the lines $P'G$, $P'T$ and $P'L$, $P'M$ are harmonic lines through the point P'. It thus appears that in the particular case where the points L, M are the foci of the conic U', the line $P'G$ is the normal at the point P'; and we may say in general that $P'G$ is the quasi-normal at the point P' of the conic U'.

541.

ON THE RECIPROCAL OF A CERTAIN EQUATION OF A CONIC.

[From the *Messenger of Mathematics*, vol. I. (1872), pp. 120, 121.]

THE following formula is useful in various problems relating to conics: the reciprocal equation of the conic

$$\lambda\,(ax+by+cz)\,(a'x+b'y+c'z)-\mu\,(a''x+b''y+c''z)\,(a'''x+b'''y+c'''z)=0$$

may be written indifferent in either of the forms

$$\left\{\lambda \begin{vmatrix} \xi, & \eta, & \zeta \\ a', & b', & c' \\ a, & b, & c \end{vmatrix} + \mu \begin{vmatrix} \xi, & \eta, & \zeta \\ a'', & b'', & c'' \\ a''', & b''', & c''' \end{vmatrix} \right\}^2 + 4\lambda\mu \begin{vmatrix} \xi, & \eta, & \zeta \\ a, & b, & c \\ a''', & b''', & c''' \end{vmatrix} \begin{vmatrix} \xi, & \eta, & \zeta \\ a', & b', & c' \\ a'', & b'', & c'' \end{vmatrix} = 0,$$

and

$$\left\{\lambda \begin{vmatrix} \xi, & \eta, & \zeta \\ a', & b', & c' \\ a, & b, & c \end{vmatrix} - \mu \begin{vmatrix} \xi, & \eta, & \zeta \\ a'', & b'', & c'' \\ a''', & b''', & c''' \end{vmatrix} \right\}^2 + 4\lambda\mu \begin{vmatrix} \xi, & \eta, & \zeta \\ a, & b, & c \\ a'', & b'', & c'' \end{vmatrix} \begin{vmatrix} \xi, & \eta, & \zeta \\ a', & b', & c' \\ a''', & b''', & c''' \end{vmatrix} = 0.$$

In fact, in the reciprocal equation, seeking for the coefficient of ξ^2, it is

$$\{\lambda\,(bc'+b'c)-\mu\,(b''c'''+b'''c'')\}^2-(2\lambda bb'-2\mu b''b''')\,(2\lambda cc'-2\mu c''c'''),$$

viz. this is

$$\lambda^2\,(bc'-b'c)^2+\mu^2\,(b''c'''-b'''c'')^2+2\lambda\mu \left\{ \begin{array}{c} 2bb'c''c'''+2b''b'''cc' \\ -(bc'+b'c)(b''c'''+b'''c'') \end{array} \right\},$$

or, as it may be written,

$$\{\lambda(bc' - b'c) \pm \mu(b''c''' - b'''c'')\}^2 + 2\lambda\mu \left\{\begin{array}{c} 2bb'c''c''' + 2b''b'''cc' \\ -(bc' + b'c)(b''c''' + b'''c'') \\ \mp(bc' - b'c)(b''c''' - b'''c'') \end{array}\right\}.$$

Taking the upper signs, this is

$$\{\lambda(bc' - b'c) + \mu(b''c''' - b'''c'')\}^2 + 4\lambda\mu \left(\begin{array}{c} bb'c''c''' + b''b'''cc' \\ - bc'b''c''' - b'cb'''c'' \end{array}\right),$$

viz. the term in $\lambda\mu$ is

$$+ 4\lambda\mu(bc''' - b'''c)(b'c'' - b''c').$$

Taking the lower signs, it is

$$\{\lambda(bc' - b'c) - \mu(b''c''' - b'''c'')\}^2 + 4\lambda\mu \left(\begin{array}{c} bb'c''c''' + b''b'''cc' \\ - bc'b'''c'' - b'cb''c''' \end{array}\right),$$

viz. the term in $\lambda\mu$ is

$$+ 4\lambda\mu(bc'' - b''c)(b'c''' - b'''c').$$

And it is thence easy to infer the forms of the other coefficients, and to arrive at the foregoing result.