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# AUGMENTED LAGRANGIAN APPROACH for WHEEL - RAIL THERMOELASTIC CONTACT PROBLEM with FRICTION and WEAR

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## ABSTRACT

This paper deals with the numerical solution of wheel - rail rolling contact problems. The unilateral dynamic contact problem between a viscoelastic body and a rigid foundation is considered. The contact with Coulomb friction law occurs at a portion of the boundary of the body. The contact condition is described in velocities. The friction coefficient is assumed to be bounded and suitable small. A frictional heat generation and heat transfer across the contact surface as well as Archard's law of wear in contact zone are assumed. The equlibrium state of this contact problem is described by the coupled hyperbolic variational inequality of the second order and a parabolic equation. To solve numerically this contact problem we will decouple it into mechanical and thermal parts. Finite difference and finite element methods are used to discretize the contact problem. The Augmented Lagrangian technique combined with the active set method are employed to solve the discretized contact problem. Numerical examples are provided.

*Keywords:* rolling contact, wear, heat flow, Augmented Lagrangian methods

## 1 INTRODUCTION

The paper is concerned with the numerical solution of a dynamic contact problem for a viscoelastic body. The contact with Coulomb friction and wear occurs at a portion of the boundary of the body. The nonpenetration condition governing the contact phenomenon is formulated in velocities. This first order approximation seems to be physically realistic for the case of the small distance between the body and the obstacle as well as for the small time intervals. The friction coefficient is assumed to be bounded. Moreover, a frictional heat generation and heat transfer across the contact surface are assumed. The existence of the wear process can be identified as wear debris [1, 2]. This debris is assumed to disappear immediately at the point were it is formed. In the model the wear is identified as an increase in the gap between bodies. Moreover, the dissipation energy is being changed due to wear. We employ the Archard's law of wear, where the wear rate is proportional to the normal contact pressure and the sliding velocity.

The equilibrium state of this contact problem is described by the coupled system consisting of the hyperbolic variational inequality of the second order governing the displacement field and the parabolic equation governing the heat transfer. From the assumption of viscoelasticity of contacting body as well as the contact condition formulated in velocities follows the existence and suitable regularity of solutions to this inequality. This inequality belongs to the class of hemivariational inequalities. These inequalities are employed in modelling of rigid body dynamics problems in robotics or in a non - smoth mechanics including friction and impact [6, 8]. The elastic rolling contact problem was considered by many authors (see literature in [1, 2]).

This paper extends results presented in [1]. Using results concerning the existence of solutions to the dynamic contact problems (see [5]) we solve numerically this dynamic thermoviscoelastic contact problem. After brief introduction of the thermoviscoelastic model of the rolling contact problem in the framework of two-dimensional linear elasticity theory [1, 2, 5] the general coupled parabolic - hyperbolic system describing this physical problem is formulated. Finite difference and finite element methods are used to discretize the contact problem [6]. To solve numerically the discretized system we will decouple it into mechanical and thermal parts (see [1]). First, for a given temperature field we solve the mechanical part. In order to solve the mechanical part of this system we introduce a regularization of the friction conditions. Moreover, we replace the solving the hyperbolic inequality by solving an auxiliary optimization problem to calculate the displacement and stress fields in the whole domain. Augmented Lagrangian method combined with active set strategy is used to solve this auxiliary optimization problem [4]. Newton method is employed to calculate tangent contact stress from regularized friction conditions. In the second step for the calculated displacement field we solve the thermal part of the system using the Newton method. The applications are for wheel - rail systems. The numerical results are discussed.

## 2 PROBLEM FORMULATION

Consider deformations of a viscoelastic strip lying on a rigid foundation (see Fig. 1). The strip has constant height h and occupies domain $\Omega \in R^2$ with the boundary $\Gamma$. A

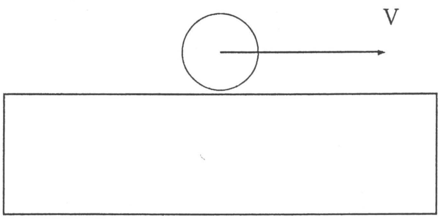


Figure 1: The wheel rolling over the strip.

wheel rolls along the upper surface $\Gamma_C$ of the strip. The wheel has radius $r_0$, rotating speed $\omega$ and linear velocity V. The axis of the wheel is moving along a straight line at a constant altitude $h_0$ where $h_0 < h + r_0$, i.e., the wheel is pressed in the viscoelastic strip. It is assumed, that the head and tail ends of the strip are clamped, i.e., we assume that the lenght of the strip is much bigger than the radius of the wheel. Moreover it is assumed, that there is no mass forces in the strip. The body is clamped along a portion $\Gamma_0$ of the boundary $\Gamma$ of the domain $\Omega$. The contact conditions are prescribed on a portion $\Gamma_C$ of the boundary $\Gamma$. Moreover, $\Gamma_0 \cap \Gamma_C = \emptyset$, $\Gamma = \Gamma_0 \cup \Gamma_C$.

We denote by $u = (u_1, u_2)$, $u = u(t,x)$, $t \in [0,T]$, $T > 0$, $x \in \Omega$, a displacement of the strip and by $\theta = \theta(t,x)$ the absolute temperature of the strip. We shall consider viscoelastic bodies obeying Kelvin - Voigt law [5]:

$$\sigma_{ij}(u) = c^0_{ijkl}(x)e_{kl}(u) + c^1_{ijkl}e_{kl}(\dot{u}), \quad x \in \Omega, \tag{1}$$

i,j,k,l=1,2, $\dot{u} = du/dt$. $c^0_{ijkl}$ and $c^1_{ijkl}$ are components of Hook's tensor satysfiying usual symmetry, boundedness and ellipticity conditions [6]. We use here and throughout the paper the summation convention over repeated indices [6]. The strain tensor $e_{kl}$ is defined by,

$$e_{kl} = (1/2)(u_{k,l} + u_{l,k}), \tag{2}$$

where $u_{k,l} = \frac{\partial u_k}{\partial x_l}$. In an equilibrium state a displacement field $u$ and a temperature field $\theta$ satisfy [1, 2, 6, 8] the system of equations:

$$\ddot{u}_i = \sigma_{ij}(u)_{,j} + b_{ij}\nabla\theta, \quad \text{in } (0,T)\times\Omega, \quad i,j = 1,2, \tag{3}$$

where $\sigma_{ij}(u)_{,j} = \frac{\partial \sigma_{ij}(u)}{\partial x_j}$, $i,j = 1,2$, $\ddot{u}_i = \frac{d^2 u_i}{dt^2}$. $b_{ij}$, $i,j = 1,2$, denotes a thermal expansion tensor, symmetric and bounded. The temperature flow is governed by [5]

$$\dot{\theta} - (c_{ij}\theta_{,j})_{,i} + b_{ij}\dot{u}_{i,j} = 0 \quad \text{in } (0,T)\times\Omega\ , \tag{4}$$

where $c_{ij}$, $i,j = 1,2$, is a tensor of thermal conductivity stisfying usual symmetry, boundedness and ellipticity conditions. The following initial conditions are given

$$u_i(0,x) = u_0, \quad \text{and} \quad \dot{u}_i(0,x) = u_1, \quad i = 1,2, \quad x \in \Omega, \tag{5}$$

$$\theta(0,x) = \theta_0 \quad \text{in } \Omega. \tag{6}$$

$u_0$, $u_1$, $\theta_0$ are given functions. The following boundary conditions are given

$$u_i(x) = 0, \quad \text{on } (0,T)\times\Gamma_0,\ i = 1,2, \tag{7}$$

$$\sigma_{ij}(u) = F, \quad \text{on } (0,T)\times\Gamma_C, \quad i,j = 1,2. \tag{8}$$

$F$ denotes a surface traction vector on the boundary $\Gamma_C$. The surface traction vector $F = (F_N, F_T)$ on the boundary $\Gamma_C$ is a priori unknown and is given by conditions of contact and friction. Under the assumptions, that the strip displacement is small the contact conditions take a form [1]:

$$g_r = r - r_0,, \tag{9}$$

$$\dot{u}_N + \dot{g}_r + \dot{w} \le 0, \quad F_N \le 0, \quad (\dot{u}_N + \dot{g}_r + \dot{w})F_N = 0, \quad \text{on } (0,T)\times\Gamma_C, \tag{10}$$

$$\dot{u}_T = 0 \Rightarrow \mid F_T \mid \le \mu \mid F_N \mid, \text{ and } \dot{u}_T \neq 0 \Rightarrow \mid F_T \mid = -\mu \mid F_N \mid \frac{\dot{u}_T}{\mid \dot{u}_T \mid}, \quad \text{on } (0,T)\times\Gamma_C, \tag{11}$$

where $u_N$, $F_N = \sigma_N$ and $u_T$, $F_T = \sigma_T$ denote normal and tangential components of the displacement and stress on the boundary $\Gamma_C$, respectively. $\mu$ is a friction coefficient and r is the distance between the center of the wheel and a point $x \in \Gamma_C$ lying on the boundary $\Gamma_C$ of the strip $\Omega$. Under suitable assumptions $g_r = h - h_0 + \sqrt{r_0^2 - (u_1 + x_1)^2}$. $w = w(x,t)$ denotes the distance between the bodies due to wear [1] and satisfies the Archard law [1],

$$\dot{w} = kVF_N \tag{12}$$

$w = w(x,t)$ is an internal state variable to model the wear process taking place at the contact interface [1]. $k$ is a wear constant. The wear process can be identified as wear debris, i.e. the removal of material particles from the contacting surfaces. The wear process between contacting surfaces may be caused by adhesion, abrasion, corrosion or surface fatigue [6]. In the considered model the wear is described as an increase in the gap in the normal direction between the contacting bodies. We assume, that in the contact area, the heat is generated due to friction and the heat flow rate is transformed completely into heat. Moreover, we assume the wear debris disappear immediately at the point where it is formed influencing the contact conditions by increasing the gap between the contacting bodies only. Since the wear debris will be warm due to conduction from heated contacted bodies as well as due to wear processes the disappearing wear debris will carry away also the heat energy [1]. Moreover the heat flux on the boundary of the strip $\Omega$ is equal to:

$$\frac{\partial \theta}{\partial n}(t,x) = \kappa(\theta_g - \theta), \quad \text{on } (0,T) \times \Gamma_0, \tag{13}$$

$$\frac{\partial \theta}{\partial n}(t,x) = \kappa(\theta_g - \theta) + \mu \mid F_N \mid\mid \dot{u}_T \mid, \quad \text{on } (0,T) \times \Gamma_C. \tag{14}$$

$\kappa$ is a given constant and $\theta_g$ is a given external temperature.

### 2.1 Variational Formulation

For the sake of numerical solution let us formulate system (1) - (14) in variational form. Let us denote:

$$V = \{v \in H^{\frac{1}{2}}(0,T; H^1(\Omega)) \ : \ v = 0 \text{ on } (0,T) \times \Gamma_0 \}, \tag{15}$$

$$K = \{v \in V \ : \ \dot{v}_N + \dot{g}_r + \dot{w} \leq 0 \text{ on } (0,T) \times \Gamma_C\}, \tag{16}$$

Let us introduce the bilinear forms: $a^i(.,.) \ : \ V \times V \to R$, $i = 0,1$, given by

$$a^0(u,v) = \int_Q c^0_{ijkl} e_{ij}(u) e_{kl}(v) dx dt, \ \ a^1(u,v) = \int_Q c^1_{ijkl} e_{ij}(u) e_{kl}(v) dx dt, \tag{17}$$

where $Q = (0,T) \times \Omega$ denotes a cylinder. The problem (1) - (14) is equivalent to the following system:

$$\int_Q \ddot{u}(v - \dot{u}) dx dt + a^0(u, v - \dot{u}) + a^1(\dot{u}, v - \dot{u}) - \int_Q b_{ij}\theta e_{ij}(v - \dot{u}) dx dt +$$
$$\int_0^T \int_{\Gamma_C} j(\mu \mid F_N \mid, (\mid v_T \mid - \mid \dot{u}_T \mid)) ds dt \geq 0 \ \ \forall v \in K, \tag{18}$$

$$\int_Q \dot{\theta}\varphi dx dt + \int_Q (c_{ij}\theta_{,j}\varphi_{,i} + b_{ij}\dot{u}_{i,j}\varphi) dx dt +$$
$$\int_0^T \int_\Gamma \kappa(\theta_g - \theta)\varphi ds dt = \int_0^T \int_{\Gamma_C} j(\mu \mid F_N \mid, \mid \dot{u}_T \mid)\varphi ds dt \ \ \forall \varphi \in V \tag{19}$$

Problem (18) - (19), without wear, has a unique solution [6]. The regular enough function $j(.,.)$ replace the term describing tangential friction and frictional heat generation.

## 3 FINITE DIMENSIONAL MODEL

The contact problem (18) - (19) has been discretized using the finite difference method for time derivatives and the finite element method for spatial derivatives [6].
Let us denote by $k > 0$ the discretization parameter of the time variable. Let the interval $[0, T]$ be devided into r subintervals of the length k=T/r. The function u dependent on time variable t is approximated by a piecewise constant functions $\lambda^{i+1}$ on each subiterval $\Delta_k = (ik, (i+1)k]$. The time derivatives $\dot{u}$ and $\ddot{u}$ will be approximated by the following finite difference quotiens:

$$\dot{u} \simeq \frac{[u(t+k) - u(t)]}{k}, \; \ddot{u} \simeq \frac{[u(t+k) - 2u(t) + u(t-k)]}{k^2}, \tag{20}$$

If $\{u^i\}_{i=0}^r$ is the set of values of a sufficiently smooth function $u$ at time levels $t_i = ik$, $u^i = u(ik)$, then the symbol $u^{i+\vartheta}$, $\vartheta \in [0,1]$, denotes the convex combination of the values at two succesive time steps $i$ and $i+1$, i.e.,

$$u^{i+\vartheta} = (1-\vartheta)u^i + \vartheta u^{i+1}, \;\; i = 0, 1, 2, ..., r-1. \tag{21}$$

The selection $\vartheta = 1/2$ corresponds to Crank - Nicholson scheme. The finite element method is used to approximate the domain $\Omega$ and the functions defined on it. Let $h > 0$ be a discretization parameter of the domain $\Omega$ and its boundary. By $\Omega_h$ we denote the approximation of the domain $\Omega$ consisting of triangular elements $T_l$, $l = 1, ..., I$. The function $u_k^i$ is approximated by the piecewise linear functions $\xi_j$ on each triangle $T_l$. For details concerning the discretization of the domain $\Omega$ and its boundary $\Gamma$ see [6]. By $u_{h,k}^{\vartheta}$ and $\theta_{h,k}^{\vartheta}$ we denote the functions approximating the functions $u$ and $\theta$, respectively, depending on t and x, i.e.,

$$u_{h,k}^{\vartheta} = \sum_{i=0}^{r-1} u_{h,k}^{i+\vartheta} \lambda^{i+1}, \;\; \theta_{h,k}^{\vartheta} = \sum_{i=0}^{r-1} \theta_{h,k}^{i+\vartheta} \lambda^{i+1} \tag{22}$$

Using (22) the system (18) - (19) takes the form,

$$\int_{Q_{h,k}} \frac{(u_{h,k}^{i+1} - 2u_{h,k}^{i} + u_{h,k}^{i-1})}{k^2} \xi_h dxdt + a_h^0(u_{h,k}^{i+\vartheta}, \xi_h) + a_h^1(\frac{(u_{h,k}^{i+1} - u_{h,k}^{i-1})}{2k}, \xi_h) - \int_{Q_{h,k}} b_{ij} \theta_{h,k}^{i+\vartheta} e_{ij}(\xi_h) dxdt + \int_0^T \int_{\Gamma_{Ch}} \mu \mid F_{Nh} \mid (\mid (\frac{(u_{h,k}^{i+1} - u_{h,k}^{i-1})}{2k})_T \mid) \xi_h dsdt \geq 0 \;\; \forall \xi_h \in K_{h,k}, \tag{23}$$

$$\int_{Q_{h,k}} \frac{\theta_{h,k}^{i+1} - \theta_{h,k}^{i}}{2k} \varphi_h dxdt + \int_{Q_{h,k}} (c_{ij} \theta_{,j}^{i+\vartheta} \varphi_{h,i} + b_{ij}(\frac{(u_{h,k}^{i+1} - u_{h,k}^{i-1})}{2k})_{i,j} \varphi_h) dxdt + \int_0^T \int_{\Gamma_{Oh}} \kappa(\theta_g - \theta_{h,k}^{i+\vartheta}) \varphi_h dsdt = \int_0^T \int_{\Gamma_{Ch}} j_h(\mu \mid F_N \mid, \mid (\frac{(u_{h,k}^{i+1} - u_{h,k}^{i-1})}{2k})_T \mid) \varphi_h dsdt \;\; \forall \varphi_h \in V_{h,k} \tag{24}$$

For the sake of brevity the system (23) - (24) may be written in the matrix form: find $u_{h,k}^{i+\vartheta}$, $\theta_{h,k}^{i+\vartheta}$ and $w_{h,k}^{i+\vartheta}$ satisfying:

$$Au_{h,k}^{i+\vartheta} + K_2(u_{h,k}^{i+\vartheta}) + B\theta_{h,k}^{i+\vartheta} = F_{h,k} \,, \tag{25}$$

$$A\theta_{h,k}^{i+\vartheta} + Bu_{h,k}^{i+\vartheta} = G_{h,k} \,, \tag{26}$$

$$w_{h,k}^{i+\vartheta} = Du_{h,k}^{i+\vartheta} + C_{h,k} \,, \tag{27}$$

and such that,

$$u_{h,k}^{i+\vartheta} + g_{h,k}^{i+\vartheta} + w_{h,k}^{i+\vartheta} \leq 0 \ \text{ on } \Gamma_{Ch}. \tag{28}$$

$A$, $B$, $C$, $D$, and $F_{h,k}$, $G_{h,k}$, $C_{h,k}$ denote suitable matrices and vectors, respectively.

# 4 THE SOLUTION ALGORITHM

Problem (18) - (19) is a coupled thermoviscoelastic problem since the contact traction will depend on the thermal distortion of the bodies and wear process. On the other hand, the amount of heat generated due to friction will depend on the contact traction. The main solution strategies for coupled problems are global solution algorithms where the differential systems for the different variables are solved together or operator splitting methods. In this paper we employ operator split algorithm.

The conceptual algorithm for solving (18) - (19) is as follows [1]:

Step 1 : Choose $\theta = \theta^0$ and $w = w^0$. Choose $\eta \in (0,1)$. Set $k = 0$.
Step 2 : For given $\theta^k$ and $w^k$ find $u^k$ and $\sigma_N^k$ satisfying system (25) and boundary conditions (4) - (8).
Step 3 : For given $u^k$ and $\sigma_N^k$ find $w^{k+1}$ as well as $\theta^{k+1}$ satisfying equations (26), (27) respectively.
Step 4 : If $\| \theta^{k+1} - \theta^k \| \leq \eta$ , Stop. Otherwise : set $k = k+1$, go to Step 2.

For the convergence of the operator split algorithm using Fixed Point Theorem see literature in [1, 6]. Let us present in details the algorithms for solving disrete mechanical and thermal subproblems.

## 4.1 Solution of the mechanical and thermal subproblems

In order to solve numerically problem (25) we reformulate it as an optimization problem. The inequality (25) is a necessary optimality condition for a functional,

$$J(u_{h,k}^{i+\vartheta}) = 0.5u_{h,k}^{i+\vartheta}Au_{h,k}^{i+\vartheta} + K_2(u_{h,k}^{i+\vartheta})u_{h,k}^{i+\vartheta} + u_{h,k}^{i+\vartheta}B\theta_{h,k}^{i+\vartheta} - F_{h,k}u_{h,k}^{i+\vartheta} \tag{29}$$

to reach minimum on the set $K_{k,h}$. We apply the Augmented Lagrangian method with the active set strategy [4] to solve this optimization problem. Let us introduce the Augmented Lagrangian associated with the functional (29), i.e.,

$$L(u_{h,k}^{i+\vartheta}, \alpha) = J(u_{h,k}^{i+\vartheta}) + \alpha(u_{h,k}^{i+\vartheta} + g_{h,k}^{i+\vartheta} + w_{h,k}^{i+\vartheta}) + \frac{\varepsilon}{2} \| u_{h,k}^{i+\vartheta} + g_{h,k}^{i+\vartheta} + w_{h,k}^{i+\vartheta} \|^2, \tag{30}$$

where $\alpha$ is a Lagrange multiplier associated with the boundary condition (10) and $\varepsilon > 0$ denotes a penalty coefficient. Using (30) we can write the necessary optimality condition in the form:

$$\nabla J(u_{h,k}^{i+\vartheta}) + \alpha + \varepsilon(u_{h,k}^{i+\vartheta} + g_{h,k}^{i+\vartheta} + w_{h,k}^{i+\vartheta}) = 0, \tag{31}$$

$$\alpha = \varepsilon \max(0, u_{h,k}^{i+\vartheta} + \frac{\alpha}{\varepsilon}) \tag{32}$$

The employed algorithm will solve the system of optimality conditions (31) - (32). The algorithm is as follows [4, 7]:

Step 0. Set $n = 1$. Choose initial values of $\alpha$, $u_0$ and $u_1$.
Step 1. Determine the following subsets of $\Gamma_C$: $\mathcal{A}_n = \{x \in \Gamma_{Ch} : u_{h,k}^{i+\vartheta} + \frac{\alpha}{\varepsilon} > 0\}$, and $\mathcal{I}_n = \{x \in \Gamma_{Ch} : u_{h,k}^{i+\vartheta} + \frac{\alpha}{\varepsilon} \leq 0\}$.
Step 2. If $n \geq 2$ and $\mathcal{A}_n = \mathcal{A}_{n-1}$, then Stop.
Step 3. For given $\theta_{h,k}^{i+\vartheta}$ find $u_{h,k}^{i+\vartheta}$ satisfying (31).
Step 4. Set $\alpha_{n+1} = \alpha_n - \varepsilon u_{h,k}^{i+\vartheta}$, update $n = n + 1$, and go to Step 2.

For details of the above algorithm see [1, 4]. Having calculated $u_{h,k}^{i+\vartheta}$ and $\sigma_{Nh}$ we can solve the equations (26) - (27). The equation (26) is solved using Choleski algorithm

## 5 NUMERICAL RESULTS

Problem (18)-(19) was solved numerically using the described in the previous section algorithms. Polygonal domain $\Omega$ given by

$$\Omega = \{(x_1, x_2) \in R^2 \ : \ x_1 \in (-2, 2), \ x_2 \in (0, 1)\} \qquad (33)$$

was divided into 192 triangles. The contact boundary $\Gamma_C$ is modeled by 13 nodes. The Lame constants were $\lambda = 11.66 \cdot 10^{10}$ [N/$m^2$], $\gamma = 8.2 \cdot 10^{10}$ [N/$m^2$], the density $\rho = 7.8 \cdot 10^3$ [kg/$m^3$], the velocity $V = 10$ [m/s] , radius of the wheel $r = 0.46$ [m]. The penetration of the wheel was taken as $\delta = 0.1 \cdot 10^{-3}$ [m]. The heat capacity $c = 460$ J/kgK, thermal diffusitivy coefficient $\kappa = 1,4410^{-5}\ m^2/s$, thermal expansion coefficient $\gamma = 1210^{-6}$. The friction coefficient $\mu = 0.4$, the thermal resistance coefficient $r = 1000$ KNs/J, the wear constant $k = 0.510^{-6}\ MPa^{-1}$. $\varepsilon = 0.001$. $\bar{u}_0$ and $\bar{u}_1$ in (3) as well as $\bar{\theta}$ in (3) are equal to 0. $T = 5s$. Time interval was divided into 20 subintervals. The wear gap is shown on Fig 2.

Normal traction $F_N$ has its peak in the middle of the contact area. Tangent traction $F_T$ has different shapes in front and behind of the rolling wheel. The proposed algorithm converges quickely. Its speed of convergence depends on the choice of the parameter $\varepsilon$ value. For $\varepsilon$ very small we obtain much more accurate results than for big values of $\varepsilon$ at a cost of increase in computational time.

## 6 CONCLUDING REMARKS

The dynamic rolling contact problem was solved numerically using Augmented Lagrangian approach. The obtained numerical results are in accordance with physical reasoning [6]. Performance of the optimization method is being improved.

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Fig. 2. Wear gap evolution

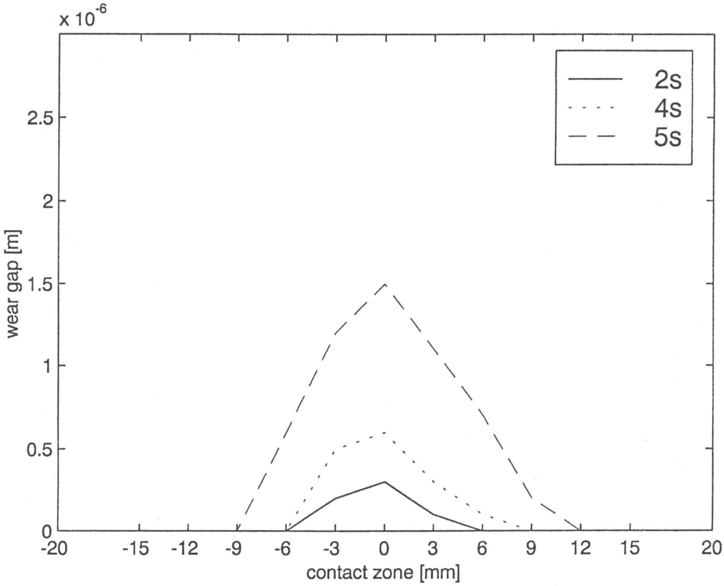

Fig. 2. Wear gap evolution