form of the second group of these equations (in the case of $\boldsymbol{\chi}^{(1)} \equiv \mathbf{E}^a$), we shall mention those proposed in [6]:

(1.8)
$$\dot{\mathbf{E}}^a = \mathbf{B}(\tilde{\mathbf{T}}, \theta, \mathbf{E}^a)f(\Delta z),\ \dot{\mathbf{E}}^a \neq 0 \quad \text{for} \quad \Delta z = z - z_0 > 0,$$
$$\dot{\mathbf{E}}^a = 0 \quad \text{for} \quad \Delta z \leqslant 0,$$

or those proposed in [7]

$$\dot{\mathbf{E}}^a = \gamma(\theta)\langle\Phi(F)\rangle\,\mathbf{M}(\tilde{\mathbf{T}},\theta,\mathbf{E}^a), \quad \begin{array}{ll}\langle\Phi\rangle = \Phi, & F>0,\\ \langle\Phi\rangle = 0, & F\leqslant 0.\end{array} \tag{1.9}$$

B and **M** are tensors characterizing the direction of the anelastic strain rate in the stress space; f and Φ are functions characterizing the viscoplastic properties of the material; $z(\tilde{\mathbf{T}},\theta,\mathbf{E}^a)$ is the free enthalpy per unit mass; z_0 is the boundary value of z; $F(\tilde{\mathbf{T}},\theta,\mathbf{E}^a) = 0$ is the static yield condition; γ is the viscosity coefficient.

It is convenient to consider the thermomechanical process in the space D, consisting of the elements $M \equiv \{M_{\mathbf{X}}(t)\}$, at all $\mathbf{X}\in\Omega_0$ and $t\in(t_0,t_1)$. The sequential positions of $M(t)$ where $t\in(t_0,t_1)$ describe the trajectory of the process in the space D. The trajectory begins at the initial point $M_0 = M(t_0)$. We introduce the following metrics in the space D. The distance R between any two elements $M_1, M_2\in D$ is defined by

$$R^2(M_1,M_2) = R^2_{\mathrm{I}}(\mathbf{a}_1,\mathbf{a}_2)+R^2_{\mathrm{II}}(\mathbf{A}_1,\mathbf{A}_2)+R^2_{\mathrm{III}}(c_1,c_2)+\sum_{\alpha=1}^{n}R^2_{\alpha}(\mathbf{Y}_1^{(\alpha)},\mathbf{Y}_2^{(\alpha)}), \tag{1.10}$$

where

$$R^2_{\mathrm{I}}(\mathbf{a}_1,\mathbf{a}_2) = \int_V \mathbf{a}_1\cdot\mathbf{a}_2\,dm,$$

$$R^2_{\mathrm{II}}(\mathbf{A}_1,\mathbf{A}_2) = \int_V \operatorname{tr}\{\mathbf{A}_1\mathbf{A}_2\}\,dm, \qquad R^2_{\mathrm{III}}(c_1,c_2) = \int_V c_1c_2\,dm, \tag{1.11}$$

$$R^2_{\alpha}(\mathbf{Y}_1^{(\alpha)},\mathbf{Y}_2^{(\alpha)}) = \int_V \operatorname{tr}\{\mathbf{Y}_1^{(\alpha)}\cdot\mathbf{Y}_2^{(\alpha)}\}\,dm \qquad (\alpha = 1,2,\ldots,n);$$

$\mathbf{a}_1$ and $\mathbf{a}_2$ are vector functions; $\mathbf{A}_1$ and $\mathbf{A}_2$ are tensor functions, c_1 and c_2 are scalar functions, $\mathbf{Y}_1^{(\alpha)}$ and $\mathbf{Y}_2^{(\alpha)}$ are tensor functions of the order of $\boldsymbol{\chi}^{(\alpha)}$. They all depend on $\mathbf{X}$ and t.

We shall investigate the stability of the thermomechanical process, using the norm $\|M\|_D^2 \equiv R^2(M,M)$ in the space D.

2. Stability criterion

In order to obtain a stability criterion for the process $M^\circ(t)$, we consider another thermo-viscoplastic dynamic process, described also by means of the Eqs. (1.5) and (1.6). For each $t\in(t_0,t_1)$, the second process obeys boundary conditions (1.1) with initial condition M_0' which is not very different from M_0° for the basic process $M^\circ(t)$. The process $M'(t)$ leads to a trajectory in the space D, differing little from $M^\circ(t)$ at $t\geqslant t_0$. The assumption that $M'(t)$ is a process with viscoplastic deformations is similar to Shanly and Hill's assumptions in the theory of plasticity [11]. The disturbed process $M'(t)$ as well as the basic process $M^\circ(t)$ are really possible, obeying the thermodynamic principles (1.7).

Let us introduce a fictitious process $M^*(t) \equiv \{M^*_{\mathbf{X}}(t)\}$ where $M^*_{\mathbf{X}}(t) = M'_{\mathbf{X}}(t)-M^\circ_{\mathbf{X}}(t)$ at $t\in(t_0,t_1)$. $M'(t) = M^\circ(t)$ if $M^*(t)\equiv\overline{M} = 0$. According to the second Liapunov's method we reduce the stability problem for the process $M^\circ(t)$ to a stability problem for the fictitious equilibrium state $\overline{M}$ with initial disturbances $M_0^* = M_0' - M_0^\circ$. The fictitious process $M^*(t)$ is locally described by means of the following differential equations system:

$$(2.1)\quad \begin{aligned} &\varrho_0 \dot{\mathbf{v}}^* = \operatorname{Div} \mathbf{T}^*, \quad \mathbf{T}^* = \mathbf{T}' - \mathbf{T}^\circ, \quad \mathbf{F}^* = \mathbf{F}' - \mathbf{F}^\circ, \\ &\dot{\mathbf{F}}^* = \partial_{\mathbf{X}} \mathbf{v}^* \qquad (\text{or } 2\dot{\mathbf{E}}^* = \dot{\mathbf{F}}^{\circ T}\mathbf{F}^* + \dot{\mathbf{F}}^{*T}\mathbf{F}^\circ + \mathbf{F}^{\circ T}\dot{\mathbf{F}}^* + \dot{\mathbf{F}}^{*T}\mathbf{F}^* + \mathbf{F}^{*T}\dot{\mathbf{F}}^*), \\ &\varrho_0 \dot{\eta}^* = \operatorname{Div}\left(\frac{\mathbf{Q}'}{\theta'} - \frac{\mathbf{Q}^\circ}{\theta^\circ}\right) + \Delta\sigma, \quad \Delta\sigma = \varrho_0 \frac{\delta'}{\theta'} - \varrho_0 \frac{\delta^\circ}{\theta^\circ}, \\ &\dot{\boldsymbol{\chi}}^{(\alpha)*} = \Delta \boldsymbol{\Phi}^{(\alpha)}, \quad \Delta \boldsymbol{\Phi}^{(\alpha)} = \boldsymbol{\Phi}^{(\alpha)}(\mathbf{E}', \eta', \boldsymbol{\chi}^{(\alpha)'}) - \boldsymbol{\Phi}^{(\alpha)}(\mathbf{E}^\circ, \eta^\circ, \boldsymbol{\chi}^{(\alpha)\circ}). \end{aligned}$$

The corresponding boundary conditions are:

$$(2.2)\quad \begin{aligned} \mathbf{T}^*\mathbf{N} &= 0, \quad \mathbf{X} \in S_P, \quad t \in (t_0, t_1), \\ \mathbf{v}^* &= 0, \quad \mathbf{X} \in S_v, \quad t \in (t_0, t_1), \\ \mathbf{Q}^*\mathbf{N} &= -\theta^* h_T, \quad \mathbf{X} \in S, \quad t \in (t_0, t_1), \end{aligned}$$

where

$$(2.3)\quad \mathbf{Q}^* = \mathbf{Q}' - \mathbf{Q}^\circ, \quad \theta^* = \theta' - \theta^\circ.$$

The initial conditions are M_0^*. The fictitious process obeys total relations, obtained after substracting the expressions in (1.7) for the disturbed and the basic processes.

$$(2.4)\quad \begin{aligned} &\frac{d}{dt}\int_V \left(\Delta u + \frac{1}{2}\mathbf{v}^*\cdot\mathbf{v}^* + \mathbf{v}^\circ\cdot\mathbf{v}^*\right)\varrho_0\, dV = \int_{S_P} \mathbf{v}^*\cdot\hat{\mathbf{T}}\mathbf{N}\, dS + \int_{S_v} \hat{\mathbf{v}}\cdot\mathbf{T}^*\mathbf{N}\, dS - \int_S \theta^* h_T\, dS, \\ &\frac{d}{dt}\int_V \varrho_0 \eta^*\, dV = -\int_S \frac{\theta^*}{\tau} h_T\, dS + \int_V \Delta\sigma\, dV, \end{aligned}$$

where:

$$(2.5)\quad \tau = \frac{\theta'\theta^\circ}{\hat{\theta}} > 0, \quad \Delta u = u(\mathbf{E}', \eta', \boldsymbol{\chi}^{(\alpha)'}) - u(\mathbf{E}^\circ, \eta^\circ, \boldsymbol{\chi}^{(\alpha)\circ}).$$

From $(2.4)_1$ and $(2.4)_2$ we obtain

$$(2.6)\quad \begin{aligned} \frac{d}{dt}\int_V \left(\varrho_0 \Delta u + \frac{1}{2}\varrho_0 \mathbf{v}^*\cdot\mathbf{v}^* + \varrho_0 \mathbf{v}^\circ\cdot\mathbf{v}^* - \varrho_0\hat{\theta}\eta^*\right) dV = -\int_V \hat{\theta}\Delta\sigma\, dV \\ + \int_{S_P} \mathbf{v}^*\cdot\hat{\mathbf{T}}\mathbf{N}\, dS + \int_{S_v} \hat{\mathbf{v}}\cdot\mathbf{T}^*\mathbf{N}\, dS + \int_S \left(\frac{\hat{\theta}}{\tau} - 1\right)\theta^* h_T\, dS. \end{aligned}$$

The energy dissipation is the only heat supply in the case under consideration and therefore $\theta' > \hat{\theta}$, $\theta^\circ > \hat{\theta}$ and $\left(\dfrac{\hat{\theta}}{\tau} - 1\right) \leqslant 0$.

We expand the internal energy per unit mass for the disturbed process $u(\mathbf{E}^\circ + \mathbf{E}^*, \eta^\circ + \eta^*, \boldsymbol{\chi}^{(\alpha)\circ} + \boldsymbol{\chi}^{(\alpha)*})$ in Taylor's series in the neighbourhood of $M_{\mathbf{X}}^\circ$. Taking into account that $M^*_{\mathbf{X}}$ were assumed to be of small values, the terms in the series, consisting of $M^*_{\mathbf{X}}$ of the order higher than two, could be neglected and in the case of small second order interactions we obtain:

$$(2.7)\qquad \varrho_0 \Delta u = \operatorname{tr}\{\tilde{\mathbf{T}}^\circ \mathbf{E}^*\} + \varrho_0 \theta^\circ \eta^* - \sum_{\alpha=1}^{n} \operatorname{tr}\{\mathbf{P}^{(\alpha)\circ} \boldsymbol{\chi}^{(\alpha)*}\} + \frac{1}{2}\operatorname{tr}\{\boldsymbol{\Gamma}^\circ_u \mathbf{E}^* \mathbf{E}^*\} + \frac{1}{2}\vartheta^\circ_u (\eta^*)^2 + \frac{1}{2}\sum_{\alpha=1}^{n} \operatorname{tr}\{\mathbf{B}^{(\alpha)\circ}_u \boldsymbol{\chi}^{(\alpha)*} \boldsymbol{\chi}^{(\alpha)*}\},$$

where

$$(2.8)\qquad \begin{aligned} \boldsymbol{\Gamma}^\circ_u &= \varrho_0\, \partial_{\mathbf{E}}(\partial_{\mathbf{E}} u)|_{M^\circ_{\mathbf{X}}}, \qquad \vartheta^\circ_u = \partial_\eta(\partial_\eta u)|_{M^\circ_{\mathbf{X}}}, \\ \mathbf{B}^{(\alpha)\circ}_u &= \varrho_0\, \partial_{\boldsymbol{\chi}^{(\alpha)}}(\partial_{\boldsymbol{\chi}^{(\alpha)}} u)|_{M^\circ_{\mathbf{X}}}. \end{aligned}$$

Substituting (2.7) into (2.6)

$$(2.9)\qquad \dot{I} = \Delta \dot{D} + \dot{W}^N,$$

where

$$(2.10)\qquad \begin{aligned} I &= \frac{1}{2}\int_V \Big(\operatorname{tr}\{\boldsymbol{\Gamma}^\circ_u \mathbf{E}^*\mathbf{E}^*\} + \vartheta^\circ_u(\eta^*)^2 + \sum_{\alpha=1}^{n} \operatorname{tr}\{\mathbf{B}^{(\alpha)\circ}_u \boldsymbol{\chi}^{(\alpha)*}\boldsymbol{\chi}^{(\alpha)*}\} \\ &\quad + \varrho_0 \mathbf{v}^*\cdot\mathbf{v}^* + 2\operatorname{tr}\{\tilde{\mathbf{T}}^\circ \mathbf{E}^*\} + 2\varrho_0 \Delta\theta^\circ \eta^* - 2\sum_{\alpha=1}^{n}\operatorname{tr}\{\mathbf{P}^{(\alpha)\circ}\boldsymbol{\chi}^{(\alpha)*}\} + 2\varrho_0 \mathbf{v}^\circ\cdot\mathbf{v}^*\Big) dV, \\ \Delta\dot{D} &= \int_S \left(\frac{\hat{\theta}}{\tau} - 1\right)\theta^* h_T\, dS - \int_V \hat{\theta}\,\Delta\sigma\, dV, \\ \dot{W}^N &= \int_{S_p} \mathbf{v}^*\cdot\hat{\mathbf{T}}\mathbf{N}\, dS + \int_{S_v} \hat{\mathbf{v}}\cdot\mathbf{T}^*\mathbf{N}\, dS, \qquad \Delta\theta^\circ = \theta^\circ - \hat{\theta}. \end{aligned}$$

An expression is obtained, consisting of the measure of alternation of the total energy of the fictitious process I, the power of the non-conservative forces during the fictitious process $\dot{W}^N$ and the difference $\Delta\dot{D}$ between the dissipative and thermal terms of the basic and disturbed processes. This result is similar to the result corresponding to the case of equilibrium state stability of bodies, loaded with non-conservative forces [9]. As the load in the considered case is time-dependent, the term $\dot{W}^N$ appears in the Eq. (2.9). All $\{M^*_{\mathbf{X}}(t)\}$ obeying conditions (2.2) form the manifold $K \subset D$. We assume that the basic process $\{M^\circ_{\mathbf{X}}(t)\}$ is known; I is then a functional of $M^*_{\mathbf{X}}(t)$, having the range of definition K. We consider the r-neighbourhood of the fictitious equilibrium state $\overline{M}$, determined by $\|\{M^*_{\mathbf{X}}(t)\}\|^2_D < r$. According to Liapunov's stability definition, the fictitious equilibrium state will be locally stable if for each $0 < \varepsilon < r$ there exists a $0 < \delta(\varepsilon) < r$ such, that $\|\{M^*_{\mathbf{X}}(t)\}\|^2_D < r$ if $\|M^*_0(t)\|^2_D < \delta(\varepsilon)$ at each $t \in (t_0, t_1)$. In that case the basic process $M^\circ(t)$ will be stable too.

The value of r is to be chosen according to the specific character of the process. If the condition of the definition is violated with respect to the norm $\|M^*(t)\|^2_D > r$, at the moment $t = t_k$ the process is thermomechanically and structurally unstable at that moment. If $R_{\rm I}^2(\mathbf{v}, \mathbf{v}) > r$ at $t = t_k$, the body loses its stability of motion at the moment t_k; if $R_{\rm II}^2(\mathbf{E}^*, \mathbf{E}^*) > r$, it loses stability of the form; if $R_{\rm III}^2(\eta^*, \eta^*) > r$, thermal instability occurs; if $\sum_{\alpha=1}^{n} R_\alpha^2(\boldsymbol{\chi}^{(\alpha)*}, \boldsymbol{\chi}^{(\alpha)*}) > r$, internal structural instability occurs. (The internal structural equilibrium state instability with constant $\mathbf{E}$ and η and $\mathbf{v} = 0$ is investigated in [10, 7]).

In order to establish sufficient conditions for stability of the considered process, Theorem 12 [8] will be applied. In order to fulfil the condition of the theorem, the following requirements have to be proved:

a) The functional $I(M_{\rm X}^*(t))$ must be continuous and upper bounded;

b) The functional $I(M_{\rm X}^*(t))$ must be positive definite with respect to the norm $\|\{M_{\rm X}^*(t)\}\|_D$ at each $t \in (t_0, t_1)$, e.g.

$$I \geqslant A\|\{M_{\rm X}^*\}\|_D, \quad A > 0; \tag{2.11}$$

c) The functional $I(M_{\rm X}^*(t))$ must be decreasing with respect to t in the interval (t_0, t_1), e.g.

$$\Delta\dot{D} + \dot{W}^N \leqslant 0. \tag{2.12}$$

If anyone of the above-mentioned conditions is violated at a certain moment $t_k \in (t_0, t_1)$, the process is unstable at that moment.

Thus, an energetic criterion is obtained for the local stability of viscoplastic bodies under dynamic thermomechanical processes.

References

1. J. POMEY, *Aperçu sur la plasticité adiabatique*, Annales du CIRP, **13**, 2, 93–103, 1966.
2. A. BALTOV, P. T. VINH, *Sur la formation des bandes adiabatiques plastiques dans un tube mince soumis à une torsion dynamique*, C.R. Acad. Sc. Paris, **275**, ser. A, 291–294, 1972.
3. J. L. ERICKSEN, *A thermo-kinetic view of elastic stability theory*, Int. J. Eng. Sci., **2**, 573–580, 1966.
4. W. T. KOITER, *On thermodynamic background of elastic stability theory*, Problems of Hydrodynamics and Continuum Mechanics, M., 277–285, 1969.
5. P. M. NAGHDI, J. A. TRAPP, *On the general theory of stability for elastic bodies*, Arch. Rat. Mech. Anal., **51**, 3, 165–191, 1973.
6. D. KOLAROV, A. BALTOV, *On the visco-plasticity at large deformations* [in Bulgarian]. Theor. and Appl. Mech., **4**, 2, 21–30, 1973.
7. P. PERZYNA, *Thermodynamic theory of viscoplasticity*, Adv. Appl. Mech., **11**, 313, 1971.
8. V. I. ZUBOV, *Stability of the motion* [in Russian], M. 1973.
9. S. NEMAT-NASSER, *On thermomechanics of elastic stability*, ZAMP, **21**, 4, 538–558, 1970.
10. B. D. COLEMAN, M. E. GURTIN, *Thermodynamics with internal state variables*, J. Chem. Phys., **47**, 2, 597–613, 1967.
11. R. HILL, *A general theory of uniqueness and stability in elastic-plastic solids*, J. Mech. Phys. Sol., **6**, 3, 236–249, 1958.

BULGARIAN ACADEMY OF SCIENCES,
INSTITUTE OF MATHEMATICS AND MECHANICS
WITH COMPUTER CENTER, SOFIA.