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Effective material constants for orthotropically damaged elastic solid

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THE AIM of the paper is the formulation of the constitutive equation of elasticity for damaged solid with regular array of rectilinear narrow cracks. The current state of the damaged material was described by means of a symmetric second order damage tensor entering the constitutive equation as an independent variable. The validity of the constitutive equation derived in this paper and the definition of the damage tensor proposed were verified experimentally. To this end the elastic constants calculated theoretically were compared with those measured experimentally employing the models simulating the damage solid.

Celem pracy jest sformułowanie równania konstytutywnego sprężystości dla materiału uszkodzonego z regularnym układem prostoliniowych pęknięć. Do opisu bieżącego stanu materiału uszkodzonego wykorzystano symetryczny tensor drugiego rzędu, wchodzący do równania konstytutywnego jako zmienna niezależna. Poprawność sformułowanego równania konstytutywnego oraz zaproponowanej definicji tensora uszkodzenia została zweryfikowana doświadczalnie. W tym celu stałe sprężystości obliczone na drodze teoretycznej zostały porównane z wartościami zmierzonymi doświadczalnie przy zastosowaniu modeli symulujących materiał uszkodzony.

В работе представлены определяющие уравнения упругости для материала с регулярно расположенными прямолинейными трещинами. Для описания текущего состояния поврежденного материала использован симметричный тензор второго порядка, входящий в состав определяющего уравнения как независимая переменная. Правильность сформулированного определяющего уравнения и предложенного определения тензора повреждения проверена экспериментально. С этой целью постоянные упругости, вычисленные теоретически, сравнены со значениями измеренными экспериментально при применении моделей, имитирующих поврежденный материал.

1. Introduction

THE NUCLEATION and growth of regularly distributed microcracks and voids at grain boundaries of the initially homogeneous and isotropic solids results in overall mechanical behaviour of the material far from being isotropic. Development of such an internal oriented damage affects the mechanical response of the material in elastic and plastic ranges as well as at rupture. When dealing with the problem of the constitutive equation of continuum damage mechanics, two groups of equations should be formulated. The first one specifies in the elastic range the overall strain in terms of the stress and damage tensor. The second group of relations concerns the variation of the damage with the applied stress. The main subject of numerous scientific papers on damage mechanics is the first group of equations accounting for the stiffness and strength reduction as well as the specific symmetry of the damaged solids due to cracks development.

Various attempts to formulate such equations for damaged solids employing the concept of the damage tensor were presented by VAKULENKO and KACHANOV [1, 2, 3], DRAGON [4], KACHANOV [5], MURAKAMI and OHNO [6], KRAJCINOVIC [7, 8], and BETTEN [9].

The considerations presented in this note follow that general line but the attention is focused on the elastic behaviour of the cracked solid. The specific case of the damage induced elastic anisotropy and formulation of the respective constitutive equation was presented by KACHANOV [2, 3]. CHABOCHE [10] and CORDEBOIS and SIDOROFF [11]. The constitutive equation of elasticity formulated by VAKULENKO and KACHANOV [1] and discussed in details by KACHANOV [2, 3] includes several unknown functions and constants. It means that, when utilizing this theory some additional data are necessary which can be obtained, for example, from suitable experiments. Thus the continuum damage theory proposed in [1, 2, 3] is not self-contained in spite of possibility to determine those unknown functions on the basis of the results presented in more recent papers by HOENIG [12], BUDIANSKY [13, 14] and SALGANIK [15] concerning the theoretical homogenization of cracked solids.

An alternative method of formulation of the constitutive equation for damaged solids is presented in this note. The constitutive equation of elasticity derived, employing the theory of tensor function representations, contains as an independent variable the damage tensor describing the current state of the solid and being fully responsible for the symmetry of the material structure as well as for the modification of the material constants. The analysis of the damage evolution was restricted to some well-defined stages of the cracks development, and the damage tensor components were calculated according to the definition proposed in the paper. The equation of the damage tensor evolution being the separate problem has not been formulated here. However, if such an equation is derived, it could easily be incorporated into the constitutive equation proposed.

The effective elastic constants for damaged solid entering the constitutive equation were expressed in terms of the material constants for the undamaged, original material and the damage tensor components. The numerical values calculated for a specific case of two different regular arrays of rectilinear cracks were compared with the effective elastic constants measured experimentally, employing the models simulating the damaged material.

2. Constitutive equation

Due to the introduced oriented damage, the overall material behavior is anisotropic and, thus the constitutive equation of elasticity should account for the specific material symmetry. It can be done by formulating this equation in the form of a tensor function

$$\mathbf{E} = \mathbf{F}(\mathbf{T}, \mathbf{D}), \tag{2.1}$$

where $\mathbf{E}$ and $\mathbf{T}$ are strain and stress tensors, respectively, and $\mathbf{D}$ is the second order symmetric tensor describing the internal damage of the material. It seems reasonable to assume an independent variable $\mathbf{D}$ in the form of such a tensor, although it is obvious that the symmetric second rank tensor possessing three mutually perpendicular principal directions can describe the symmetry of orthotropic material only.

Employing the tensor function representations theory, the most general polynomial form of the relation (2.1) can be written as follows

$$\mathbf{E} = \alpha_1\mathbf{I}+\alpha_2\mathbf{T}+\alpha_3(\mathbf{TD}+\mathbf{DT})+\alpha_4\mathbf{D}+\alpha_5\mathbf{T}^2 + \alpha_6\mathbf{D}^2+\alpha_7(\mathbf{TD}^2+\mathbf{D}^2\mathbf{T})+\alpha_8(\mathbf{T}^2\mathbf{D}+\mathbf{DT}^2)+\alpha_9\mathbf{T}^2\mathbf{D}^2, \tag{2.2}$$

where α_i ($i = 1, 2, \ldots 9$) are the polynomial functions of the scalar invariants of the stress and damage tensor.

To derive an explicit form of the function (2.2), the stress-strain law of elasticity

$$E_{ij} = A_{ijkl} T_{kl} \tag{2.3}$$

was considered. The fourth order anisotropy tensor A_{ijkl} is a function of only one independent variable D_{ij}. According to the theory of tensor function representation [16] this function, used also by MURAKAMI and IMAIZUMI [17], has the form

$$\begin{aligned} A_{ijkl} = {} & A\delta_{ij}\delta_{kl} + B(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \alpha\delta_{ij}D_{kl} \\ & + \beta D_{ij}\delta_{kl} + \gamma(\delta_{ik}D_{jl} + \delta_{il}D_{jk} + \delta_{jk}D_{il} + \delta_{jl}D_{ik}) + \gamma_1 D_{ij}D_{kl} \\ & + \gamma_2\delta_{ij}D_{kp}D_{pl} + \gamma_3 D_{ip}D_{pj}\delta_{kl} + \gamma_4(\delta_{ik}D_{jp}D_{pl} \\ & + \delta_{il}D_{jp}D_{pk} + \delta_{jk}D_{ip}D_{pl} + \delta_{jl}D_{ip}D_{pk}) \\ & + \gamma_5 D_{ij}D_{kp}D_{pl} + \gamma_6 D_{jp}D_{pl}D_{kl} + \gamma_7 D_{ip}D_{pj}D_{kq}D_{ql}, \end{aligned} \tag{2.4}$$

where α, β, γ, and γ_i ($i = 1, 2, \ldots 7$) are functions of invariants of D_{ij}, and A, B are constants.

It seems justified to reduce the function (2.4) to the form linear in damage

$$\begin{aligned} A_{ijkl} = {} & A\delta_{ij}\delta_{kl} + B(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \beta(D_{ij}\delta_{kl} + \delta_{ij}D_{kl}) \\ & + \gamma(\delta_{ik}D_{jl} + \delta_{jl}D_{ik} + \delta_{il}D_{jk} + \delta_{jk}D_{il}), \end{aligned} \tag{2.5}$$

where it was assumed that $\alpha = \beta$ due to the symmetry $A_{ijkl} = A_{klij}$. To simplify the further considerations, the functions β and γ entering the equation (2.5) are assumed to be constants. The other constants in (2.5) are expressed by the Young modulus E and the Poisson ratio ν of the matrix material

$$A = -\frac{\nu}{E}, \quad B = \frac{1+\nu}{2E}.$$

In the case of undamaged material when $D_{ij} = 0$, the expression (2.5) becomes the isotropic fourth order tensor and the function (2.3) represents the well known equation of linear elasticity for isotropic material.

Inserting (2.5) into (2.3), the following form of the function (2.1) was obtained

$$E = (A\,\mathrm{tr}\,\mathbf{T} + \beta\,\mathrm{tr}\,\mathbf{DT})\mathbf{I} + 2B\mathbf{T} + 2\gamma(\mathbf{TD} + \mathbf{DT}) + \beta\mathbf{D}\,\mathrm{tr}\,\mathbf{T}. \tag{2.6}$$

Comparing this equation with that represented by (2.2) it is seen that functions of invariants have a form

$$\begin{aligned} \alpha_1 &= A\,\mathrm{tr}\,\mathbf{T} + \beta\,\mathrm{tr}\,\mathbf{DT}, \\ \alpha_2 &= 2B, \\ \alpha_3 &= 2\gamma, \\ \alpha_4 &= \beta\,\mathrm{tr}\,\mathbf{T}, \\ \alpha_5 &= \alpha_6 = \alpha_7 = \alpha_8 = \alpha_9 = 0. \end{aligned}$$

Thus the constitutive equation (2.6) contains two unknown constants β and γ and the damage tensor D. If those two constants are determined and if the suitable definition of the damage tensor is formulated, all the effective elastic constants entering the constitutive equation (2.6) will be easily calculated.

3. Damage tensor

Usually it is assumed that the principal effect of the material damage consists in the net area reduction caused by nucleation and growth of fissures and grain boundary cavities. However, such an internal damage has, in general, directional properties resulting also in the development of an overall material anisotropy. As it was mentioned in the previous section, the material symmetry in the specific case of orthotropic solids can be described by the symmetric second rank tensor called the damage tensor. The well known definitions of such a tensor proposed by VAKULENKO and KACHANOV [1], MURAKAMI and OHNO [6], KRAJCINOVIC [8] or BETTEN [9] being the generalization of the scalar parameter introduced by KACHANOV [18] to describe the internal damage, proved to be insufficient when utilizing the equation (2.6). Thus it is advisable to derive a modified form of the damage tensor suitably describing the mechanical response of the cracked material.

The considerations presented in this paper are restricted to the case of the damaged material possessing three mutually perpendicular planes of symmetry π_1, π_2, π_3 shown

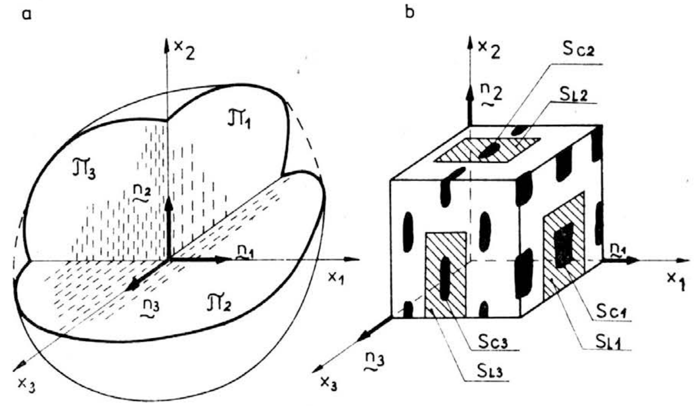

FIG. 1. Internal structure of regularly cracked solid.

in Fig. 1a. The principal directions of the damage tensor should be associated with the directions normal to those planes defined by unit vectors $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$. Thus the damage tensor can be defined as follows:

$$\mathbf{D} = D_1 \mathbf{n}_1 \otimes \mathbf{n}_2 + D_2 \mathbf{n}_2 \otimes \mathbf{n}_2 + D_3 \mathbf{n}_3 \otimes \mathbf{n}_3, \tag{3.1}$$

where D_1, D_2 and D_3 are the principal values of the damage tensor depending on the net area reduction due to fissuration determined on the planes perpendicular to the axes x_1, x_2, x_3. This definition of the damage tensor is similar to that proposed by KACHANOV [2] and MURAKAMI and OHNO [6], but the meaning of the principal values D_1, D_2, D_3 is different. It was found in calculating the elastic constants from equation (2.5) that the components of the damage tensor must be expressed not only by the cracks area as it was suggested in [1, 6, 8, 9], but also by the area of the ligament between the adjacent cracks, that means by the net area of the matrix material remained. Thus it is proposed to calculate the principal values of the damage tensor as follows

$$D_i = \frac{S_{Ci}}{S_{Li}}, \quad i = 1, 2, 3, \tag{3.2}$$

where S_{Ci} and S_{Li} stand for the respective crack and ligament areas on the plane π_i normal to the coordinate axis x_i shown in Fig. 1b. In the limit case of absence of fissuration on the given plane π_i, the appropriate principal component is equal zero what results in the overall strength for loading in the direction perpendicular to this plane equal to that observed for original undamaged material. On the other hand, if the damage evolution reduces the ligament area to zero on the considered plane, the respective component D_i increases indefinitely and this means that the Young modulus for this direction vanishes. Thus the damage tensor defined by (3.1) and (3.2) fully describes not only the material symmetry but also the stiffness reduction of the solid.

4. Effective elastic constants

The overall mechanical response of the cracked solid shown in Fig. 1 is that observed in orthotropic material. It means that its mechanical behavior in the elastic range can be described by nine elastic constants: three Young moduli E_1, E_2, E_3, three shear moduli G_{12}, G_{23}, G_{31} and three Poisson ratios $\nu_{12}, \nu_{23}, \nu_{31}$. All those material constants are expressed by the equation (2.5) as functions of the damage tensor **D**. However, their numerical values cannot be calculated directly from (2.5) because this equation contains two unknown constants β and γ. These two constants could be determined if two of nine above mentioned elastic constants were known. Considering, for example, the uniaxial loading of the damaged material with crack orientation presented in Fig. 2, one obtains from (2.5) for uniaxial tension in the direction parallel to longitudinal axes of cracks the following set of two linear equations

$$(4.1)\qquad \begin{aligned} 2D_2\beta+4D_2\gamma &= \frac{1}{E_2}-\frac{1}{E},\\ (D_1+D_2)\beta &= -\frac{\nu_{21}}{E_2}+\frac{\nu}{E}, \end{aligned}$$

enabling to determine β and γ. The numerical values of the Young modulus E_2 and Poisson ratio ν_{21} required to calculate β and γ can easily be measured experimentally, thus in order to determine nine elastic constants for orthotropic solids, a very simple experiment of uniaxial tension is necessary. However, it appears that there exists another, more efficient method to determine β and γ. For a specific case of loading in the direction parallel to longitudinal axis of relatively narrow cracks, that means if their width t is small in comparison with the distance P_1 between two adjacent cracks, it is possible to estimate E_2 and ν_{21} from very simple theoretical considerations. It is obvious that the state of stress in the vicinity of single crack is very complicated and the stress concentration strongly depends on the crack geometry. Thus the theoretical determination of the exact values of elastic constants E_2 and ν_{21} by considering the deformations of a unit cell of the material structure requires the solution of a fairly complicated boundary value problem. However, it is justified to assume that the values of those elastic constants for a damaged material with cracks of a given length l and width t are bounded by the limit values

$$E_2 = E, \qquad \nu_{21} = \nu$$

for $l = t = 0$, that means for material without cracks and by

$$E_2 = \left(1 - \frac{t}{P_1}\right)E, \quad \nu_{21} = \left(1 - \frac{t}{P_1}\right)\nu$$

for $l = P_2$. In the considered case of narrow cracks where the dimensionless crack width $\tau = t/P_1 < 0.1$, those two limit values are very close to each other, therefore it seems unreasonable to look for the exact solution of the problem. To obtain an approximate solution furnishing the results very close to those probably obtainable from the exact solution, the deformations of the unit cell of the material structure are analysed under the following assumptions: a) irregular contour of each crack is approximated by the rectangle of dimensions l and t, b) the state of stress in the unit cell of material structure uniaxially loaded in the direction parallel to the longitudinal axes of cracks is uniaxial and homogeneous; c) the value of uniaxial stress in portion I of the material structure (Fig. 2) is equal to the homogenized overall uniaxial stress T_{22} applied to the damaged

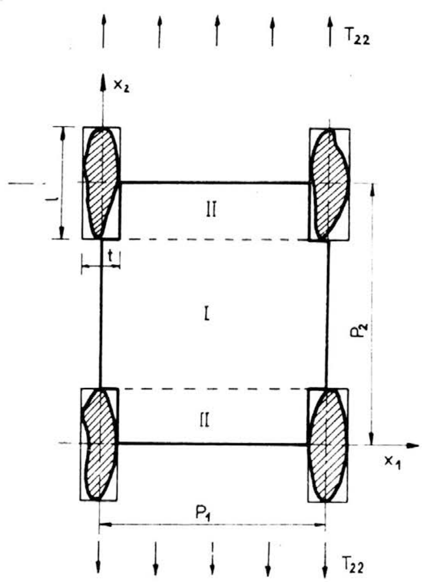

FIG. 2.

material, d) the value of uniaxial stress in portion II is equal to $T_{22}/(1-\tau)$. This approximation gives the following values of the Young modulus E_2 and the Poisson ratio ν_{21}:

$$E_2 = \frac{(1-\tau)E}{1-\tau+\lambda\tau}, \quad \nu_{21} = \frac{(1-\tau)\nu}{1-\tau+\lambda\tau}, \tag{4.2}$$

where λ is the dimensionless crack length $\lambda = l/P_2$. Inserting (4.2) into (4.1), the following values of β and γ are obtained

$$\beta = 0,$$
$$\gamma = \frac{\lambda\tau}{4(1-\tau)D_2E}, \tag{4.3}$$

and, finally, all the elastic constants for orthotropic equivalent material can be calculated from the equation

$$(4.4)\qquad A_{ijkl} = -\frac{\nu}{E}\delta_{ij}\delta_{kl} + \frac{1+\nu}{2E}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) + \frac{\lambda\tau}{4(1-\tau)D_2E}(\delta_{ik}D_{jl}+\delta_{jl}D_{ik}+\delta_{il}D_{jk}+\delta_{jk}D_{il}).$$

Thus the homogenized overall elastic behavior of the damaged material described by the equivalent elastic constants A_{ijkl} is expressed by the material constants E and ν of the original undamaged material and by the components of the damage tensor D_{ij} accounting for the current state of the damaged solids.

5. Experimental results

To verify the validity of the theoretical considerations concerning the effective elastic constants of the damaged material, the suitably designed experiments were performed.

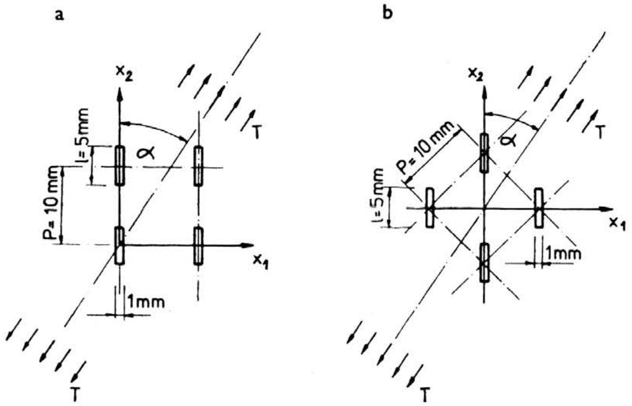

FIG. 3.

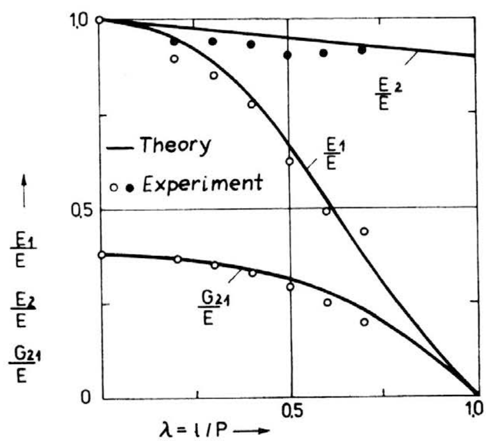

FIG. 4. Young moduli and shear modulus versus crack length for damaged solid with cracks arranged in pitch direction.

Two types of the damaged materials were simulated by means of flat specimens having the sets of regularly distributed rectangular openings arranged in square patterns, as shown in Fig. 3. In the first case presented in Fig. 3a, the cracks are oriented in the pitch direction, and in the second one shown in Fig. 3b — in diagonal direction. The uniaxially loaded specimens with a given cracks arrangement were cut out from the aluminium alloy metal sheets of thickness 0.7 mm. The total length of the specimen was equal to 400 mm with cracked portion 210 mm long and 70 mm wide. The details concerning the specimens preparation and experimental technique are presented in [19]. The process of the damage evolution in the models tested was simulated by changing length of the openings expressed by the rations $1/P = 0.2, 0.3, 0.4, 0.5, 0.6$ and 0.7, where P is the pitch of the square pattern

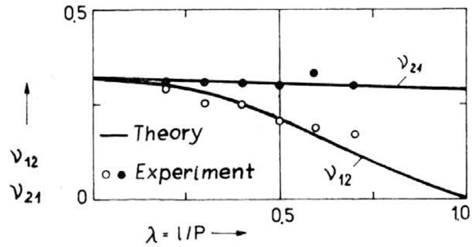

FIG. 5. Poisson ratio versus crack length for damaged solid with cracks arranged in pitch direction.

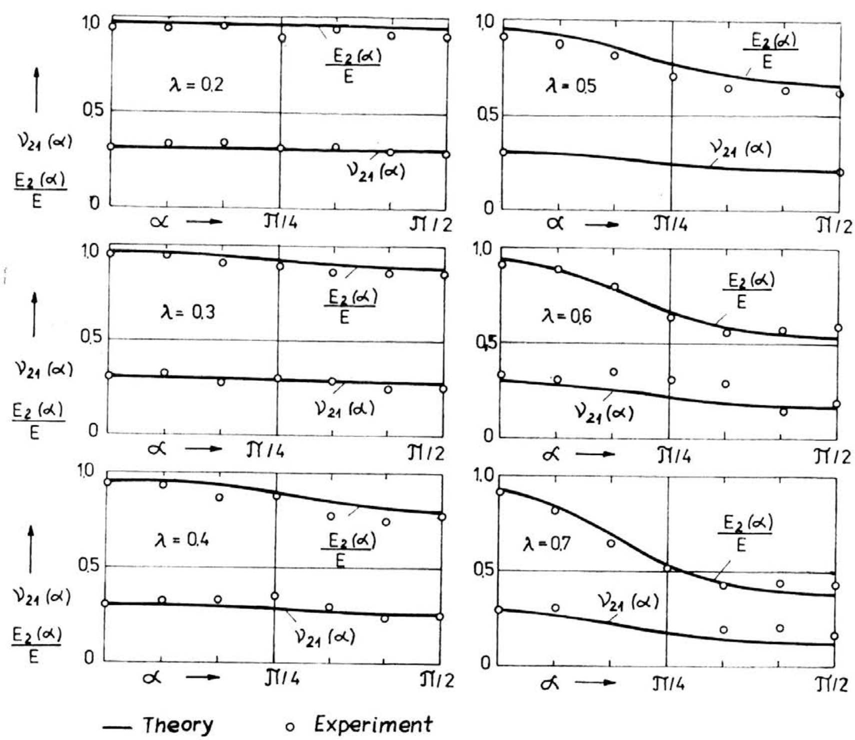

FIG. 6. Dependence of Young modulus and Poisson ratio on the loading orientation for damaged solid with cracks arranged in pitch direction.

of cracks. The width of rectangular openings t remained constant and equal to 1 mm. To study the directional properties of the material, the uniaxial loading applied to the specimens was inclined at angles $\alpha = 0, \pi/12, \pi/6, \pi/4, \pi/3, 5\pi/12$ and $\pi/2$ with respect to the cracks longitudinal axes parallel to the coordinate axis x_2, as shown in Fig. 3.

The longitudinal and transversal strains of the specimens were measured by means of electric strain gauges 50 mm long. This enabled determining four elastic constants E_1, E_2, G_{12} and ν_{21} or ν_{21}, so that the experiments furnished the results necessary to analyse the plane stress problems. Wide variety of loading orientations defined by the angle α made it possible to study the dependence of the effective Young modulus $E_2(\alpha)$ and Poisson ratio $\nu_{21}(\alpha)$ determined for the specimens loaded in various directions with respect to the symmetry axes of the crack pattern. All the experimental results concerning the elastic range are shown in Fig. 4, 5, 6, 7, 8 and 9. The specimens were loaded up to

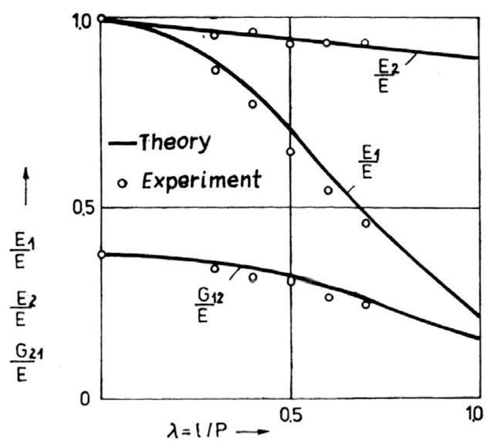

FIG. 7. Young moduli and shear modulus versus crack length for damaged solid with diagonal cracks arrangement.

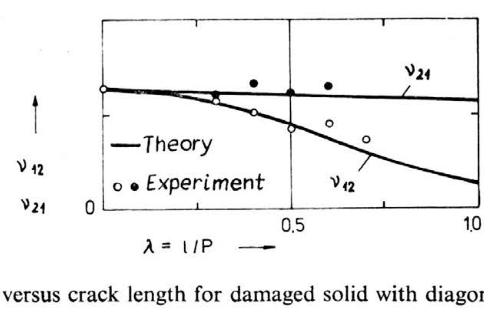

FIG. 8. Poisson ratio versus crack length for damaged solid with diagonal cracks arrangement.

rupture and the mechanical characteristics in the plastic range as well as at failure were studied. However, those problems are out of scope of this note and will not be presented here. The preliminary discussion of plastic behavior and failure modes of the models simulating the damaged materials are presented in [20, 21, 22].

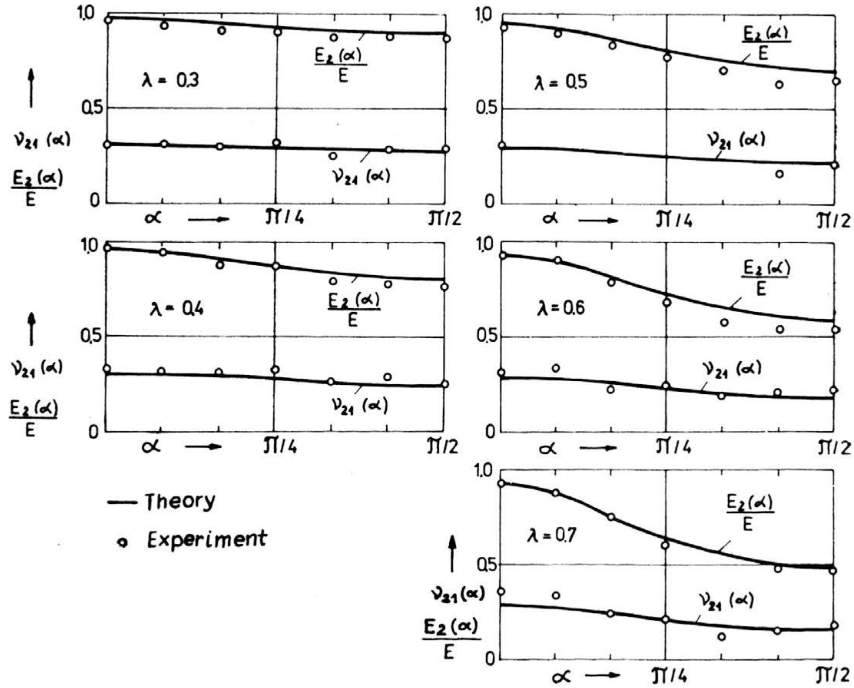

FIG. 9. Dependence of Young modulus and Poisson ratio on the loading orientation for damaged solid with diagonal cracks arrangement.

6. Discussion of results

In the case of the models simulating the damaged material used in the experiments, the principal values of the damage tensor calculated from (3.2) are

$$D_1 = \frac{\lambda}{1-\lambda} \quad \text{and} \quad D_2 = \frac{\tau}{1-\tau}, \tag{6.1}$$

where $\lambda = l/P$ and $\tau = 0.1$ for cracks arranged in the pitch direction, and $\lambda = l/\sqrt{2}P$ and $\tau = 0.1/\sqrt{2}$ for diagonal orientation of cracks. The equation (4.4) written in terms of the damage tensor components (6.1) made it possible to calculate all the elastic constants for an orthotropic equivalent material. Comparison of these values with those measured experimentally is shown in Fig. 4, 5, 7 and 8.

To study the directional properties of the damaged material, the dependence of the material constants on the loading orientation was also analysed. The comparison of the Young modulus $E_2(\alpha)$ and Poisson ratio $\nu_{21}(\alpha)$ measured experimentally for the specimens loaded at various directions (defined by angle α) with the corresponding values obtained from (4.4) suitably transformed according to the rotation of the coordinate system is shown in Fig. 6 and 9. To calculate the theoretical values of $E_2(\alpha)$ and $\nu_{21}(\alpha)$, well known expressions discussed for example by LEKHNICKIJ [23] were used.

Good agreement of the numerical results obtained theoretically and experimentally can be considered as an evidence that very simple constitutive equation of elasticity derived in this paper describes satisfactorily the mechanical behavior of the cracked solid. Moreover, it is seen from the results presented in Fig. 4, 5, 6, 7, 8 and 9 that the damage tensor defined by equations (3.1) and (3.2) fully determines not only the symmetry of the material, but also the variation of the material constants for cracked solid.

7. Conclusions

The constitutive equation of elasticity for damaged material with the regular crack array possessing three mutually perpendicular planes of symmetry was formulated employing Rivlin's and Ericksen's representation theory [16]. The effective material constants entering the constitutive equation were expressed in terms of the elastic constants of matrix material and the damage tensor components accounting for the current state of the cracked solid. Comparison of the theoretical results with the effective elastic constants measured experimentally on models simulating the damaged material confirmed the validity of the definition of the damage tensor proposed and the constitutive equation derived.

Acknowledgments

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