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LEARNING ALGORITHMS AND UNCERTAIN VARIABLES FOR A CLASS OF KNOWLEDGE-BASED ASSEMBLY SYSTEMS

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Abstract: A class of uncertain assembly systems described by a relational knowledge representation is considered. The formal model of a system, the statement of the decision (control) problem and the solution in the form of a sequence of sets of assembly operations in successive stages of the assembly process under consideration are presented. The algorithms of learning consisting in *step by step* knowledge validation and updating are given. The structure of the decision making system is described. The application of uncertain variables to the determination of optimal decisions is presented.

Keywords: learning algorithms, intelligent control systems, assembly systems, uncertain variables

1. INTRODUCTION

In many practical situations there exist uncertainties in the description of an assembly process: the sequence of assembly operations is not *a priori* determined and the relationships between the successive operations, states and features describing the assembly plant are nondeterministic [see e.g. 1,2,3,4]. This is a frequent situation in small batch production processes with changes in the relationships describing the assembly plant in different cycles. In such cases it is reasonable to apply artificial intelligence tools and methods of decision making in uncertain systems to the planning and control of the assembly process.

The paper deals with a class of knowledge-based assembly systems described by a relational knowledge representation consisting of relations between the operations, states and features, i.e. variables characterizing the current effect of the assembly process (e.g. dimensions or sizes evaluating the precision, accuracy or tolerance in the placement and fastening of elements). The problem of choosing assembly operations from the given sets of operations on each stage is considered as a specific multistage decision process for a relational plant. For the knowledge-based systems with unknown parameters in the knowledge representation, algorithms of learning consisting in *step by step* knowledge validation and updating have been presented [5,6,7,8]. Another approach consists in the application of so called uncertain variables introduced and elaborated for decision making problems in a class of uncertain systems [8-11,13,15,16]. The purpose of this paper is to show how these approaches may be applied to a class of assembly systems under consideration.

2. KNOWLEDGE REPRESENTATION

Let us consider an assembly process as a sequence of assembly operations $O_n \in \{O_{n1}, O_{n2}, \ldots, O_{nl_n}\}$ executed on the successive stages n. On each stage the assembly plant is characterized by a state $s_n \in \{S_{n1}, S_{n2}, ..., S_{nm_n}\}$. The state s_{n+1} depends on the state s_n and the operation O_n (Fig.1). To formulate the description of the assembly process let us introduce the following notation:

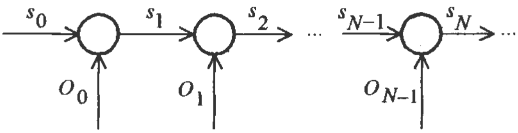

Fig.1.

i_n – index of the operation, e.g. if $i_n = 4$ then $O_n = O_{n4}$,

$i_n \in \{1, 2, ..., l_n\} \triangleq L_n$ – set of the operations, $n \in \overline{1, N}$,

j_n – index of the state, e.g. if $j_n = 2$ then $s_n = S_{n2}$,

$j_n \in \{1, 2, ..., m_n\} \triangleq M_n$ – set of the states,

$y_n \in Y_n$ – real number vector of features describing the assembly plant on n-th stage. The evaluation of the result of n-th operation and the quality of the process concern the components of y_n, e.g. $a \le y_n^{(\nu)} \le b$ may denote the requirement concerning the size $y_n^{(\nu)}$ (a component of y_n).

The knowledge representation consists of relations between the variables i_n, j_n, j_{n+1} and y_n, and may be divided into two parts:

I. A relation between i_n, j_n and j_{n+1}

$$R_{1n}(i_n, j_n, j_{n+1}) \subset L_n \times M_n \times M_{n+1}.$$

II. A relation between j_{n+1} and y_{n+1}

$$R_{IIn}(j_{n+1}, y_{n+1}) \subset M_{n+1} \times Y_{n+1}.$$

The relations R_I and R_{II} may be presented in the form of families of sets:

I. $$D_{j,n+1}(i_n, j_n) \subset M_{n+1} \quad (1)$$

for all pairs $(i_n, j_n) \in L_n \times M_n$,

II. $$D_{y,n+1}(j_{n+1}) \subset Y_{n+1} \quad (2)$$

for all $j_{n+1} \in M_{n+1}$.

Consequently, the relational knowledge representation (or the knowledge base in the computer implementation) consists of $l_n \cdot m_n$ sets (1) and m_{n+1} sets (2). The decision making (control) in the assembly process should be based on the knowledge representation (1), (2) given by an expert.

3. DECISION PROBLEM

On each stage the decision consists in the proper choosing of the assembly operation O_n (i.e. the index i_n) from the given set of possible operations (i.e. from the set L_n), satisfying the requirement concerning the features y_{n+1} presented in the form $y_{n+1} \in D_{y,n+1}$ where the set $D_{y,n+1} \subset Y_{n+1}$ is given by a user. For making the decision a knowledge of the state j_n in the form $j_n \in D_{j,n} \subset M_n$ is used.

The decision problem on n-th stage may be formulated as follows: For the given knowledge representation, the set $D_{j,n}$ (the knowledge of the state j_n, determined on the stage $n-1$) and the set $D_{y,n+1}$ (a user's requirement), one should find the set of all operations i_n

satisfying the requirement, i.e. the largest set $D_{i,n} \subset L_n$ such that the implication

$$i_n \in D_{i,n} \rightarrow y_{n+1} \in D_{y,n+1}$$

is satisfied. Consequently, the assembly operation should be chosen from the set $D_{i,n}$. The decision problem may be decomposed into two parts.

A. For the given sets (2) and Y_{n+1} find the largest set $D_{j,n+1} \subset M_{n+1}$ such that the implication $j_{n+1} \in D_{j,n+1} \rightarrow y_{n+1} \in D_{y,n+1}$ is satisfied.

B. For the given sets (1) and the sets $D_{j,n}$, $D_{j,n+1}$ find the largest set $D_{i,n}$ such that the implication $i_n \in D_{i,n} \rightarrow j_{n+1} \in D_{j,n+1}$ is satisfied for each $j_n \in D_{j,n}$.

For the problem solving a general solution of the decision problem based on relational knowledge representation may be used [7]. It is easy to note that:

A.

$$D_{j,n+1} = \{ j_{n+1} \in M_{n+1} : D_{y,n+1}(j_{n+1}) \subseteq D_{y,n+1} \}, \tag{3}$$

B.

$$D_{i,n} = \{ i_n \in L_n : \bigwedge_{j_n \in D_{j,n}} D_{j,n+1}(i_n, j_n) \subseteq D_{j,n+1} \}. \tag{4}$$

Using (3) and (4) for $n = 0, 1, ..., N-1$, we can determine an *assembly plan* in the form of a sequence $D_{i,1}$, $D_{i,2}$, ..., $D_{i,N-1}$. The set of initial states $D_{i,0}$ must be known and on n-th stage we use $D_{i,n}$ determined on the former stage. The decision process is then performed in an open-loop control systems: on each stage the assembly operation i_n may be chosen randomly from the set $D_{i,n}$ according to the assembly plan.

The decision process may be performed in a closed-loop control system if it is possible to measure y_n. We can introduce then the *knowledge-based recognition* of the state j_n, which consists in the determination of the set $\overline{D}_{j,n}$ of the possible indexes j_n:

$$\overline{D}_{j,n}(y_n) = \{ j_n \in M_n : \; y_n \in D_{y,n}(j_n) \} \tag{5}$$

where the sets $D_{y,n}(j_n)$ are given in the knowledge representation. Consequently, in (4) we use $\overline{D}_{j,n}$ instead of $D_{j,n}$ which may give less restricted set of the possible decisions $D_{i,n}$. The block scheme of the closed-loop control system is presented in Fig.2 where G denotes the generator of random numbers for the random choosing of the assembly operation i_n from $D_{i,n}$.

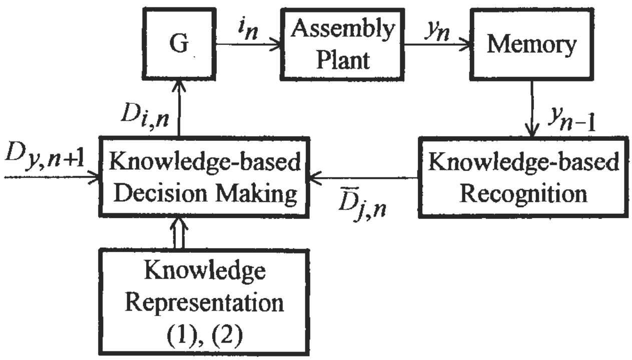

Fig.2.

4. KNOWLEDGE VALIDATION AND UPDATING

Let us consider the knowledge representation with unknown parameters in the second part. The relation $R_{II}(j_n, y_n, c_n)$, and consequently the sets $D_{y,n}(j_n, c_n)$ in (2) and the set $D_{j,n}(c_n)$ in (3) rewritten with n in the place of $n+1$, depend on the vector parameter

$c_n \in C_n$. Assume that the parameter c_n has the value $c_n = \bar{c}_n$ and $\bar{c}_n$ is unknown. We shall present the algorithm of *step by step* estimation of the unknown value $\bar{c}_n$ based on the results of observations. On each step one should prove if the current observation "belongs" to the knowledge representation determined to this step **(knowledge validation)** and if not – one should modify the current estimation of the parameters in the knowledge representation **(knowledge updating)**. The successive estimations will be used in the determination of the decision concerning the assembly operations, based on the current knowledge in the learning system. According to the general approach [5], the validation and updating may concern the knowledge in the form of $D_{y,n}(j_n, c_n)$ or directly the knowledge in the form of $D_{j,n}(c_n)$. The second version is used here, taking into account the specific form of the knowledge representation for the assembly process. Let us assume that the assembly process consisting of N operations is repeated in successive cycles and denote by i_{pn}, j_{pn}, y_{pn} the variables in p-th cycle. One cycle corresponds to one step of the estimation process. When the parameter $\bar{c}_n$ is unknown then for the fixed value i_{n-1} it is not known if i_{n-1} is a correct decision, i.e. if $j_n \in D_{j,n}(c_n)$ and consequently $y_n \in D_{y,n}$. Our problem may be considered as a classification problem with two classes. The index j_n should be classified to class $k = 1$ if $j_n \in D_{j,n}(c_n)$ or to class $k = 2$ if $j_n \notin D_{j,n}(c_n)$. Assume that we can use the *learning sequence*

$$(j_{1n}, k_{1n}), (j_{2n}, k_{2n}), \dots, (j_{vn}, k_{vn}) \triangleq S_{vn}$$

where $k_{pn} \in \{1,2\}$ are the results of the correct classification given by an external trainer or obtained by testing the property $y_{pn} \in D_{y,n}$ at the output of the assembly plant. Let us denote the index j_{pn} by $\bar{j}_{pn}$ if $k_{pn} = 1$ and by $\hat{j}_{pn}$ if $k_{pn} = 2$, and introduce the following sets in C_n:

$$\overline{D}_{c,n}(p)=\{c_n \in C_n : j_{pn} \in D_{j,n}(c_n) \text{ for every } j_{pn}=\bar{j}_{pn} \text{ in } S_{vn}\}, \tag{6}$$

$$\hat{D}_{c,n}(p)=\{c_n \in C_n : j_{pn} \notin D_{j,n}(c_n) \text{ for every } j_{pn}=\hat{j}_{pn} \text{ in } S_{vn}\}. \tag{7}$$

The set

$$\overline{D}_{c,n}(p) \cap \hat{D}_{c,n}(p) \triangleq \Delta_{c,n}(p) \tag{8}$$

is proposed here as the estimation of $\bar{c}_n$ after p assembly cycles. The value c_n may be chosen randomly from $\Delta_{c,n}(p)$ and put into $D_{j,n}(c_n)$. The determination of $\Delta_{c,n}(p)$ may be presented in the form of the following *recursive algorithm*:

If $k_{pn}=1$:

Prove whether

$$\bigwedge_{c_n \in \overline{D}_{c,n}(p-1)} [j_{pn} \in D_{j,n}(c_n)]$$

(knowledge validation).

If yes then $\overline{D}_{c,n}(p)=\overline{D}_{c,n}(p-1)$. If not – determine new $\overline{D}_{c,n}(p)$ **(knowledge updating)**

$$\overline{D}_{c,n}(p)=\{c_n \in \overline{D}_{c,n}(p-1): \ j_{pn} \in D_{j,n}(c_n)\}.$$

Put

$$\hat{D}_{c,n}(p)=\hat{D}_{c,n}(p-1).$$

If $k_{pn}=2$:

Prove whether

$$\bigwedge$$

$$\bigwedge_{c_n \in \hat{D}_{c,n}(p-1)} [j_{pn} \notin D_{j,n}(c_n)]$$

(knowledge validation).

If yes then $\hat{D}_{c,n}(p) = \hat{D}_{c,n}(p-1)$. If not – determine new $\hat{D}_{c,n}(p)$ **(knowledge updating)**

$$\hat{D}_{c,n}(p) = \{c_n \in \hat{D}_{c,n}(p-1):\ j_{pn} \notin D_{j,n}(c_n)\}.$$

Put

$$\overline{D}_{c,n}(p) = \overline{D}_{c,n}(p-1)$$

and

$$\Delta_{c,n}(p) = \overline{D}_{c,n}(p) \cap \hat{D}_{c,n}(p).$$

Let us note that for the determination of $\Delta_{c,n}(p)$ it is necessary to observe the states j_{pn}.

5. EXAMPLE

Let the sets $D_{y,n}(j_n)$ be described by inequalities

$$c_n a(j_n) \le y_n^{\mathrm{T}} y_n \le 2c_n a(j_n),\ \ c_n > 0,\ \ a(j_n) > 0, \tag{9}$$

given by an expert. It is then known that the value $y_n^{\mathrm{T}} y_n$ (where y_n is a vector of the features characterizing the assembly plant) satisfies the inequality (9) and the bounds are the coefficients a depending on the state, i.e. for the different assembly operations i_n and consequently – the different states j_n we have the different bounds for the value $y_n^{\mathrm{T}} y_n$ which denotes a quality index. The requirement concerning the quality (i.e. the set $D_{y,n}$) is the following

$$\beta \le y_n^{\mathrm{T}} y_n \le \alpha\,, \quad \beta > 0\,, \quad \alpha \ge 2\beta\,.$$

Then, according to (3)

$$D_{j,n}(c_n) = \{\, j_n \in M_n : \beta \le c_n a(j_n) \le \frac{\alpha}{2} \}\,.$$

Using (6) and (7) we obtain

$$\overline{D}_{c,n}(p) = \{ c_n : \frac{\beta}{a(j_{pn})} \le c_n \le \frac{\alpha}{2a(j_{pn})} \ \text{ for every } \ j_{pn} = \bar{j}_{pn} \ \text{ in } \ S_{vn} \},$$

$$\hat{D}_{c,n}(p) = \{ c_n : \ c_n < \frac{\beta}{a(j_{pn})} \ \text{ or } \ c_n > \frac{\alpha}{2a(j_{pn})} \ \text{ for every } \ j_{pn} = \hat{j}_{pn} \ \text{ in } \ S_{vn} \}.$$

Hence,

$$\overline{D}_{c,n}(p) = [\bar{c}_{\min,p}\,, \bar{c}_{\max,p}\,] \tag{10}$$

where

$$\bar{c}_{\min,p} = \max_p \frac{\beta}{a(j_{pn})}\,,$$

$$\bar{c}_{\max,p} = \min_p \frac{\alpha}{2a(j_{pn})}$$

for all p such that $j_{pn} = \bar{j}_{pn}$, and

$$\hat{D}_{c,n}(p) = (0, c_{\max,p}) \cup (c_{\min,p}, \infty) \tag{11}$$

i.e.

$$0 < c_n < c_{\max,p} \qquad \text{or} \qquad c_n > c_{\min,p}$$

where

$$c_{\max,p} = \min_p \frac{\beta}{a(j_{pn})}, \qquad c_{\min,p} = \max_p \frac{\alpha}{2a(j_{pn})}$$

for all p such that $j_{pn} = \hat{j}_{pn}$. Finally c_n should be chosen randomly from the set

$$\Delta_{c,n}(p) = \overline{D}_{c,n}(p) \cap \hat{D}_{c,n}(p)$$

where $\overline{D}_c$ and $\hat{D}_c$ are given by (10) and (11).

6. LEARNING ALGORITHM FOR DECISION MAKING IN CLOSED-LOOP SYSTEM

The estimations of $\bar{c}_n$ may be performed for the successive cycles p and used for the proper choosing of the assembly operations i_{pn} in a closed-loop learning system. Before the cycle (p+1) the values c_{pn} (n = 1, 2, ... , N) should be chosen randomly from the sets $\Delta_{c,n}(p)$ obtained after the cycle p and put into the sets $D_{j,n}(c_n)$ in the place of c_n. The decisions i_n in the cycle (p+1) should be chosen randomly from the sets $D_{i,n}(c_n)$ determined according to (4) for $c_n = c_{pn}$. The decision making (control) algorithm in the learning system is then the following:

1. Execute the assembly operations i_{pn} (n = 0, 1, ... , N – 1) and measure y_{pn} (n = 1, 2, ... , N).
2. Determine $\Delta_{c,n}(p)$ (n = 1, 2, ... , N) using the procedure presented in the previous section.
3. Choose randomly c_{pn} from $\Delta_{c,n}(p)$ and put c_{pn} into $D_{j,n}(c_n)$.
4. Determine the sets $D_{i,n}(c_{pn}) \stackrel{\Delta}{=} D_{i,p+1,n}$ (n = 0, 1, ... , N – 1) using $D_{j,n}(c_{pn})$ and the procedure presented in Sec. 3.

5. Choose randomly the assembly operations $i_{p+1,n}$ (n = 0, 1, ... , N – 1) from the sets $D_{i,p+1,n}$.

The sequence $i_{p+1,0}$, $i_{p+1,1}$, ... , $i_{p+1,n-1}$ forms the assembly plan for the cycle (p+1). The operations i_{pn} may be chosen randomly from the set $D_{i,pn}$ with the same probability for each operation equal to $[d(p,n)]^{-1}$ where $d(p,n)$ denotes the number of operations in the set $D_{i,pn}$. Assume that the points c_{pn} are chosen randomly from $\Delta_{c,n}(p)$ with probability density $f_{pn}(c)$. It may be shown that if $f_{pn}(c) > 0$ for every $c \in \Delta_{c,n}(p)$ then $\Delta_{c,n}(p)$ converges with probability 1 to $\{\bar{c}_n\}$, i.e. to the value of the unknown parameter, and the sequence $D_{i,pn}$ converges to the set D_{in} obtained for $c_n = \bar{c}_n$. The proof is based on the theorem concerning the convergence of the learning process, presented in [5]. The block scheme of the closed-loop control system with learning is presented in Fig.3 where i_{pn}, y_{pn}, c_{pn} etc. denote the respective sequences of variables for different n in the cycle p, e.g. i_{pn} denotes the sequence of operations i_{p0}, ..., $i_{p,N-1}$ and c_{pn} denotes the sequence c_{p1}, ... , c_{pN}. Two generators of random variables are used in the system: G_1 for the random choosing of c_{pn} from $\Delta_{c,n}(p)$ and G_2 for the random choosing of i_{pn} from $D_{i,pn}$.

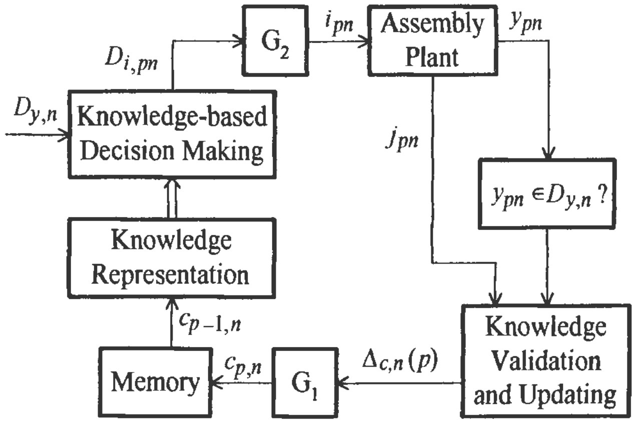

Fig.3.

7. APPLICATION OF UNCERTAIN VARIABLES

The *uncertain variable* $\bar{x}$ is defined by a set of values X and the certainty distribution $h_x(x) = v(\bar{x} \cong x)$ where v denotes the certainty index that $\bar{x}$ is *approximately* equal to x [8-11]. The certainty distribution is given by an expert. For the given set $D_x \subset X$, the certainty index that $\bar{x}$ approximately belongs to D_x is defined as follows

$$v(\bar{x} \,\tilde{\in}\, D_x) = \max_{x \in D_x} h_x(x).$$

Let us consider the assembly process described by the knowledge representation with constant unknown parameters $x \in X$ and $c \in C$ in the first and the second part, respectively. Now the sets $D_{j,n+1}(i_n, j_n; x)$ in (1) depend on x and the sets $D_{y,n+1}(j_{n+1}; c)$ in (2) depend on c. Consequently, the set of the decisions $D_{i,n}(x,c)$ in (4) depend on (x, c). The values (x, c) are assumed to be values of uncertain variables $(\bar{x},\bar{c})$ with the certainty distribution

$$h(x,c) = v[(\bar{x} \cong x) \wedge (\bar{c} \cong c)]$$

given by an expert. The decision problem consists in the determination of the optimal decisions i_n^* maximizing the certainty index that i_n approximately belongs to $D_{i,n}(\bar{x},\bar{c})$ (precisely speaking: that i_n belongs to $D_{i,n}(x,c)$ with the approximate values x and c), i.e.

$$i_n^* = \arg \max_{i_n \in I_n} v(i_n)$$

where

$$v(i_n) = v[i_n \,\tilde{\in}\, D_{i,n}(\bar{x},\bar{c})].$$

It is easy to note that

$$v[i_n \,\tilde{\in}\, D_{i,n}(\bar{x},\bar{c})] = v[(\bar{x},\bar{c}) \,\tilde{\in}\, D_{xc}(i_n)]$$

where

$$D_{xc}(i_n) = \{(x,c) : i_n \in D_{i,n}(x,c)\}.$$

Then

$$v(i_n) = \max_{(x,c) \in D_{xc}(i_n)} h(x,c).$$

As a result we obtain an *assembly plan* in the form of a sequence of the assembly operations i_n^*. We can apply another versions of the decision problem described in [10,15,16]. In particular, $(\bar{x},\bar{c})$ may be considered as so called C-uncertain variables. In this case

$$i_n^* = \arg \max_{i_n \in I_n} v_C(i_n)$$

where

$$v_C(i_n)=\frac{1}{2}\{v[i_n \tilde{\in} D_{i,n}(\bar{x},\bar{c})]+1-v[i_n \tilde{\in} \overline{D}_{i,n}(\bar{x},\bar{c})]\} \text{ and } \overline{D}_{i,n}=I_n-D_{i,n}.$$

8. CONCLUSIONS AND RELATED PROBLEMS

The learning algorithms and the application of the uncertain variables have been presented for a class of uncertain assembly processes described by a relational knowledge representation with unknown parameters. In the successive cycle of the assembly process a set of possible values of the unknown parameters is determined and used for the proper choosing of the assembly operation in the next cycle. The computer control system consists of the following blocks (subprograms): knowledge base, evaluation of the features, knowledge evaluation and updating, generation of the decisions (the current assembly operations) and two generators of random variables. This system has been used for simulations which show the significant influence of the system parameters on the convergence and quality of the learning process.

It is worth to indicate two related problems which may be the subject of further works:

1. The application of the presented approach to complex assembly process (in particular, to parallel processes), similar to the control of parallel operations presented in [6,12,13].
2. The determination of the learning algorithm for the assembly process described by a distributed knowledge representation. In such a case a special decomposition for the determination of the control decisions may be applied [14].

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