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A study of some physical characteristics of liquid foams including resistance to bodies of revolution from subsonic to high supersonic Mach numbers (*)

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DRAG MEASUREMENTS were conducted in a dry liquid foam from subsonic to high supersonic Mach numbers. A shooting range was set up and spherical bullets were used crossing a foam column. Because of a very low velocity of sound in liquid foams, high Mach numbers could be reached with very low projectile velocities. The results in the range from subsonic to high supersonic speeds justified the method giving numerical values identical to those in the air. Sound wave attenuation in a visco-elastic fluid is discussed and compared to expressions in Newtonian fluids. For a liquid foam which is visco-elastic, an equation is derived giving the attenuation constant as function of the surface tension, velocity of the fluid, diameter of the bubbles, frequency of the oscillations and the velocity of sound in the liquid foam. The experimental results confirm this approach and give attenuation constants considerably higher than for a polyurethane dry foam.

Przeprowadzono pomiary sił oporu w suchych pianach ciekłych w zakresie prędkości opływu od poddźwiękowych do naddźwiękowych o dużych liczbach Macha. Stanowisko strzałowe pozwalało na wystrzeliwanie pocisków sferycznych poprzez walec pianki. Wobec niskich prędkości dźwięku w ciekłych piankach można, przy bardzo niskich prędkościach pocisku, osiągnąć bardzo wysokie liczby Macha. Wyniki osiągnięte w całym przedziale prędkości potwierdziły metodę, w której przyjmuje się wartości numeryczne takie same jak dla powietrza. Przedyskutowano tłumienie fali dźwiękowej w płynie lepkosprężystym i przeprowadzono porównanie z cieczami newtonowskimi. Dla ciekłej pianki lepkosprężystej wyprowadzono równanie, w którym stała tłumienia jest funkcją napięcia powierzchniowego, lepkości płynu, średnicy pęcherzyków, częstości drgań i prędkości dźwięku w piance ciekłej. Wyniki doświadczalne potwierdzają słuszność przyjętej metody i prowadzą do stałych tłumienia znacznie wyższych niż w przypadku suchej pianki poliuretanowej.

Проведены измерения сил сопротивления в сухих жидких пенах в интервале скоростей обтекания от дозвуковых до сверхзвуковых с большими числами Маха. Взрывная установка позволяет выстреливать сферические снаряды через цилиндр пены. Из-за низких скоростей звука в жидких пенах можно, при очень низких скоростях снарядя, достигнуть очень высоких чисел Маха. Реаультаты, достигнутые в целом интервале скоростей, подтвердили метод, в котором принимаются численные значения аналогично как для воздуха. Обсуждено затухание звуковой волны в вязко-упругой жидкости и проведено сравнение с ньютоновскими жидкостями. Для жидкой вязко-упругой пены выведено уравнение, в котором постоянная затухания является функцией поверхностного натяжения, вязкости жидкости, диаметра пузырьков, частоты колебаний и скорости звука в жидкой пене. Экспериментальные результаты подтверждают справедливость принятого метода и приводят к постоянным затухания значительно более высоким, чем в случае сухой полиуретановой пены.

1. Introduction

SOME UNUSUAL properties of liquid dry foams were reported previously [1] mainly related to shock wave attenuation. More research was continued chiefly on the attenuation of

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sound waves and the drag of bodies in liquid from through the trans-sonic into the high supersonic range. Because of the extremely low velocity of sound in foam, bodies moving is stationary foam at comparatively low velocities with reference to an observer on the ground can reach a high supersonic speed in the foam medium.

Tests were also conducted in liquid foams to assess the sound attenuation characteristics which were found to be considerably higher than in polyurethene solid foams. Formulae were derived to predict this behaviour.

2. Drag tests

2.1. Physical considerations

Wave formation at supersonic speeds is the major cause of the drag forces resulting from a pressure disequilibrium around bodies even in an ideal fluid. The linearized theory of airfoil drag above $M = 1.0$ gives results very close to the measured ones although it disregards boundary layer effects. A projectile moving in the liquid foam will experience drag mainly due to the pressure forces and to some extent friction forces. At a high speed the bubbles surrounding the projectile will most likely disintegrate and form a sheath of vapour, the friction forces will be more similar to those in a gas than those in a visco-elastic fluid, which is the foam. These considerations seem to justify drag tests by shooting bullets into liquid foam with high dryness fraction, with the advantage that for a given Mach number the speed of the bullet can be about eight times smaller than would be in the air. This reduces considerably the costs of installation, the needs for sophisticated equipment [2] and increases the precision of measurements.

If a body of mass m hits the air-foam interface with the velocity U_1, crosses a foam column of length l, and leaves the foam with the velocity U_2, it can be shown by elementary mechanics and using linear approximations that the drag is

$$D = \frac{m(U_1^2 - U_2^2)}{2l} \tag{2.1}$$

and the drag coefficient reduced to the reference surface A of the body

$$C_D = \frac{4m(U_1 - U_2)}{\varrho l A(U_1 + U_2)}. \tag{2.2}$$

For a projectile of diameter D, with the reference surface $\frac{\pi}{4} D^2$

$$C_D = \frac{16m(U_1 - U_2)}{\pi \varrho D^2 l(U_1 + U_2)}, \tag{2.3}$$

where ϱ is the density of the foam. If the velocity of sound in the foam is a_c, the average Mach number in the foam is

$$\bar{M} = \frac{U_1 + U_2}{2a_c}. \tag{2.4}$$

Further refinement is possible by assuming a nonlinear variation of velocity with distance in the foam. An error analysis indicated that with the available testing rig and precision of measurements such a refinement was not justifiable. As the velocity of the bullet before hitting and after leaving the foam can be assessed using appropriate techniques, the drag coefficient in the foam can be estimated. It was thought that spherical bullets would be the most appropriate ones to measure because the drag results are well known and also because the radius of curvature is easily measurable and constant. As the drag coefficient is very sensitive to the radius of curvature of the nose of the body which is not easily measurable for small ogives, most of the tests were conducted with spherical bullets.

2.2. Details of tests

The schematic outline of the test rig for projectile drag tests is shown in Fig. 1. An "air gun" using capsules of compressed CO_2 was mounted and aligned in front of a tube ($D = 50$ mm) containing liquid foam. The length of the foam column was $l = 450$ mm or $l = 260$ mm. Each capsule was capable of giving approximately 30 firings with velocities ranging from about 150 m/s, when new, to 30 m/s at the lower limit of its pressure level. In this manner the Mach number could be varied over a wide range. As explained above, the bullets were spherical, their diameter was $D = 4.35$ mm and mass 0.345 gr.

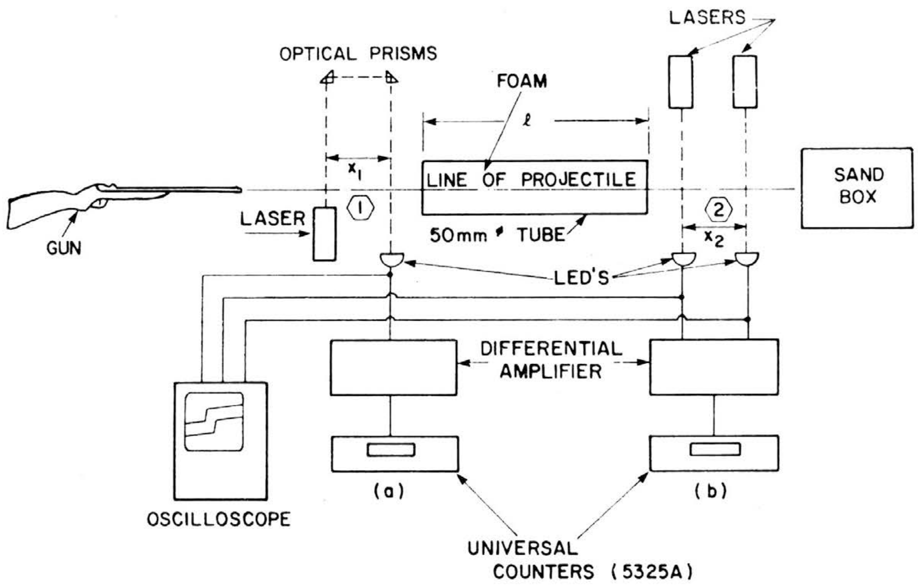
Although various foams could be generated in the laboratory using a whipper mechanism and various foam forming liquids, it was found that the most stable one requiered for these tests was a commercial Gillette shaving foam which had a dryness fraction of $\alpha = 0.934$. The definition of the dryness fraction is — (the volume of gas): (the volume of liquid and gas). As shown in [1] the velocity of sound in a liquid foam can vary from about 23 m s^{-1} with $\alpha = 0.5$ to about 45 m s^{-1} for a stable dry foam. The velocity of sound of the Gillette foam estimated from weak shock waves (with pressure ratios extrapolated to unity) was $a_c = 46$ ms^{-1}. More details of this foam is given in Tabl. 1.

Table 1.

Type of foam	bubble diam. *mm*	dryness fraction	μ liquid viscosity kg/m sec	ϱ liquid denisty kg/m^3 approx.
Commercial (Gillette)	0.02 to 0.035	0.934	0.0148	1000

This foam was exteremely stable and could remain in the tube for several hours without changing its dryness fraction. The same could not be said about whipper generated foams in the laboratory which would easily form a thin layer of liquid at the bottom of the tube during the tests adding to the uncertainty of the results.

Two laser beams distant $X_1 = 51.5$ mm and $X_2 = 55$ m were aligned perpendicularly to the path of the bullet as shown in Fig. 1. The time interval required to cross the distance between the laser beams could be recorded with an exactitude of about 1.5 μs. The record-



PROJECTILE DRAG SET-UP

FIG. 1. Drag measurement range for projectiles in liquid foam.

ding was done by means of universal counters connected to differential amplifiers and an oscilloscope.

The tests were conducted within the laboratory and the limits of velocity of the bullet were set by safety regulations. Also it was found be error analysis that increase in the velocity of the bullet resulted in a decrease in the exactitude of the recorded time intervals. There was not much point to continue systematic tests in the hypersonic range. Nevertheless ogive bullets were chosen, of one shape only and with one strength of explosive charge resulting in the Mach number in the foam $M = 8.3$.

2.3. Results of the tests

Figure 2 shows the drag coefficient C_D defined in Sect. 2.1 as function of the average Mach number. In spite of the scatter of measured points, most likely due to the experimental errors in assessing the time interval over the limited length of the bullet trajectory, the curve follows very closely the results obtained for spheres in the subsonic and supersonic range ([3] Vol. I). It shows the typical hump above $M = 1.0$, which seems to be a little larger than expected. Although the reflected shock and recorded shock on the walls of the tube were weak, one might expect an interference effect between the reflected shock and the near-wake of the bullet after crossing $M = 1.0$ which might be responsible for these results. Also the development of base pressures in the foam as compared to tests in gases is not clear. Tests at $M = 8.3$ gave a moderately dispersed cluster of points with the mean $C_D = 0.97$, what compares favourably with the asymptotic values of measurements on various ogives (ogive No. II) with varying shapes of the noses as reported by A. SHAPIRO

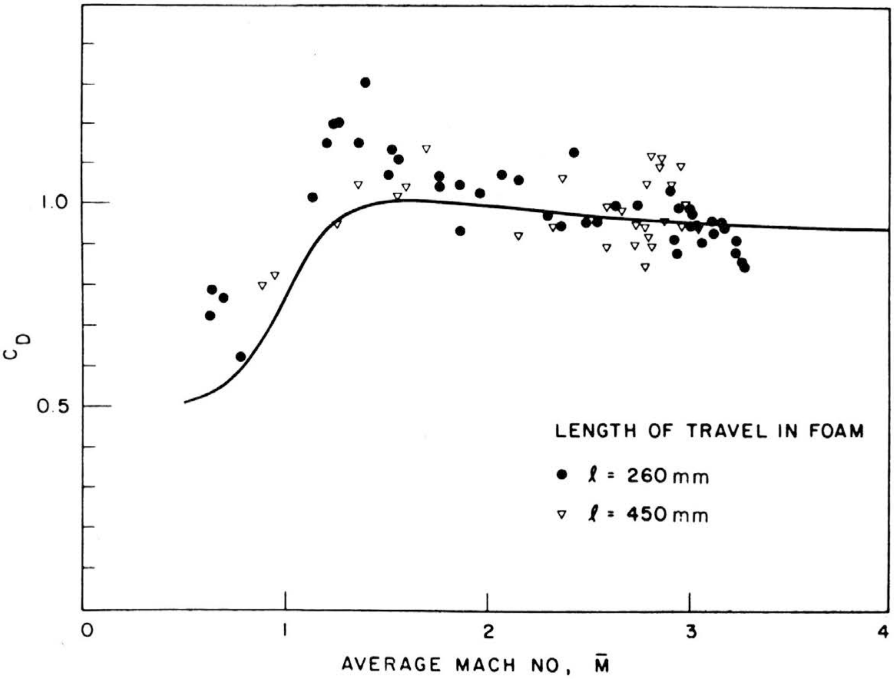


FIG. 2. Drag coefficient against Mach No. measured in foam on spheres.

([3] Vol. II). As the radius of curvature and the exactitude of time signals was not easy to assess and the measured drag depends critically upon them, the tests were discontinued. A more sophisticated rig would be required and a larger range should be built for hypersonic tests. It appears that this method can yield very useful results for drag measurements at high speeds at a very reduced cost.

3. Acoustic attenuation in liquid foams

3.1. Preliminary observations

A liquid foam of high dryness fraction is a typical viscoelastic fluid. Thus a heavy object laid on the foam surface will oscillate although the mass of the displaced foam is an order of magnitude less than the mass of the object itself. If the mass of the solid is further increased beyond the first Newtonian region of elastic behaviour, it will slowly sink, given sufficient time. In such fluids the shear stresses are accompanied by additional normal stresses. The τ_{ij}, τ_{ji}.. components of the stress tensor are associated with additional σ_{ii}, σ_{jj}.. components higher than in the Newtonian fluids partly due to the rate of strain and partly to the strain itself required to compress the fluid ([5], Chap. 1 and 2). When dealing with acoustic attenuation of longitudinal sound waves in liquid foams this aspect should be taken into account.

Dissipation of mechanical energy in sound waves in gases can be attributed to four causes: i) viscous effects, ii) heat transfer, iii) heat radiation and iv) intermolecular exchanges of energy either between the molecules of different gases or between different degrees of freedom of the molecules.

Most likely in liquid foams the viscous dissipation effects and energy exchange between different degrees of freedom of gaseous bubbles play a dominant role, while the irreversible heat transfer associated with an increase of entropy and the heat radiation play a secondary role at higher frequencies as shown by K. OSWAITICH [6]. His study of dispersion and absorption of sound in clouds where the condensation and evaporation of water takes place is similar to the propagation of sound waves in a liquid foam. Considerable sound absorption occurs at 54 Hz and afterwards diminishes rapidly. It appears, therefore, that measured absorption of sound in liquid foams would be lower at small frequencies than predicted on the basis of viscous dissipation of energy only.

3.2. Acoustic attenuation in real gases with Newtonian viscosity

The classical equation for a plane sound wave in ideal gases (see for example [4], Chap. V) is

$$\begin{gathered} \varrho \frac{\partial^2 \xi}{\partial t^2} - K \frac{\partial^2 \xi}{\partial x^2} = 0, \\ \frac{K}{\varrho} = c^2, \end{gathered} \tag{3.1}$$

where ϱ — density, ξ — displacement, t — time, K — bulk modulus, x — distance, c — velocity of sound.

If viscous losses are taken into account, another damping term must be added and the equation is

$$\varrho \frac{\partial^2 \xi}{\partial t^2} + R \frac{\partial \xi}{\partial t} - K \frac{\partial^2 \xi}{\partial x^2} = 0, \tag{3.2}$$

where R is the damping coefficient. STOKES [7] and RAYLEIGH [8] obtained a differential equation for a plane sound wave in the form

$$\varrho \frac{\partial^2 \xi}{\partial t^2} - \frac{4}{3} \mu \frac{\partial^3 \xi}{\partial x^2 \partial t} - K \frac{\partial^2 \xi}{\partial x^2} = 0, \tag{3.3}$$

μ — viscosity coefficient.

In the second term one recognizes the presence of the σ_{ii} component of the stress tensor for Newtonian fluids, namely:

$$\frac{4}{3} \mu \frac{\partial u_i}{\partial x_i}.$$

Equations (3.2) and (3.3) can be made identical if

$$\begin{gathered} R\xi = -\frac{4}{3} \mu \frac{\partial^2 \xi}{\partial x^2} = \frac{4}{3} \mu \frac{\omega^2}{c^2} \xi \quad \text{i.e.,} \\ R = \frac{4}{3} \mu \frac{\omega^2}{c^2}, \end{gathered} \tag{3.4}$$

where ω = cicrular frequency, because

$$\frac{\partial^2 \xi}{\partial x^2} = -\frac{\omega^2}{c^2}\xi.$$

It follows that the solution of this differential equation is in the complex form

$$\xi = ae^{-\alpha x}e^{i\omega\left(t-\frac{x}{c}\right)}, \tag{3.5}$$

where a is the amplitude, α is the attenuation constant per unit distance. The attenuation constant

$$\alpha = \frac{R}{2\varrho c} = \frac{2}{3}\frac{\mu}{\varrho}\frac{\omega^2}{c^3} = \frac{2}{3}\nu\frac{\omega^2}{c^3} \tag{3.6}$$

and

$$\nu = \frac{\mu}{\varrho}.$$

The physical meaning of the nondimensional exponent αx can be understood as: (work per second due to viscous friction): (work per second due to compressibility) what leads to the ratio of (viscous stress): (bulk modulus).

Similar reasoning can now be applied to a non-Newtonian fluid like the liquid foam.

3.3. Acoustic attenuation in liquid foams

In a liquid foam the relative motion of the bubbles with the liquid between them will produce a τ_{ji} component in the x-direction which will be also accompanied by an additional σ_{ii} component discussed in Sect. 3.1 due to the non-Newtonian character of the fluid and the compressibility of the bubbles. Although the magnitude of these components would be difficult to establish, one can again assume that the work done per second due to viscous friction for one bubble is proportional to

$$(\mu\omega)d^2\omega d = \mu(\omega d)^2 d.$$

If the wave crosses in one second n bubbles covering a distance $nd = l$, the work per second is

$$\sim \mu(\omega d)^2 l.$$

On the other hand, if the surface tension is σ, then the work/s to overcome the surface tension of the bubbles is proportional to $(\sigma d)\omega dn = (\sigma d)\omega l$ but $l \sim c/\omega$, therefore work/s $\sim \sigma dc$.

The ratio becomes

$$\sim \frac{\mu\omega^2 d^2 l}{\sigma dc} = \frac{\mu\omega^2 ld}{\sigma c}.$$

The attenuation constant per unit distance

$$\alpha \sim \frac{\mu\omega^2 d}{\sigma c} = k\frac{\mu}{\sigma}\frac{\omega^2 d}{c} \tag{3.7}$$

with a proportionality factor k to be found experimentally; μ — viscosity of the fluid forming the bubbles, c — velocity of sound in the foam, d — average diameter of a bubble, σ — surface tension of the fluid (N/m).

If the reasoning is correct, one should obtain varying attenuation characteristics of foams in which there is a range of diameters and viscosities of the fluid. In general, bigger bubbles and high viscosity should enhance attenuation. Very low velocities of sound observed in liquid foams will also contribute substantially in the attenuation.

It follows that the ratio of amplitudes of acoustic signals along the foam column should be

$$\frac{y_2}{y_1} = e^{-k\frac{\mu}{\sigma}\left(\frac{d}{c}\right)\omega^2 l}, \tag{3.8}$$

where ω — circular frequency, μ — viscosity of the liquid forming the foam, l — trajectory.

3.4. Experimental details and results

Figure 3 shows the experimental set-up to measure acoustic attenuation in foams. A Hewlett Packard 5423A Structural Dynamic analyzer was used. A signal generator giving a pure sinusoidal wave was conducted through a power amplifier to an "acoustic

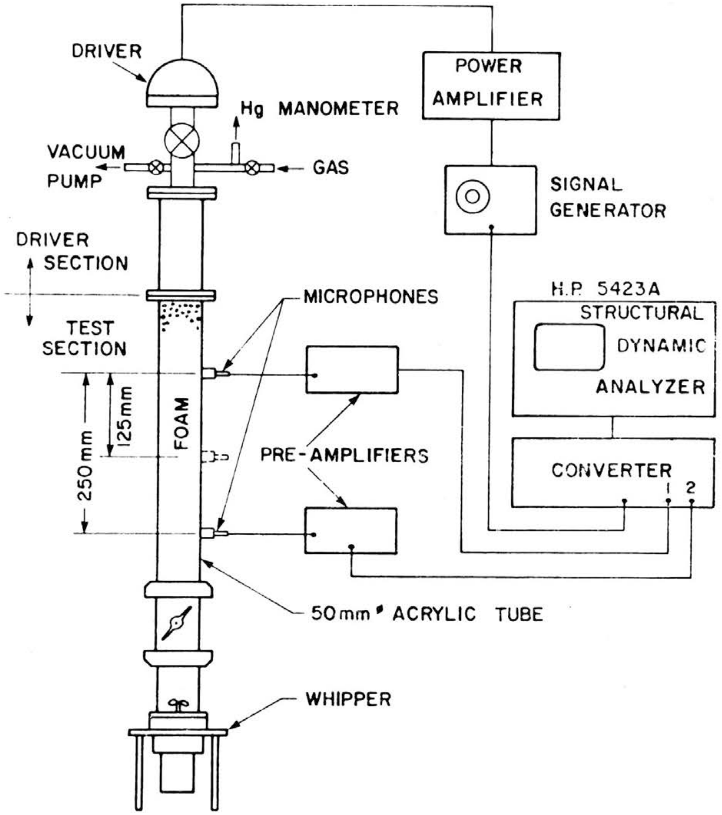


FIG. 3. The experimental set-up for measurements of acoustic attenuation.

driver"; the test section consisted of a transparent acrylic tube $D = 50$ mm at the bottom of which a whipper was installed which could generate various foams using liquids with different viscosities. Two microphones connected through pre-amplifiers could be set at distances l of 250 and 125 mm and the signals reached the H. P. Analyzer through a converter. Also a ready made foam could be fed into the tube. The vacuum pump was used when other gases than air had to be tested. By putting a specified quantity of liquid, the height of the column after using the whipper determined the dryness fraction. A butterfly valve

Table 2.

Foam	bubble diameter mm	dryness fraction	Vel. of sound m/s	liquid viscosity kg/m · s	liquid density kg/m³ (approx)
Gillette (ready made)	0.02 to 0.035	0.934	51	0.0148	1000
water-shampoo air	0.44 to 0.446	0.9–0.95	71.8	0.00105	1000
water-detergent CO_2	0.44 to 0.46	0.9–0.95	72–78	0.00121	1000
shampoo-glycerine air	0.44 to 0.46	0.9–0.95	72–78	0.00337	1000

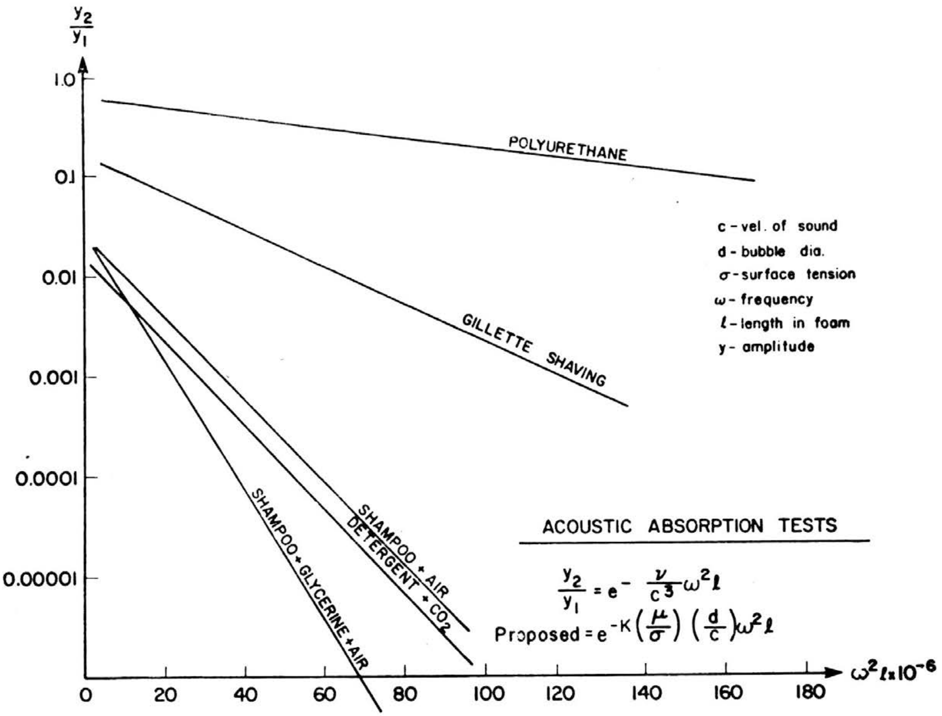


FIG. 4. Experimental results of acoustic attenuation in foams and in polyurethane.

separated the whipper from the tests section. The surface tension, σ was, approximately constant $\sigma \cong 0.025$ N/m. The liquids and gases used for the experiments and their chief characteristics are tabulated in Tabl. 2.

The velocities of sound in the foam were assessed for each test. The experimental results were plotted against $\omega^2 l \times 10^{-6}$ on a semi-log paper, and the mean values are shown in Fig. 4. Various foams formed well defined families of straight lines indicating the exponential nature of the sound decay. The lines followed well Eq. (3.8) (with $k \simeq 0.18$) within the range bubble diameter variation. One observes that the sequence of the lines follows the trends indicated by the proposed approach, the biggest attenuation is observed for comparatively large bubbles and high liquid viscosity. For lower frequencies these lines do not reach unity what could be attributed to thermal effects discussed in Sect 3.1.

Also polyurethane dry foam was used as a comparison (density $\varrho = 19.2$ kg/m^3), a material well known for its strong sound attenuation. One observes that the amplitude ratio practically reaches unity for very low frequencies and also that its absorption characteristics are of several orders of magnitude worse than of the liquid foams. This is of great practical value. As no thermal effects are present in a polyurethane foam the sound attenuation is worse at low frequencies than in liquid foams; this confirms the trend predicted in Sect. 3.1.

4. Conclusions

More study was made on dry liquid foams as a continuation of the previous one reported in [1]. Very low velocities of sound which characterizes such foams were taken advantage of to test the drag of projectiles moving at comparatively low speed but at a high Mach number in the foam. The maximum Mach number of the tests was $M = 8.3$. The C_D curves as F(M) were practically identical with classical tests in high speed ranges.

Also acoustic attenuation was studied in various foams. A formula is proposed, based on physical consideration to assess the acoustic attenuation characteristics of various liquid foams. The results seem to confirm the derived equation indicating the advantages of using a visco-elastic non-Newtonian with low velocity of sound to attenuate sound waves.

Drag measurements of spheres in foam justify the assumption that the wave drag component is prevalent and as it is a function of the Mach number, the results are comparable between air and foam. The advantage of using liquid foam is that the velocities of the projectiles can be almost an order of magnitude lower than in the air for the same Mach number.

The acoustic attenuation of plane sound wave in liquid foam is considerably larger than a dry foam (polyurethane). This is explained on the basis of the non-Newtonian character of liquid foams and low velocities of sound in them. A formula is derived for the exponential attenuation constant per unit distance which takes into account the bubble diameter, viscosity of the fluid forming the foam, surface tension and the velocity of sound in the foam. The experimental results confirm this expression.

Both results, high Mach number drag tests in liquid foams and their remarkable sound attenuation characteristics are of great practical value.

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