XX.

TWO LETTERS TO PROFESSOR PHILLIPS ON THE CONSTRUCTION OF OBJECT GLASSES

[Note Book 28, pp. 88–92.]

Observatory,
23 *December*, 1843.

My dear Phillips,*

If it be not too late to do what you proposed, respecting the construction of your new telescope on a plan suggested by me, you may be interested in hearing that I have at last found (among my long accumulating masses of papers) the chief investigations which I made upon the subject several years ago. It was early in 1832 that I concluded from the expressions which were afterwards published in the Third Report of the British Association, (pages 362, 363,)† that if, in addition to the formula of achromaticity, which in the notation of that Report would be

$$\delta P = 0; \tag{1}$$

(δ referring to variations of colour) and the condition of aplanaticity for the case of rays incident parallel to the axis, which condition, in the same notation, is

$$Q = \tfrac{1}{4} P; \tag{2}$$

we make also, in the same notation,

$$Q_{,} = 0: \tag{3}$$

then, a set of rays incident parallel to each other, but slightly inclined to the axis of the instrument, will emerge from that instrument, even if the aperture be not extremely small, so as all to pass, very nearly, through *two focal lines*, one in the diametral plane of the system, and the other perpendicular thereto.‡ The *instrument* is here supposed to be one of revolution, symmetric in all respects (geometrical and optical) about an axis, but otherwise quite arbitrary, so far at least that its surfaces need not be spherical, nor its lenses ordinary refractors; nor need these lenses be thin, nor few, nor close together; but it is supposed to be in vacuo, that is, the first incident and last emergent rays are each in an optical void. *This* theorem of the two focal lines, is not to be confounded with one which sounds very like it, and which Mr. Airy has employed in his Memoir on the Spherical Aberrations of eyepieces. Mr. Airy's focal lines are merely a certain pair of tangents to the two Caustic Surfaces, which surfaces were discovered

* [John Phillips, F.R.S. (1800–1874), was a geologist with wide physical interests. He was in turn Professor of Geology at King's College, London, at Trinity College, Dublin (1843–1845), and at Oxford. For further details, see *Dictionary of National Biography*, xlv, pp. 207–8. Mr. D. Baxandall, of the Science Museum, South Kensington, to whom the Editors are indebted for information, remarks that Phillips, having been engaged in detail sketching work in connection with the preparation of geological maps, must have been familiar with the advantages which would be given, in field work, by the use of a good telescope.]

† [Pp. 298, 299 of the present volume.]

‡ [The condition $Q = \frac{1}{4}P$ corrects for spherical aberration, the condition $Q_{,} = 0$ for coma; the defect of astigmatism remains uncorrected. But, while coma depends linearly on the small initial inclination of the rays to the axis of the instrument, astigmatism depends on the square of that small inclination. The geometrical discussion of coma and astigmatism will be found in No. XIX, while the calculations underlying the results stated in these letters are developed in No. XXI, p. 406; see also [33.] to [36.] inclusive.]

long before by Malus, and which are touched by all the rays of the emergent system; mine are, at least approximately, the Caustic Surfaces themselves, which under the conditions (2) and (3) of the present letter, degenerate, very nearly, for some extent, into the two straight lines described, or referred to, above. Mr. Airy's lines are not intersected, or approached to, at all so nearly, by rays at a moderate distance from what he takes for the axis of his pencil, as mine are by all the rays of a moderately broad emergent system. Such at least, is my recollection of the result of my old researches on this subject; but it is fair to say that he studied chiefly eye-glasses, I chiefly object glasses. Still, I think that my theory must apply to eyepieces also.

I have nothing new (worth mentioning) to add to the known theories of achromaticity and aplanaticity, for direct parallel incident rays; the equations (1) and (2) of this letter are merely my own modes of concisely expressing what is, for instance, already stated in Sir J. Herschel's Treatise on Light; and no doubt, the artist, your friend, is well acquainted therewith. But I shall be glad, (if the whole matter be not ere this disposed of,) to know of what two colours he proposes, as the result of his experience, to correct the chromatic separation; or, in short, to know *what focal lengths he will give to the two lenses*, so as to avoid, in the way he judges best, chromatic aberration (thereby in fact fulfilling in his own way the equation (1) of this letter,) and at the same time to supply the focal length required by you for the combination. It will then not be requisite for me to know the dispersive powers, but merely *the refractive indices of the two lenses for a mean ray*; and I shall endeavour to determine the two anterior curvatures, so as to satisfy the two equations (2) and (3) of the present letter; after which you or he can try (as a verification) whether the usual condition of aplanaticity is not satisfied, as it ought to be: and then perhaps you may be tempted to make the actual experiment, whether an object glass so constructed will have any advantage in distinctness, for a star somewhat out of the centre. You see, of course, that *both* lenses should be figured in conformity to my theory. I have your first letter safe, but cannot at present lay my hand upon the second. Perhaps you will favour me (though undeserving) with an early answer, and in the meantime believe me to be, my dear Phillips,

Very sincerely yours,
WILLIAM R. HAMILTON.

Observatory, Dublin.
3d. Jan. 1844.

MY DEAR PHILLIPS,

I found your second letter, in the course of a settlement of papers on New Year's Day, and find that you state the indices and dispersive ratio to be

1,533; 1,630; 0,620;

focal length 37,5 inches; aperture 2,4 inches.

With these data, or rather with the 4 first, I yesterday computed the 4 successive radii to be*

$$+23{,}16;\quad -11{,}30;\quad -11{,}56;\quad -57{,}25.$$

The signs are here so chosen (according to the plan of Herschel's Essay on Light) as to be positive when the surface is convex, but negative when it is concave, to the incident light. The

* [Equations (A) and (B) of p. 406 yield another set of radii

$$+5{\cdot}147,\quad +15{\cdot}97,\quad +6{\cdot}134,\quad +4{\cdot}309,$$

but these values are obviously unsuitable from a practical point of view.]

crown lens would therefore be double convex, the radius of the outer surface being about double that of the inner; which latter would very nearly fit into the concave surface of the flint, but would be slightly more curved than it; and the last surface of the flint would be convex towards the eye, and would have a radius about five times as great as the radius of the inner surface. The double object glass would thus have a great *general* similarity to that represented in Herschel's plate 6, fig. 108; but I have to-day computed from your data that on Herschel's construction the radii ought to be, in inches,

$$+25{,}47\,;\quad -10{,}82\,;\quad -11{,}05\,;\quad -46{,}67\,;$$

(interpolation from his Table gave me

$$+25{,}52\,;\quad -10{,}81\,;\quad -11{,}03\,;\quad -46{,}32\,;)$$

thus my first surface is more curved than his, my middle ones a little flatter, and my last one a good deal so. Indeed, I can prove that the two constructions *can never coincide*, though I think that they will always have some resemblance to each other. On each plan, your compound focal length and dispersive ratio give, for the respective focal lengths of the two separate lenses,

$$\text{crown, } +14{,}25 \text{ in.}; \quad \text{flint, } -22{,}98 \text{ in.}$$

I wrote to you, to York, a letter on the 23d. of December, but as I had not then found yours of the 28th. of September, which had unluckily been mislaid, I thought myself obliged to trouble you with some questions which it may now be unnecessary for you to answer. In that late letter of mine, I stated that having recently found various old papers of optical investigations of my own, I had recovered a precise statement of my theorem of the existence of *two focal lines*, (quite different from Mr. Airy's,) for rays which emerge from an object glass, after being incident parallel to each other, but with a moderate inclination to the axis; provided that besides the usual condition of aplanaticity for direct parallel incident rays, a certain other condition be satisfied, which in the notation of the Third Report of the British Association, (Cambridge Meeting,) pages 362, 363, is

$$Q_{,} = 0.$$

I have lately perceived that to those who are familiar with Herschel's excellent Treatise on Light, already referred to, (published in the Encyclopædia Metropolitana,) my rule may be very simply stated by saying that in his formula (f), page 425, Art. 469, the first member is to be changed from 0 to 1, if the power of the combination be unity; or more generally, and more fully, to the square of the power of the whole double lens, that is to $(L' + L'')^2$.* This form of the rule, indeed, ought to be sufficient for your friend, or for any scientific optician, without his being obliged to study, or inspect, a single paper of my own; supposing him to be acquainted with Herschel's Essay. With Herschel's formula (f), Art. 469, thus modified, he may combine the formula (v), Art. 313, of the same work, taking care to read the last term of (v) as $\frac{2+3\mu''}{\mu''} L'^2 L''$. He will thus have two equations between the two anterior curvatures, called R' and R''' in (v), but r' and r'' in (f), one equation being linear and the other quadratic, and consequently conducting to an ordinary quadratic equation with only one anterior curvature involved in it, when the indices μ' and μ'' are known, and when the powers L' and L'' of the

* [Cf. No. XXI, p. 439.]

two component lenses have been deduced from the total power and the dispersive ratio. Such is my rule, in the simplest form in which I can state it to a student of Herschel; I do not quite like that it should *yet* be made public, but if it be not too late for your purpose and his, you are at liberty to show it (or to send this letter) to Mr. Cooke.* I have only to add that my changing 0 to $(L'+L'')^2$ in (v) arises from my having aimed at an object different from that which Sir John Herschel had in view; not from any error of calculation on his part, nor, I hope, on mine. And though I have, as I conceive, sufficiently stated my process, yet I shall be very glad to spend a day (or more if necessary) on any further numerical calculations which may make your friend sure that, if he uses other indices, he knows my curvatures to match them. I am, my dear Phillips,

Very truly yours,
W. R. Hamilton.

I should like for curiosity to know the thickness of your flint glass at the middle; I have, like Herschel, treated both lenses as infinitely thin. As I am not sure of your being at home, a line would be a satisfaction. I think I understand the phenomenon which you describe, and that it is connected with my theory.

* [Messrs. Cooke, Troughton and Simms, Ltd., of York, state that they are unable to trace any reference to the construction of the telescope in question in the books of the firm of T. Cooke and Son of this date.]