492.

NOTE ON A SYSTEM OF ALGEBRAICAL EQUATIONS.

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CONSIDER the system of equations

$$a + b(y+z)^2 + cy^2z^2 = 0,$$

$$a + b(z+x)^2 + cz^2x^2 = 0,$$

$$a + b(x+y)^2 + cx^2y^2 = 0,$$

which is a particular case of that belonging to the porism of the in-and-circumscribed triangle. We have y and z the roots of

$$a + bx^2 + 2u\,.\,bx + u^2(b+cx^2) = 0;$$

consequently

$$y + z = \frac{-2bx}{b+cx^2},$$

$$yz = \frac{a+bx^2}{b+cx^2},$$

or substituting in the equation between y and z, this becomes

$$(ac+b^2)(a+4bx^2+cx^4) = 0,$$

so that if $ac+b^2$ is not $=0$, we have

$$a + 4bx^2 + cx^4 = 0,$$

and moreover

$$(x-y)(x-z) = x^2 + \frac{2bx^2}{b+cx^2} + \frac{a+bx^2}{b+cx^2}, = \frac{1}{b+cx^2}(a+4bx^2+cx^4) = 0,$$

so that $x=y$ or else $x=z$. If $x=z$, the three equations reduce themselves to the two

$$a + bx^2 + 2y \,.\, bx + y^2(b+cx^2) = 0,$$
$$a \quad + \quad 4bx^2 + \quad cx^4 \quad = 0,$$

giving $y=x$, or else $y = -\frac{3bx+cx^3}{b+cx^2}$; and it hence appears that if from this last equation and $a+4bx^2+cx^4=0$ we eliminate x, the result must be $a+4by^2+cy^4=0$. For in the same way that the elimination of y, z from the original three equations gives $a+4bx^2+cx^4=0$, the elimination of x, z from the same three equations will give $a+4by^2+cy^4=0$, so that in any case y is a root of this equation.