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Description of thermo-mechanical properties of viscoelastic irradiated materials

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A NEW concept of a description of coupling between the process of neutron transport and thermo-mechanical effects in viscoelastic irradiated materials is presented. A mixed material structure of a differential type is proposed with an irradiation dose as a certain kind of history. A thermo-radiation-mechanical coupling is discussed. For anisotropic materials new theorem is proved on the vanishing of a heat flux under zero temperature gradient and on the positive definitness of a heat conduction tensor. A quasi-linear theory is considered which is characterized by uncoupled equations of thermo-radiation problem and mechanical problem "embedded" in the known fields of temperature and neutron flux.

W pracy przedstawiono nową koncepcję opisu sprzężenia efektów transportu neutronów z efektami termo-mechanicznymi w napromieniowanych materiałach lepkosprężystych. Zaproponowano mieszaną strukturę materialną ciała typu różniczkowego z pewnym rodzajem historii, którą jest doza napromieniowania. Przedyskutowano sprzężenia termo-radiacyjno-mechaniczne. Udowodniono nowe twierdzenie dla materiałów anizotropowych o znikaniu strumienia ciepła przy zerowym gradiencie temperatury oraz o dodatniej określoności tensora przewodnictwa cieplnego. Wyprowadzono równania teorii quasi-liniowej. W efekcie uzyskano rozsprzężone równania problemu termoradiacyjnego oraz problemu mechanicznego "zanurzonego" w znanym polu temperatury i strumienia neutronów.

В работе представлена новая концепция описания сопряжения эффектов переноса нейтронов с термо-механическими эффектами в облучаемых вязкоупругих материалах. Предложена смешанная материальная структура тела дифференциального типа с некоторого рода историей, которой является доза облучения. Обсуждены термо-радиационно-механические сопряжения. Доказана новая теорема для анизотропных материалов об исчезновении потока тепла при нулевом градиенте температуры, а также о положительной определенности тензора теплопроводности. Выведены уравнения квазилинейной теории. В эффекте получены распряженные уравнения термо-радиационной задачи, а также механической задачи „погруженной" в известном поле температуры и потока нейтронов.

1. Introduction

THE paper is devoted to the development of a new concept of a phenomenological description of thermo-mechanical properties of non-fissionable crystalline materials irradiated by a neutron flux.

A similar problem has first been considered by PERZYNA who established a mathematical structure of thermodynamic theory of plasticity [12, 14] and viscoplasticity [15] for irradiated materials. Using an internal parameter description of dissipation mechanisms for time-dependent plasticity, constitutive equations were developed for irradiated materials in the viscoplastic range.

In the present paper, restricting considerations to the viscoelastic range, the method suggested by PERZYNA is extended by introducing a new description of irradiation effects and considering coupling between the process of neutron transport and thermo-mechanical effects.

In Sect. 2 assumptions are discussed under which general equations of the neutron transport can be reduced to the diffusion equation. A general theory of irradiated viscoelastic materials (Sect. 3) is developed in the framework of the thermodynamics of materials of differential type [6]. Known concepts of a state and a method of preparation [13] are also incorporated in the present approach. In the description of the method of preparation two sources of the internal dissipation are considered: one connected with viscous properties of the material and the other with internal structural changes generated by irradiation. It is further assumed that the latter source is governed by the irradiation dose in the particle X at time t.

In Sect. 4 a detailed discussion on the nature of thermo-radiation-mechanical coupling is presented. Possibilities of certain simplifications of the problem are indicated. Quasi-linear equations of thermo-radiation problem and mechanical problem "embedded" in the known fields of temperature and neutron flux are presented in Sect. 5, cf. PERZYNA [12]. For an inelastic anisotropic material new theorem is also proved that a heat flux vanishes when the temperature gradient is zero and that the heat conduction tensor is positive definite.

2. Basic assumptions

Consider a deformable body $\mathscr{B}$ composed of particles X subjected to neutron radiation. Assume that the body can conduct heat. The radiation effects in the material will be described using a theory of neutron transport. We shall show how this theory couples with a thermodynamic description of viscoelastic materials.

The motion of neutrons are governed by the Boltzmann transport equation (1)

$$(2.1)\qquad \frac{1}{v}\partial_t\hat{\Phi}(X,\Omega,E,t)+\Omega\cdot\nabla\hat{\Phi}(X,\Omega,E,t)+\mu_t(X,E)\hat{\Phi}(X,\Omega,E,t)$$
$$=\int_E\int_\Omega \mu_s(X,\varepsilon')\hat{\Phi}(X,\omega',\varepsilon',t)\,g_t(\omega',\varepsilon',\omega,\varepsilon)\,d\varepsilon'd\omega'+\hat{S}(X,\Omega,E,t),$$

where $\hat{\Phi}(X,\Omega,E,t)$ denotes an amount of neutrons moving with the velocity $V=v\Omega$ and carrying the energy E per unit area and per second in the particle X at time t, $\partial_t\hat{\Phi}$ is a spatial derivative of the function $\hat{\Phi}$ with respect to time and $\mu_t(X,E)$ denotes an energy dependent total macroscopic cross-section interaction coefficient of neutrons with a medium in the particle X

$$(2.2)\qquad \mu_t(X,E)=\mu_s(X,E)+\mu_a(X,E)=\mu_e(X,E)+\mu_i(X,E)+\mu_a(X,E),$$

where the macroscopic cross-section coefficients μ_s, μ_a, μ_e and μ_i describe respectively scattering, absorption, elastic scattering and inelastic scattering. The function $g_s(\Omega',E',\Omega,E)$ denotes a relative probability of a change in kinematic parameters of neutrons $(\Omega',E')\rightarrow(\Omega,E)$ due to scattering and $\hat{S}(X,\Omega,E,T)$ is an intensity of external and internal

(1) Derivation of this equation and extensive discussion of approximation methods can be found, e.g., in [17].

neutron sources whose velocity and energy in the particle X and time t is respectively $V = v\Omega$ and E.

A general boundary value problem of the neutron transport is rather complex and difficult to solve and usually various approximations are introduced to the equation (2.1)(²).

A thermo-radiation-mechanical coupling can be influenced significantly by the internal heat sources generated in the material by neutron radiation. The transport equation (2.1) can be conveniently approximated by a coupled system of neutron diffusion equations using a multigroup method. Such a possibility was discussed by A. B. CHILTON [5].

It is reasonable to study first a one-group approximation in which the neutron transport in the body $\mathscr{B}$ is characterized by a single diffusion equation.

As a starting point in the derivation of a neutron diffusion equation is the transport equation in the one-velocity theory(³)

$$(2.3)\qquad \frac{1}{v}\partial_t\tilde{\Phi}(X,\Omega,t)+\Omega\cdot\nabla\tilde{\Phi}(X,\Omega,t)+\mu_t(X)\tilde{\Phi}(X,\Omega,t) = \mu_e(X)\int_{\Omega}\tilde{\Phi}(X,\omega',t)g_e(\omega',\omega)d\omega'+\tilde{S}(X,\Omega,t),$$

where

$$(2.4)\qquad \tilde{S}(X,\Omega,t)=\mu_i(X)\int_{\Omega}\tilde{\Phi}(X,\omega',t)g_i(\omega,\omega')d\omega' + S^{\text{ext}}(X,\Omega,t)+[\nu\mu_f(X)/4\pi]\int_{\Omega}\Phi(X,\omega',\omega)d\omega',$$

$g_i(\Omega,\Omega')$ is a relative probability of a change in the direction of motion due to inelastic scattering, ν denotes a number of prompt neutrons generated in the fission process, μ_f is a macroscopic coefficient of fission in the particle X and $S^{\text{ext}}(X,\Omega,t)$ is an intensity of external neutron sources.

We shall consider non-fissionable materials in which effects of radioactivity caused by irradiation can be neglected. Under these assumptions the last term in the Eq. (2.4) becomes unimportant.

In a one-velocity theory the neutron diffusion equations can be obtained from transport equations as a first approximation in the spherical harmonic method [3 and 7]. It is assumed that the material is isotropic with respect to the properties of neutron transport and homogeneous in sufficiently large subregions.

The main idea of a spherical harmonic method consists in the expansion of the functions $\tilde{\Phi}$ and $\tilde{S}$ in the series of spherical functions and the function g_e in a series of Legendre polynomials.

It can be shown that a two-term expansion of the function has the form

$$(2.5)\qquad \tilde{\Phi}(X,\Omega,t)\approx\left(\frac{1}{4}\pi\right)[\Phi(X,t)+3J(X,t)\cdot\Omega].$$

The quantity $\Phi(X,t)$, called a neutron flux in the particle X at time t, is a measure of an amount of neutrons per unit area and per unit time.

(²) Discussion of various approximation methods of the transport equation can be found in the papers by H. SOODAK [17] and M. CLARK and K. F. HANSEN [7].

(³) Cf. H. SOODAK [17].

The vector quantity $J(X, t)$ called a neutron current in the particle X at time t defines an amount of neutrons per unit area of an oriented surface and per unit time.

According to (2.5), the first approximation of the Eq. (2.3) is the desired linearized diffusion equation (4) in the particle X

$$\frac{1}{v}\partial_t\Phi(t) = D\nabla^2\Phi(t)-\mu_a\Phi(t)+s(t), \tag{2.6}$$

where D stands for a coefficient of diffusion, $s(t)$ is an intensity of isotropic neutron source and quantities $J(t)$ and $\nabla\Phi(t)$ are related through the Fick's law

$$J(t) = -D\nabla\Phi(t). \tag{2.7}$$

In the theory of neutron diffusion the quantities $J(X, t)$ and $\Phi(X, t)$ fully describe the radiation field. The Eq. (2.7) can be regarded as the simplest example of a constitutive equation on the neutron flux in the classical diffusion theory. However, in the present thermo-radiation-mechanical problem in the material whose properties may depend on time, the constraint imposed by (2.7) appears to be too strong and the neutron absorption coefficient may be time variable. We shall thus assume the following form of the equation governing the neutron flux

$$\frac{1}{v}\partial_t\Phi(t) = -\mathrm{Div}J(t)-\mu_a(t)\Phi(t)+s(t), \tag{2.8}$$

where no particular relation is specified between the functions $J(t)$ and $\Phi(t)$. The operator Div is defined in the material coordinate system in the motion χ of the body $\mathscr{B}$.

3. General theory

The deformation, temperature and radiation fields in the material are completly specified by a right Cauchy-Green deformation tensor $C(X, t)$, temperature $\vartheta(X, t)$, neutron flux $\Phi(X, t)$ and gradients of temperature and neutron flux $\nabla\vartheta(X, t)$ and $\nabla\Phi(X, t)$, respectively, computed in the reference configuration in the particle X at time t

$$\Lambda(t) \equiv \{C(t), \vartheta(t), \nabla\vartheta(t), \Phi(t), \nabla\Phi(t)\}. \tag{3.1}$$

A local thermodynamic process $[t_0, t_k]$ in the particle X of the irradiated body $\mathscr{B}$ subjected to the motion χ is a family of functions defined for every $t \in [t_0, t_k] \subset R^1$:

$$\mathscr{P}_x \equiv \{\Lambda(t), \pi(t), r(t)\} \tag{3.2}$$

satisfying Cauchy equations with certain body forces $b(t)$

$$\mathrm{Div}[F(t)T(t)]+\varrho b(t) = \varrho\ddot{\chi}(t), \quad T(t) = T^T(t), \tag{3.3}$$

the first law of thermodynamics

$$\frac{1}{2}\mathrm{tr}[T(t)\dot{C}(t)]-\mathrm{Div}q(t)-\varrho[\dot{\psi}(t)+\vartheta(t)\dot{\eta}(t)+\dot{\vartheta}(t)\eta(t)]+\varrho r(t) = 0 \tag{3.4}$$

(4) Cf. K. M. Case and P. Z. Zweifel [3], p. 226.

the Clausius-Duhem inequality

$$(3.5) \qquad -\dot{\psi}(t)-\dot{\vartheta}(t)\eta(t)+\frac{1}{2\varrho}\operatorname{tr}[T(t)\dot{C}(t)]-\frac{1}{\varrho\vartheta(t)}\,q(t)\cdot\nabla\vartheta(t)\geqslant 0$$

and the balance equation for the neutron flux $\Phi(t)$ with certain intensity of the neutron source $s(t)$ in the particle X

$$(3.6) \qquad \frac{1}{v}\,\partial_t\Phi(t) = -\operatorname{Div} J(t)-\mu_a(t)\Phi(t)+s(t)$$

at each instant of time $t\in[t_0, t_k]$ provided the derivatives $\dot{\psi}(t)$, $\dot{C}(t)$, $\dot{\vartheta}(t)$, $\dot{\eta}(t)$, $\partial_t\Phi$ exist. A dot denotes here material differentiation with respect to time, $F(t)$ is a deformation gradient in the particle X at time t and ϱ denotes mass density in the reference configuration.

The set of functions

$$(3.7) \qquad \pi(t) \equiv \{\psi(t), T(t), \eta(t), q(t), J(t)\}$$

represents respectively the specific free energy per unit mass $\psi(t)$, second Piola-Kirchhoff stress tensor $T(t)$, specific enetropy per unit mass $\eta(t)$, heat flux vector per unit surface in the reference configuration $q(t)$ and neutron current vector $J(t)$ per unit surface in the reference configuration in the particle X at time t. The intensity of internal heat sources per unit mass generated by neutron irradiation is denoted by $r(t)$.

A thermodynamic process in the particle X of the body $\mathscr{B}$ is described by the local configuration $\Lambda(t)$ of the particle X, the dependent variables $\pi(t)$ related by $\Lambda(t)$ through constitutive equations and the independently determined quantity $r(t)$.

It should be noted that introducing a new vector field of a neutron current together with its balance equation to the definition of a thermodynamic process does not lead ot a full description of a thermo-radiation coupling. However, the simplified theory might prove useful in solving certain practical problems. Local equations describing fully a coupled thermodynamic-radiation process can only be derived by considering a global process.

A set of values which the function $\mathscr{P}_x$ takes for given values of $t\in[t_0, t_k]$ is called a thermo-radiation-mechanical state of a particle X at time t.

The material structure of the body $\mathscr{B}$ in the particle X is described by the following constitutive equation

$$(3.8) \qquad \pi(t) = \mathscr{F}(g(t)),$$

where

$$(3.9) \qquad g(t) \equiv \{\Lambda(t), \dot{C}(t), \dot{\vartheta}(t), \varphi(t)\}$$

is a value of the function describing a thermo-radiation-mechanical state of the particle X at time t. This function is determined by a local configuration $\Lambda(t)$ and a method of preparation(5) ($\dot{C}(t)$, $\dot{\vartheta}(t)$, $\varphi(t)$) in the particle X at ime t. A dot denotes a material differentiation with respect to time, $\varphi(t)$ is a value of the irradiation dose in the particle X at time t defined for an integrable function $\Phi(\tau)$, $\tau\in[t_0, t]$ belonging to the process $\mathscr{P}_x$

$$(3.10) \qquad \varphi(t) = \int_{t_0}^{t}\Phi(\tau)d\tau, \quad t\in[t_0, t_k].$$

(5) Cf. P. PERZYNA [13].

The operation

$$\mathscr{F} \equiv \{\hat{\Psi}, \hat{N}, \hat{T}, \hat{Q}, \hat{J}\} \tag{3.11}$$

represents the constitutive functions for the free energy $\hat{\Psi}$, entropy $\hat{N}$, stress $\hat{T}$, heat flux $\hat{Q}$ and neutron current $\hat{J}$.

We assume hence and after that the constitutive functions are bijections defined on the region $\mathscr{D}$ where $\mathscr{D}$ is a collection of all $\{C(t), \vartheta(t), \nabla\vartheta(t), \Phi(t), \nabla\Phi(t), \dot{C}(t), \dot{\vartheta}(t), \varphi(t)\}$. The second rank tensor $C(t) = F^T(t)F(t)$ is symmetric, the deformation gradient $F(t)$ being subjected to the condition $0 < \det F(t) < \infty$. The quantities $\nabla\vartheta(t)$ and $\nabla\Phi(t)$ represent vectors, whereas $\vartheta(t)$, $\Phi(t)$, $\dot{\vartheta}(t)$ and $\varphi(t)$ are real numbers satisfying $\vartheta(t) > 0$, $\Phi(t) > 0$, $\varphi(t) \geqslant 0$ for every $t \in [t_0, t_k]$.

We also assume that the function $\hat{Q}$ is piecewise of the class C^1 on $\mathscr{D}$ and the functions $\hat{\Psi}$, $\hat{N}$ and $\hat{T}$ are piecewise of the class C^2 on $\mathscr{D}$.

The Eqs. (3.8) define the material of differential type with irradiation effects. The internal dissipation is described by specifying the functions $\dot{C}(t)$, $\dot{\vartheta}(t)$ and $\varphi(t)$, according to the discussion presented in the introduction. While the proposed description of radiation effects through the neutron dose and flux does not provide an insight into the mechanisms responsible for dissipation, it allows to make use of the available experimental data(6) in which the change in mechanical properties of the material are usually measured as a function of the neutron dose $\varphi(t)$ or flux $\Phi(t)$.

The Clausius-Duhem inequality (3.5) imposes certain restriction on the form of constitutive equations (3.8). It can be shown that for arbitrary values of $C(t), \vartheta(t), \nabla\vartheta(t)$, $\Phi(t), \nabla\Phi(t), \dot{C}(t)$, and $\dot{\vartheta}(t)$ in the generic point of the body $X \in \mathscr{B}$ in the interval $t \in [t_0, t_k]$ one can construct functions χ, ϑ, φ such that an admissible thermodynamic process will exist, provided certain smoothness conditions are satisfied. By the admissible process it is understood a local process $\mathscr{P}_x$ consistent with the constitutive assumption (3.8).

From the Clausius-Duhem inequality and the smoothness requirement for the function ψ it follows that(7):

$$\begin{gathered}\partial_{\nabla\vartheta}\hat{\Psi} = 0, \quad \partial_{\varphi}\hat{\Psi} = 0, \quad \partial_{\dot{\vartheta}}\hat{\Psi} = 0, \quad \partial_{\nabla\Phi}\hat{\Psi} = 0, \quad \partial_{\dot{C}}\hat{\Psi} = 0,\\ \frac{1}{2}\operatorname{tr}\{[T(t)-2\varrho\partial_{C(t)}\hat{\Psi}(\cdot)]\dot{C}(t) - [\eta(t)+\partial_{\vartheta(t)}\hat{\Psi}(\cdot)]\dot{\vartheta}(t)\\ -\partial_{\varphi(t)}\hat{\Psi}(\cdot)\Phi(t) - \frac{1}{\varrho\vartheta(t)}q(t)\nabla\vartheta(t) \geqslant 0.\end{gathered} \tag{3.12}$$

The thermo-radiation-mechanical state in the particle X at time t is said to be in equilibrium if it is described by the value of the following functions in $t \in [t_0, t_k]$:

$$g^{\#}(t) \equiv \{C(t), \vartheta(t), 0(t), 0(t), 0(t), 0(t), \varphi(t)\}. \tag{3.13}$$

The thermo-radiation-mechanical state in the particle X at time t under a fixed reference configuration of the body $\mathscr{B}$ is called a deformationless equilibrium state if it is described by the values of the following functions in $t \in [t_0, t_k]$:

$$g_0^{\#}(t) \equiv \{1(t), \vartheta(t), 0(t), 0(t), 0(t), 0(t), \varphi(t)\}. \tag{3.14}$$

(6) A different concept of a description of irradiation effects through a certain internal parameter with an appropriate evolution equation was presented by PERZYNA in Refs. [12, 14 and 15]. This parameter has been identified with a defect concentration density.

(7) B. D. COLEMAN and V. J. MIZEL [6].

Using known arguments([8]) it can be proved that for an equilibrium state we get the relations

$$T^{\circ}(t) = 2\varrho\partial_{C(t)}\hat{\Psi}(g^{\#}(t)), \quad \eta^{\circ}(t) = -\partial_{\vartheta(t)}\hat{\Psi}(g^{\#}(t)), \tag{3.15}$$

where

$$T^{\circ}(t) \equiv \hat{T}(g^{\#}(t)), \quad \eta^{\circ}(t) \equiv \hat{N}(g^{\#}(t)) \tag{3.16}$$

denote respectively stress and equilibrium entropy of the irradiation body.

A difference in the values

$$\begin{aligned} \hat{T}^{d}(g(t)) &\equiv \hat{T}(g(t)) - T^{\circ}(t), \\ \hat{N}^{d}(g(t)) &\equiv \hat{N}(g(t)) - \eta^{\circ}(t) \end{aligned} \tag{3.17}$$

will be called a dissipative part of stress and entropy, respectively. It represents the change in the stress and entropy due to the temperature gradient, neutron flux, gradient of the neutron flux, strain and temperature rate and irradiation dose. The inequality $(3.12)_2$ can be written

$$\frac{1}{2\varrho}\mathrm{tr}[\hat{T}^{d}(g(t))\,\dot{C}(t)] - \hat{N}^{d}(g(t))\,\dot{\vartheta}(t) - \partial_{\varphi(t)}\hat{\Psi}(g^{\#}(t))\,\Phi(t) - \frac{1}{\varrho\vartheta(t)}\hat{Q}(g(t))\cdot\nabla\vartheta(t) \geqslant 0. \tag{3.18}$$

The internal dissipation of the material in the particle X at time t is determined by

$$\hat{\imath}(g(t)) = -\frac{1}{\vartheta(t)}\left\{-\frac{1}{2\varrho}\mathrm{tr}\,[\hat{T}^{d}(g(t))\dot{C}(t)] + \hat{N}^{d}(g(t)\dot{\vartheta}(t) + \partial^{1}_{\varphi(t)}\hat{\Psi}(g^{\#}(t))\,\Phi(t)\right\}. \tag{3.19}$$

The first two terms in the expression (3.19) are responsible for the dissipation caused by viscous properties of the material, whereas the last term describes the dissipation caused by internal structural changes due to irradiation.

The constitutive equations (3.8) take now the form

$$\begin{aligned} \psi(t) &= \hat{\Psi}(C(t), \vartheta(t), \varphi(t)), \\ \eta(t) &= \eta^{\circ}(t) + \hat{N}^{d}(g(t)), \\ T(t) &= T^{\circ}(t) + \hat{T}^{d}(g(t)), \\ q(t) &= \hat{Q}(g(t)), \qquad J(t) = \hat{J}(g(t)). \end{aligned} \tag{3.20}$$

The assumption about the isotropy of the material with respect to neutron diffusion implies that the functions $\hat{N}^{d}$, $\hat{T}^{d}$, $\hat{Q}$ are isotropic with respect to $\nabla\Phi(t)$, whereas $\hat{J}$ is an isotropic function with respect to all variables. From the definition of the isotropic function it follows that

$$\begin{aligned} &\partial_{\nabla\varphi(t)}\hat{N}^{d}(C(t), \vartheta(t), \nabla\vartheta(t), \Phi(t)\,0, \dot{C}(t), \dot{\vartheta}(t), \varphi(t) = 0, \\ &\partial_{\nabla\varphi(t)}\hat{T}^{d}(C(t), \vartheta(t), \nabla\vartheta(t), \Phi(t), 0, \dot{C}(t), \dot{\vartheta}(t), \varphi(t)) = 0. \end{aligned} \tag{3.21}$$

([8]) Cf. B. D. COLEMAN and V. J. MIZEL [6].

According to the theorem on the representation of an isotropic vector function we have [9]

$$J(t) = [\alpha_1 1+\alpha_2 C+\alpha_3 C^2+\alpha_4 \dot{C}+\alpha_5 \dot{C}^2+\alpha_6 C\dot{C}+\alpha_7 \dot{C}C]\nabla\vartheta(t) + [\beta_1 1+\beta_2 C+\beta_3 C^2+\beta_4 \dot{C}+\beta_5 \dot{C}^2+\beta_6 C\dot{C}+\beta_7 \dot{C}C]\nabla\Phi(t), \tag{3.22}$$

where $\alpha_1, \ldots, \alpha_7$; $\beta_1, \ldots, \beta_7$ are scalar functions of a complete minimal set of invariants of elements from the domain of the function $\hat{J}$. It is seen that the functions $\hat{Q}$ and $\hat{J}$ are coupled through a dependence on the temperature gradient $\nabla\vartheta(t)$ and the neutron flux gradient $\nabla\Phi(t)$. Thus, the process of heat conduction and neutron diffusion are interrelated.

4. Discussion of a thermo-radiation-mechanical coupling

A full system of governing equation for a viscoelastic body subjected to irradiation consists of constitutive equations (3.20), expersion for the internal dissipation (3.19), balance equations for energy

$$\varrho\vartheta(t)\dot{\eta}(t) = -\mathrm{Div}\,q(t)+\varrho r(t)+\varrho\vartheta(t)\hat{i}(t), \tag{4.1}$$

for neutron flux

$$\frac{1}{v}\partial_t\Phi(t) = -\mathrm{Div}\,\hat{J}(t)-\mu_a(t)\Phi(t)+s(t), \tag{4.2}$$

equation of motion and appropriate initial and boundary conditions.

Substituting into (4.1) the constitutive equation for the entropy $(3.20)_2$ and differentiating with respect to time we obtain

$$\begin{aligned}\vartheta(t)\partial_{\dot{\vartheta}(t)}\hat{N}(g(t))\dot{\vartheta}(t)+\vartheta(t)\partial_{\dot{\vartheta}(t)}\hat{N}^d(g(t))\ddot{\vartheta}(t)+\vartheta(t)\,\partial_{\nabla\vartheta(t)}\hat{N}^d(g(t))\overline{\nabla\vartheta(t)}^{\cdot} \\ +\frac{1}{\varrho}\mathrm{Div}\,q(t) = r(t)+\vartheta(t)\hat{i}(t)-\vartheta(t)\mathrm{tr}[\partial_{C(t)}\hat{N}(g(t))\dot{C}(t)] \\ -\vartheta(t)\partial_{\varphi(t)}\hat{N}(g(t))\Phi(t)-\vartheta(t)\partial_{\dot{C}(t)}\hat{N}^d(g(t))\ddot{C}(t)-\vartheta(t)\partial_{\Phi(t)}\hat{N}^d(g(t))\dot{\Phi}(t) \\ -\vartheta(t)\partial_{\nabla\Phi(t)}\hat{N}^d(g(t))\overline{\nabla\Phi(t)}^{\cdot}.\end{aligned} \tag{4.3}$$

The above equation describes the change in the temperature and heat flux fields due to the presence of internal heat sources generated by irradiation, internal dissipation and appropriate thermo-radiation-mechanical couplings.

The following terms can be identified in the Eq. (4.3):

i) the term $\vartheta(t)\mathrm{tr}[\partial_{C(t)}\hat{N}(g(t))\dot{C}(t)]$ is responsible for heat effects generated by a thermo-mechanical coupling;

ii) the term $\vartheta(t)\partial_{\varphi(t)}\hat{N}(g(t))\Phi(t)$ is responsible for heat effects generated by a thermo-radiation coupling. These effects are due to the entropy change caused by the variable concentration of defects which in turn are generated by the irradiation process;

(9) Cf. C.-C. WANG [22]. Assuming that the function $\hat{J}$ can be expressed in the form of a polynomial, we could give its representation in terms of the integrity basis. However, such a representation would be very complicated, since this is not a minimal set (Cf. A. J. M. SPENCER [18]).

iii) the term $\vartheta(t)[\partial_{\dot{C}(t)}\hat{N}^d(g(t))\ddot{C}(t)+\partial_{\Phi(t)}\hat{N}^d(g(t))\dot{\Phi}(t)]$ is responsible for heat effects generated in the material as a result of a coupling between mechanisms describing viscous properties with a temperature field;

iv) the term $\vartheta(t)\,\partial_{\nabla\Phi(t)}\hat{N}^d(g(t))\overline{\nabla\dot{\Phi}(t)}$ is responsible for heat effects generated by a thermo-diffusion coupling.

Viscous effects characterize rate-type quantities $\dot{C}(t)$, $\dot{\vartheta}(t)$ and $\Phi(t)$.

The intensity of a neutron flux $\Phi(t)$ controls the rate of change of the defect concentration which in turn can influence the process of neutron transport and thermo-mechanical properties of the material. Such an interrelation was found experimentally by T. H. BLEWITT, R. R. COLTMAN, R. E. JAMISON and J. K. REDMAN in Ref. [2] where irradiated copper monocrystals were subjected to the isothermal annealing in the temperature range 305°C –385°C. With increasing temperature the rate of decay of the yield stress was observed to increase which is explained by the rise in the rate of change of defect concentration.

We restrict our considerations to the problems in which the influence of the rate of change of defects concentration is of minor significance and thus will be disregarded in the subsequent analysis.

In a general case the Eqs. (3.20) describe also coupling of a strain field and an inhomogeneous temperature field with an inhomogeneous neutron flux field. The problem of neutron diffusion is formally similar to the classical problem of diffusion of gases in solids, where all couplings mentioned above can exist and in special circumstances become significant. For our purposes however these effects are disregarded. This assumption is justified, since the character of interaction between the neutrons and medium is here different. Thus, the coefficient of neutron diffusion is taken to be the function of temperature $\vartheta(t)$ and irradiation dose $\varphi(t)$.

With the introduced assumptions the system of the Eqs. $(3.20)_{2-4}$ and (4.2), (4.3) is reduced to the form

$$
\begin{aligned}
&\eta(t) = \eta^\circ(t)+\hat{N}^d(C(t),\vartheta(t),\nabla\vartheta(t),\dot{C}(t),\dot{\vartheta}(t),\varphi(t)),\\
&T(t) = T^\circ(t)+\hat{T}^d(C(t),\vartheta(t),\nabla\vartheta(t),\dot{C}(t),\dot{\vartheta}(t),\varphi(t)),\\
&q(t) = \hat{Q}(C(t),\vartheta(t),\nabla\vartheta(t),\dot{C}(t),\dot{\vartheta}(t),\varphi(t)),\\
&\vartheta(t)\,\partial_{\vartheta(t)}\hat{N}(g(t))\,\dot{\vartheta}(t)+\vartheta(t)\,\partial_{\dot{\vartheta}(t)}\hat{N}^d(g(t)\ddot{\vartheta}(t)+\vartheta(t)\,\partial_{\nabla\vartheta(t)}\hat{N}^d(g(t)\,\overline{\nabla\dot{\vartheta}(t)}\\
&\quad+\frac{1}{\varrho}\operatorname{Div}q(t) = r(t)+\vartheta(t)\,\hat{i}(t)-\vartheta(t)\,tr[\partial_{C(t)}\hat{N}(g(t))\,\dot{C}(t)]\\
&\qquad-\vartheta(t)\,\partial_{\varphi(t)}\hat{N}(g(t))\Phi(t)-\vartheta(t)\,\partial_{\dot{C}(t)}\hat{N}^d(g(t))\,\ddot{C}(t),\\
&\frac{1}{v}\,\partial_t\Phi(t) = \operatorname{Div}[D_0(\vartheta(t);\varphi(t))\,\nabla\Phi(t)]-\mu_a(t)\Phi(t)+s(t).
\end{aligned}
\tag{4.4}
$$

The Eq. $(4.4)_4$ takes into account effects of thermo-mechanical and thermo-radiation coupling and effects of thermal effects of internal dissipation. It is of interest to examine more closely the effect of irradiation on the change in the temperature field. The major part of the kinetic energy of the neutron flux which the material can absorb is converted into heat, leading to the creation of internal heat sources generated by thei rradiation. A detailed discussion of this problem was given by A. B. CHILTON [5] who determined the intensity of heat sources using methods of the physics of interactions between the radiation and medium and the neutron diffusion theory. The remaining part of the kinetic energy absorbed by the material goes into internal structural changes caused by irradiation. The two terms in the Eq. $(4.4)_4$ responsible for the above mentioned effects are respectively $\vartheta(t)\,\partial_{\varphi(t)}\hat{N}(g(t))\Phi(t)$ and $\vartheta(t)\,\partial_{\varphi(t)}\hat{\Psi}(g^{\#}(t))\Phi(t)$. It is convenient in practical calculations to assume that the whole kinetic energy of the neutron flux is converted into heat. Then, the Eq. $(4.4)_4$ is approximated by

$$(4.5)\qquad \vartheta(t)\,\partial_{\vartheta(t)}\hat{N}(g(t))\dot{\vartheta}(t)+\vartheta(t)\,\partial_{\dot{\vartheta}(t)}\hat{N}^d(g(t))\ddot{\vartheta}(t)+\vartheta(t)\,\partial_{\nabla\vartheta(t)}\hat{N}^d(g(t))\,\overline{\nabla\dot{\vartheta}(t)}$$

$$+\frac{1}{\varrho}\,\mathrm{Div}\,q(t) = r(t)+\frac{1}{2\varrho}\mathrm{tr}[\hat{T}^d(g(t))\,\dot{C}(t)]-\hat{N}^d(g(t))\,\dot{\vartheta}(t)$$

$$-\vartheta(t)\,\mathrm{tr}[\partial_{C(t)}\hat{N}(g(t))\,\dot{C}(t)]-\vartheta(t)\,\partial_{\dot{C}(t)}(\hat{N}^d(g(t))\,\ddot{C}(t).$$

From (4.5) one can derive a quasi-linear hiperbolic heat conduction equation which leads to a finite velocity of propagation of thermal disturbances. A similar type of equation for a rigid heat conductor in the absence of radiation was obtained D. B. BOGY and P. M. NAGHDI [1]. Assuming that the material is isotropic and the entropy and heat flux depend linearly on the rate of temperature, conditions were examined in [1] under which the velocity of thermal wave is finite.

Moreover, it was proved that both in the linear theory, in which the rate of temperature and temperature gradient are infinitesimal and in the case when an inhomogeneous temperature field with a finite value of the gradient is superposed on the infinitesimal changes of the rate of temperature, the dissipation inequality excludes the possibility of occurence of a finite velocity of propagation of thermal disturbances.

A similar results was reported in Ref. [10] by I. MÜLLER who considered a linearized form of the heat conduction equation with coefficients computed at the equilibrium state. Müller proved that when the coldness is equal to reciprocity of the absolute temperature, the heat conduction equation ceases to be of the hyperbolic type.

The Eqs. $(4.4)_{1-3}$, $(4.4)_5$ and (4.5) together with equation of motion and appropriate boundary and initial conditions furnish a full set of equations describing a coupled thermo-radiation-mechanical problem. In particular coupling exists between mechanical and thermal effects and between thermal and radiation effects. In spite of the already introduced simplifications the so formulated problem is extremely complicated. Further approximations may be obtained through a linearization of the constitutive equations and restricting consideration to small deformations, rate of deformations, temperature gradients and rate of temperature and through decoupling of the above system into the equations governing a thermo-radiation problem and equations governing the mechanical problem with thermal and radiation effects.

5. Quasi-linear theory

In many problems of modern technology structural elements are often exposed to the action of aggressive surroundings and to the influence of high temperature. For example in the problems of reactor technology it was found that the properties of many structural materials depend strongly on the temperature and irradiation dose.

A description of irradiated materials in a frame-work of a linear theory, where the flux and dose increments should be assumed infinitesimal, is not consistent with the real behaviour of materials in a reactor under the action of neutron irradiation field. The reason of this is that the mechanisms of internal structural changes accompaning weak irradiation are different than those which would be induced in a material subjected to strong reactor irradiation. This fact is fully confirmed by the experimental examinations of the effects of irradiation(10).

In view of the above arguments we shall introduce approximations of the general constitutive relations by assuming the dependence of the material functions on the temperature and the irradiation dose and retaining finite increments. Such a theory will be called the quasi-linear theory.

In a quasi-linear theory the rate of temperature $\dot{\vartheta}(t)$ will be infinitesimal. We assume that the stress, entropy and heat flux do not depend on the rate of change of temperature field.

The quasi-linear constitutive relations and solutions of boundary-value problems with the dependence of the material functions on temperature were considered by many authors. As an example, the papers by R. TROOSTEL [20, 21] and the monography by KOVALENKO [8](11) may be cited.

The formulation of the linear theory is based on a concept of the initial state of equilibrium and the properly defined δ-close state. In the quasi-linear theory we shall approximate the constitutive functions in an undeformed state of equilibrium.

The thermo-radiation-mechanical state in the particle X described by the value of the function $\hat{g}(t)$ in $t \in [t_0, t_k]$ is thermo-mechanically δ-close to the undeformed equilibrium state described by the value of the function $g_0^{\#}(t)$ in t if the following conditions are satisfied

$$|C(t)-1(t)|_6 < \delta, \quad |\nabla\vartheta(t)|_3 < \delta, \quad |\dot{C}(t)|_6 < \delta, \quad \delta > 0, \tag{5.1}$$

where

$$\hat{g}(t) \equiv \{C(t), \vartheta(t), \nabla\vartheta(t), \Phi(t), \nabla\Phi(t), \dot{C}(t), \varphi(t)\}.$$

The norms $|\cdot|_6$, $|\cdot|_3$ are the natural norms reduced to the dimensionless form.

Note that the quantities $\Phi(t)$, $\nabla\Phi(t)$ may be arbitrary, whereas the functions $\vartheta(t)$ and $\varphi(t)$ are the same as in the actual state of equilibrium.

We shall introduce approximations only for the stress and heat flux functions. The entropy equation for finite increments of temperature and dose is retained in the non-linear form.

Let us examine what restrictions are imposed on the general form of the constitutive functions $(4.4)_{1-3}$ by the assumption of δ-small temperature gradients and dissipation inequality (3.18).

(10) Cf. for example J. SILCOX and P. B. HIRSCH 16], M. J. MAKIN, A. D. WHAPHAM and F. J. MINTER[9].

(11) NOWACKI [11] gave a survey of results concerning the thermal stresses in an elastic isotropic body with nonhomogeneity which is induced by the dependence of the material function on temperature.

Consider a thermo-radiation-mechanical state described by the value of function $\hat{g}(t)$, for $\nabla\vartheta(t) = 0$ in the particle X and at time t:

$$\hat{g}_h(t) = \{C(t), \vartheta(t), 0(t), \Phi(t), \nabla\Phi(t), \dot{C}(t), \varphi(t)\}. \tag{5.2}$$

Using the assumption on the smoothness of the constitutive functions we may formulate the following

LEMMA. *If we assume that the temperature gradients are δ-small and we fix the quantities* $(C(t), \vartheta(t), \Phi(t), \dot{C}(t), \varphi(t))$, *then the function of internal dissipation* $\hat{i}(t)$ *in the particle X and at time t should have the following properties*:

$$\partial_{\nabla\vartheta(t)}\tilde{i}(0) = 0, \quad \partial^2_{\nabla\vartheta(t)}\tilde{i}(0) \geqslant 0, \tag{5.3}$$

where

$$\tilde{i}(\cdot) \equiv \hat{i}(C(t), \vartheta(t), \cdot, \Phi(t), \dot{C}(t), \varphi(t)). \tag{5.4}$$

Proof. The inequality of a general dissipation (3.18) may be written now in the form:

$$\tilde{i}(\nabla\vartheta(t)) - \frac{1}{\varrho\vartheta^2(t)} q(t)\cdot\nabla\vartheta(t) \geqslant 0. \tag{5.5}$$

Using the continuity of the function $\tilde{i}(\cdot)$ the inequality (5.5) and a theorem on the local conservation of sign by a continuous function it follows that there exist such δ-small values of temperature gradients $\nabla\vartheta(t)$ that

$$\tilde{i}(\nabla\vartheta(t)) \geqslant 0 \tag{5.6}$$

holds.

Let us expand now the function $\tilde{i}(\cdot)$ in a Taylor series at the point $\nabla\vartheta(t) = 0$ and retain second order terms

$$\tilde{i}(\nabla\vartheta(t)) = \tilde{i}(0) + \partial_{\nabla\vartheta(t)}\tilde{i}(0)\nabla\vartheta(t) + \nabla\vartheta(t)\,\partial^2_{\nabla\vartheta(t)}\tilde{i}(0)\nabla\vartheta(t) + o(\delta^2) \tag{5.7}$$

the inequality (5.6) may be rewritten now in the form:

$$\tilde{i}(0) + \partial_{\nabla\vartheta(t)}\tilde{i}(0)\nabla\vartheta(t) + \nabla\vartheta(t)\,\partial^2_{\nabla\vartheta(t)}\tilde{i}(0)\nabla\vartheta(t) + o(|\nabla\vartheta(t)|^2_3) \geqslant 0. \tag{5.8}$$

Thus the relations (5.3) follow from the requirement that the Eq. (5.8) holds for arbitrary δ-small temperature gradients $\nabla\vartheta(t)$.

Now we can prove the following

THEOREM([15]). *If we assume that the temperature gradients* $\nabla\vartheta(t)$ *are δ-small and we fix the quantities* $C(t), \vartheta(t), \dot{C}(t), \varphi(t)$ *then the constitutive function for heat flux in the particle X at time t has the following properties*:

$$\hat{Q}(C(t), \vartheta(t), 0(t), \dot{C}(t), \varphi(t)) = 0 \tag{5.9}$$

and the tensor of heat conduction

$$K(C(t), \vartheta(t), \dot{C}(t), 0(t), \varphi(t)) \equiv -\partial_{\nabla\vartheta(t)}\hat{Q}(C(t), \vartheta(t), 0(t), \dot{C}(t), \varphi(t)) \tag{5.10}$$

([15]) This is a generalization of a theorem given by D. E. CARLSON [4] for anisotropic thermoelastic materials. Similar theorems have been presented earlier for various isotropic materials and may be found in references cited by CARLSON. A lack of assumption on the isotropy of material has led to the conclusion that the heat flux is nonzero in a case of uniform temperature field, and in the case of a linear approximation direct dependence exists not only on the temperature gradient but also on other state variables. These facts cannot be justified physically.

is positive definite

$$K\big(C(t), \vartheta(t), \dot{C}(t), \varphi(t)\big) \geqslant 0. \tag{5.11}$$

Proof. Let us expand $\tilde{Q}(\cdot) \equiv \hat{Q}(C(t), \vartheta(t), \cdot, \dot{C}(t), \varphi(t))$ at fixed $(C(t), \vartheta(t), \dot{C}(t), \varphi(t))$ in Taylor series for $\nabla\vartheta(t) = 0$. Let us restrict our considerations to the linear element

$$\tilde{Q}\big(\nabla\vartheta(t)\big) = \tilde{Q}(0) - K(\cdot)\nabla\vartheta(t) + o(\delta). \tag{5.12}$$

Substituting (5.7) and (5.12) into dissipation inequality (5.5) one obtains

$$\tilde{i}(0) + \partial_{\nabla\vartheta(t)}\tilde{i}(0)\nabla\vartheta\nabla(t) + \nabla\vartheta(t)\,\partial^2_{\nabla\vartheta(t)}\nabla\vartheta(t) - \frac{1}{\varrho\vartheta^2(t)}\tilde{Q}(0)\nabla\vartheta(t) + \frac{1}{\varrho\vartheta^2(t)}\nabla\vartheta(t)K(\cdot)\nabla\vartheta(t) + o(\delta^2) \geqslant 0. \tag{5.13}$$

From the requirement that the above inequality hold for the arbitrary δ-small temperature gradients $\nabla\vartheta(t)$ and from the just proved Lemma the relations (5.9)–(5.11) follow, which complete the proof.

COROLLARY 1. *The functions $\hat{T}^d$ and $\hat{N}^d$, determined in a small neighbourhood of the homothermal state, are independent of the temperature gradient*:

$$\begin{aligned} &\partial_{\nabla\vartheta(t)}\hat{T}^d\big(C(t), \vartheta(t), 0(t), \dot{C}(t), \varphi(t)\big) = 0, \\ &\partial_{\nabla\vartheta(t)}\hat{N}^d\big(C(t); \vartheta(t); 0(t); \dot{C}(t); \varphi(t)\big) = 0. \end{aligned} \tag{5.14}$$

COROLLARY 2. *The heat flux function $\hat{Q}$ determined in a small neighbourhood of the homothermal state depends solely on the temperature gradient*

$$\partial_{\bar{g}(t)}\hat{Q}\big(\bar{g}(t), 0(t)\big) = 0, \tag{5.15}$$

where

$$\bar{g}(t) \equiv \big(C(t), \vartheta(t), \dot{C}(t), \varphi(t)\big). \tag{5.16}$$

In view of the Eqs. $(5.14)_1$ the constitutive equations $(4.4)_1$ and $(4.4)_2$ for δ-small temperature gradients take the following form:

$$\begin{aligned} \eta(t) &= \eta^\circ(t) + \hat{N}^d\big(C(t), \vartheta(t), \dot{C}(t), \varphi(t)\big), \\ T(t) &= T^\circ(t) + \hat{T}^d\big(C(t), \vartheta(t), \dot{C}(t), \varphi(t)\big). \end{aligned} \tag{5.17}$$

Using the assumption on the differentiability of the constitutive functions $\hat{T}$ and $\hat{Q}$, the definition of the dissipative part of stress and the Corollary 2, we may write the following relations for the given deformationless state of equilibrium and for corresponding thermo-mechanically δ-close state

$$\begin{aligned} T(t) &= \hat{T}^\circ\big(g_0^\#(t)\big) + \partial_{C(t)}\hat{T}^\circ\big(g_0^\#(t)\big)[C(t) - 1(t)] + \partial_{\dot{C}(t)}\hat{T}^d\big(g_0^\#(t)\big)[\dot{C}(t)] + o(\delta), \\ q(t) &= \partial_{\nabla\vartheta(t)}\hat{Q}\big(g_0^\#(t)\big)[\nabla\vartheta(t)] + o(\delta). \end{aligned} \tag{5.18}$$

The term $\hat{T}\big((g_0^\#(t)\big)$ in the Eq. $(5.18)_1$ describes the change of stress due to irradiation and temperature.

Let us denote this term by:

$$\mathscr{T}(\vartheta(t), \varphi(t)) \equiv \hat{T}^{\circ}(g_0^{\#}(t)). \tag{5.19}$$

Let us compute the rate of change of stress caused by the irradiation and tempeature:

$$\dot{T}^{\circ}(t) = \partial_{\vartheta(t)}\mathscr{T}(\vartheta(t), \varphi(t))\dot{\vartheta}(t) + \partial_{\varphi(t)}\mathscr{T}(\vartheta(t), \varphi(t))\Phi(t). \tag{5.20}$$

Integrating in time (5.20) in the interval (t_0, t_k) we get

$$T^{\circ}(t) = T^{\circ}(t_0) + \int_{t_0}^{t} \partial_{\vartheta(\tau)}\mathscr{T}(\vartheta(\tau), \varphi(\tau))\dot{\vartheta}(\tau)\,d\tau + \int_{t_0}^{t} \partial_{\varphi(\tau)}\mathscr{T}(\vartheta(\tau), \varphi(\tau))\Phi(\tau)\,d\tau; \tag{5.21}$$

$T^{\circ}(t_0)$ is the internal stress which may exist in the material at the beginning of the process. If we assume that the process of irradiation starts in time t_0 and the body is initially in a natural state then the value of $T^{\circ}(t_0)$ is equal to zero.

The full quasi-linear equation for the stress in the particle X and at time t may be written in the form

$$\begin{aligned} T(t) = C_E(\vartheta(t), \varphi(t))[E(t)] + \int_{\vartheta_0}^{\vartheta(t)} C_V(\xi(t), \varphi(t))\,d\xi \\ + \int_{0}^{\varphi(t)} C_\varphi(\vartheta(t), \zeta(t))\,d\xi + C_V(\vartheta(t), \varphi(t))[\dot{E}(t)], \end{aligned} \tag{5.22}$$

where

$$\begin{gathered} 2\partial_C \hat{T}^{\circ} \equiv C_E, \quad \partial_{\dot{C}} \hat{T}^{d} \equiv C_V, \quad \partial_\vartheta \mathscr{T} \equiv C_\vartheta, \quad \partial_\varphi \mathscr{T} \equiv C_\varphi, \\ C(t) = E(t) + o(\delta^2), \quad \dot{C}(t) = \dot{E}(t) + o(\delta^2). \end{gathered} \tag{5.23}$$

In the case of infinitesimal deformations, effects of thermo-mechanical coupling and the thermal effects of internal dissipation may be neglected for a broad class of boundary-value problems. The energy balance equation then assumes a form

$$\varrho\vartheta(t)\,\partial_{\vartheta(t)}\hat{N}\partial_t\vartheta(t) = \operatorname{div}[K(\vartheta(t), \varphi(t))\nabla\vartheta(t)] + \varrho r(t). \tag{5.24}$$

Using the assumptions that the function $\hat{N}$ is smooth and the deformation is δ-small we may write:

$$\partial_{\vartheta(t)}\hat{N}(E(t), \vartheta(t), \dot{C}(t), \varphi(t)) = \partial_{\vartheta(t)}\hat{N}(0(t), \vartheta(t), 0(t), \varphi(t)) + o(1). \tag{5.25}$$

The quantity

$$c(\vartheta(t), \varphi(t)) \equiv \vartheta(t)\,\partial_{\vartheta(t)}\hat{N}(0(t), \vartheta(t), 0(t), \varphi(t)) \tag{5.26}$$

is by definition the specific heat at a constant deformation.

The complete system of equations for the coupled thermo-radiation-mechanical problem may be reduced now to the system of quasi-linear parabolic equations of the thermo-radiation problem:

$$\begin{gathered} \varrho c(\vartheta(t); \varphi(t))\,\partial_t\vartheta(t) = \operatorname{div}[K(\vartheta(t), \varphi(t))\nabla\vartheta(t)] + \varrho r(t), \\ \frac{1}{v}\,\partial_t\Phi(t) = \operatorname{div}[D_0(\vartheta(t), \varphi(t))\nabla\Phi(t)] - \mu_a(\vartheta(t))\Phi(t) + s(t), \end{gathered} \tag{5.27}$$

with the correponding initial and boundary conditions, and to the equations of mechanical problem, with thermal and irradiation effects. These equations consist of the equation of motion, the Eq. (5.22), and correponding initial and boundary conditions. When writting the Eq. $(5.27)_2$ we have assumed that there is a dependence of absorption coefficient on time through the dependence on the temperature field ([16]).

Equations of the quasi-linear theory may constitute a basis for solving a class of boundary-value problems encountered in reactor technology.

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