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One-dimensional constitutive model of microcracked elastic solid

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A CONSTITUTIVE model is proposed to predict the quasi-plastic response of a brittle elastic solid undergoing internal damage in uniaxial tension. Following the experimental evidence and basing on a simplified reasoning as to the nature of damage process in the microscale, a functional dependence between the stress intensity factor and the microcrack length is introduced leading to an evolution law for microcrack growth. Nonlinear constitutive relations are then derived accounting for the damage-induced hardening and softening effects observable in the overal material behavior. A single microcrack as well as systems of interacting microcracks are considered.

W pracy przedstawiono jednowymiarowy model konstytutywny sprężystego materiału kruchego, którego zachowanie w testach jednoosiowego rozciągania ma quasi-plastyczny charakter, wywołany narastającym uszkodzeniem struktury wewnętrznej. Opierając się na wynikach doświadczeń i przeprowadzając uproszczone rozumowanie dotyczące mikroskopowej natury procesu uszkodzenia, wprowadzono fizykalną zależność funkcyjną między współczynnikiem intensywności naprężeń i długością mikroszczeliny, otrzymując w ten sposób przyrostowe prawo ewolucji mikroszczelin. Wykorzystując równanie ewolucji wyprowadzono nieliniowe prawa konstytutywne uwzględniające efekty wzmocnienia i osłabienia obserwowane w makroskopowym zachowaniu uszkadzającego się materiału. Rozważono wyizolowaną mikroszczelinę oraz najprostsze układy współdziałających ze sobą mikroszczelin.

В работе представлена одномерная определяющая модель упругого хрупкого материала, поведение которого в испытаниях одноосного растяжения имеет квазипластический характер, вызванный возрастающим повреждением внутренней структуры. Опираясь на результаты экспериментов и проводя упрощенное рассуждение, касающееся микроскопической природы процесса повреждения, введена физическая функциональная зависимость между коэффициентом интенсивности напряжений и длиной микротрещины, получая таким образом закон эволюции микротрещин в приростах. Используя уравнение эволюции, выведены нелинейные определяющие законы, учитывающие эффекты упрочнения и ослабления, наблюдаемые в макроскопическом поведении повреждающегося материала. Рассмотрены изолированная микротрещина и самые простые системы взаимодействующих с собой микротрещин.

1. Introduction

THE PRESENCE of microdefects such as microcracks, voids, inclusions and other stress concentrators in the brittle materials during the process of straining influences their mechanical properties and reduces considerably the ultimate strength. The microstructural damage is externally observed as a macroscopic inelastic deformation. In order to be able to predict the behavior of these materials under a variety of different loading circumstances, a rational theory must therefore reflect the influence of the type, arrangement and kinetics of the internal microdefects on the material response in the macroscale.

It is known that the concrete, ceramics and certain rocks exhibit pronounced strain softening in the compression tests. If the tests are performed in very rigid loading devices, even the concrete-like materials in the direct tension manifest distinct quasi-plastic behavior.

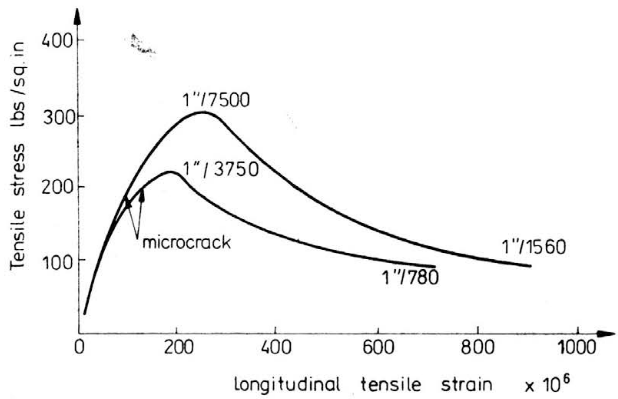

Fig. 1.

This can be seen in Fig. 1 which demonstrates the complete stress-strain curves for concrete in tension, obtained experimentally by Evans and Marathe [1] in a series of precisely conducted, strain-controlled uniaxial tests. An elastic or nearly elastic, hardening- and softening portions of the curve in question may be observed. Above the stress level which corresponds to the indicated commencement of the process of microfissuration, microcracks grow progressively with deformation resulting in the stable and unstable inelastic behavior of the specimen. Asymptotic character of the presented curves at large strains should be also mentioned. The growth and coalescence of microcracks lead to the nucleation of the macrocrack which brings the strained specimen into the final fracture while traversing across the entire cross-section.

The problem of proper analytical modelling of a material that suffers internal microfracturing when strained has been treated in the literature in a twofold manner. First, a description of an overall material behavior is conceivable without going into fine details of the damage process in the microscale. Such an approach, known as the Continuous Damage Mechanics, involves formulation of a phenomenological model based on the quantities called damage variables, characterizing structural changes in an average sense. Usually, the damage variables of scalar, vector or tensor character are being linked up with the damage-caused effects like: diminishing of an effective load-carrying cross-section due to nucleation and gradual evolution of microdefects [2, 3, 4], degradation of the elastic constants [5] or dissipation of the energy imparted to the body in a static and isothermal process of loading [6]. Extended surveys of the existing continuous damage theories can be found elsewhere [7, 8, 9]. Ignoring the physical nature of the problem, some theories have been proposed [10, 11, 12] in the framework of the Continuum Damage Mechanics that describe the damage indirectly through the effect it produces on the strains.

An alternative approach to the modelling of damage consists in embodying in the constitutive equations the most essential microstructural features of the process of damage. To this end the model derived in [13] should be mentioned.

The objective of this paper is to propose a workable constitutive model for predicting the time-independent stress-strain relations of damaged conrete-like materials subjected to uniaxial tension.

Although a phenomenological description of the material behavior in question is aimed here, we shall base, however, on some microstructural observations for an isolated microcrack as well as for systems of interacting microcracks. It seems therefore reasonable to employ some suitable notions borrowed from the Brittle Fracture Mechanics which proved to be a successful tool when accounting for the phenomena governed by a single, well-developed macrocrack. It is hoped that the intended approach combining the conventional continuum formulation and simplified microstructural reasoning will give a somewhat deeper insight into the nature of the phenomenon, and can be helpful in the identification of material parameters.

2. Theoretical setting

Consider an infinite, isotropic, elastic plate in uniaxial tension, Fig. 2. It is commonly accepted that microcracks in the tensile test nucleate and grow in the direction perpendicular to the load axis. This assumption is firmly supported by all of the experimental

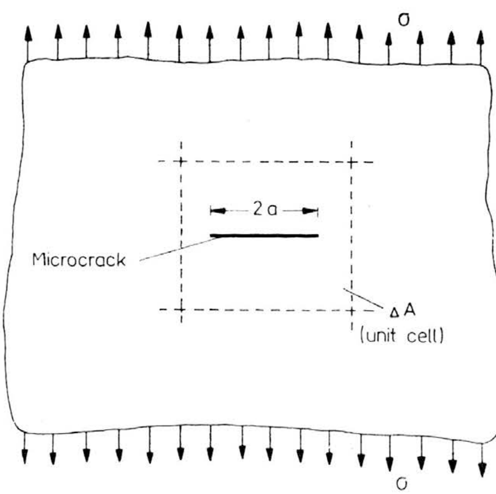

FIG. 2.

data published so far (cf. [3]). Imagine now that the plate consists of a multitude of unit cells and each cell contains one or more microcracks. The unit cell is meant to be a microcrack-attached characteristic area of a size ΔA being limited by a condition that no interaction between adjacent cells takes place. Moreover, it is assumed that the behavior of the individual cell is representative of the behavior of the whole body.

The density of the complementary energy stored in the damaged plate consists of two terms, namely that of the complementary energy density $\tilde{\pi}^e$ for the virgin elastic material being solely a function of stresses, and that of the change $\Delta\tilde{\pi} = \dfrac{\Delta\tilde{\pi}^*}{\Delta A}$ in this energy due to appearance of the microcracks depending both on the stresses and the microcrack length

$$\tilde{\pi}(\sigma, a) = \tilde{\pi}^e(\sigma) + \Delta\tilde{\pi}(\sigma, a). \tag{2.1}$$

The complementary energy density (2.1) is supposed to be the Gibbs-potential, i.e. a scalar function of stress tensor whose derivative with respect to stress component determines the corresponding strain component:

$$\varepsilon = \frac{\partial \tilde{\pi}}{\partial \sigma}\bigg|_{a=\mathrm{const}} = G(\sigma, a). \tag{2.2}$$

The length of microcrack is considered as a hidden state variable that reflects locally a current level of an internal microcracking; thus it may be treated as a damage variable. The damage variable should be provided with an evolution law (damage law) which would, in this case, be a direct condition for the microcrack growth.

At this point we postulate that the microcrack starts to grow if a certain scalar function g dependent on the stress, the microcrack length as well as on other possible parameters recording for example a geometrical configuration of microcrack pattern is satisfied:

$$g(\sigma, a, \ldots) = 0. \tag{2.3}$$

There will be no microcrack growth if $g < 0$.

In order to specify the condition (2.3), we put forward a following hypothesis justified by the findings of the Brittle Fracture Mechanics: *a microcrack begins to move if the corresponding stress intensity factor K at the microcrack tip reaches some critical level K_c*

$$K = K_c. \tag{2.4}$$

The stress intensity factor K describes the loading of a crack and depends on the stress state as well as on the configuration of the microcracks in the unit cell. It can always be expressed in the following form, no matter how complex the geometry of microcrack system is:

$$K = \sigma\sqrt{\pi a} f(a, b, c, \ldots), \tag{2.5}$$

where $b, c, \ldots$ stand for the necessary geometrical parameters. Basing on the sound microstructural premiss [14] we assume now that the K_c in the damage law (2.4) is, in general, not a material constant but a function describing the resistance of material against the

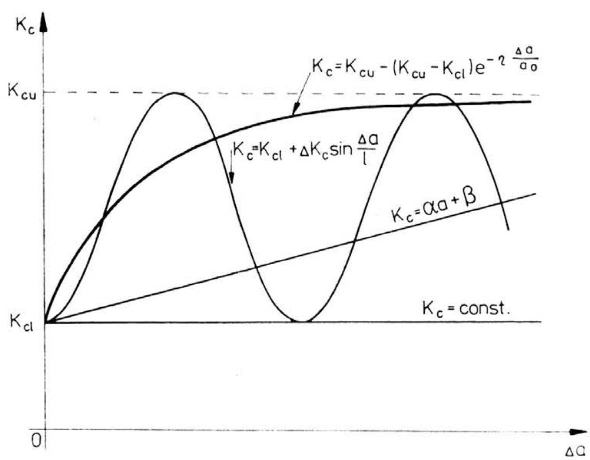

FIG. 3.

microcrack growth, and it depends essentialy on the change Δa of microcrack length. As far as the shape of the function $K_c(\Delta a)$ is concerned, various possibilities are left, Fig. 3, according to the type of material considered. A physical motivation for choosing one of the displayed functions will be given later on. If the K_c-function is specified, we may insert it together with (2.5) into the damage law (2.4) and solve (2.4) with respect to the microcrack length:

$$a = F^*(\sigma). \tag{2.6}$$

Equation (2.6) combined with the general constitutive relationship (2.2) makes possible the elimination of the internal parameter a; thus it leads to a nonlinear material law $\sigma-\varepsilon$ accounting for the presence and evolution of microcracks. It is obvious, however, that this cumbersome procedure is applicable if both the material function $K_c(\Delta a)$ and the corresponding stress intensity factor (2.5) are simple enough. In general it is difficult or even impossible to get rid of the damage parameter a in form of (2.6), mainly due to complicated transcendental equations encountered.

Reciprocally to (2.6) we have

$$\sigma = F(a), \tag{2.7$_1$}$$

or

$$g = \sigma - F(a) = 0, \tag{2.7$_2$}$$

which is straightforward on account of the algebraic structure of (2.4) and (2.5). In order to overcome the formal difficulties mentioned above, an incremental formulation of the problem at hand could be employed. An apparent gain of such an approach consists in the fact that the increment da is always derivable from the damage law $(2.7)_2$ in contrast to the parameter a itself. With the notation used in (2.2) and (2.7), we obtain

$$d\varepsilon = \frac{\partial G}{\partial \sigma} d\sigma + \frac{\partial G}{\partial a} da, \tag{2.8}$$

$$d\sigma = \frac{\partial F}{\partial a} da, \tag{2.9}$$

whence

$$d\varepsilon = \left(\frac{\partial G}{\partial \sigma} + \frac{\partial G}{\partial a} \cdot \frac{1}{\dfrac{\partial F}{\partial a}} \right) d\sigma. \tag{2.10}$$

It seems also advantageous to work henceforth with dimensionless variables introduced in the following manner:

$$\tilde{\sigma} = \frac{\sigma}{\sigma_0}, \quad \tilde{\varepsilon} = \frac{\varepsilon}{\varepsilon_0}, \quad \tilde{a} = \frac{a}{a_0}, \tag{2.11}$$

where σ_0, ε_0 are the values of stress and strain, respectively, at which the microcrack of initial length a_0 begins to grow. Having established the basic equations in the incremental form (2.8) and (2.9) it becomes possible to develop various constitutive equations of (2.10) type, depending on what the geometry of microcracks pattern is like and what kind of the material function $K_c(\Delta a)$ we adopt to account for the microcrack evolution. Some simple examples will be discussed in the next sections.

3. Examples

3.1. Single microcrack in unit cell

Suppose that the unit cell under uniaxial tension contains only one single microcrack, Fig. 2. The density of complementary energy can be expressed as follows

(3.1) $$\tilde{\pi}(\sigma, a) = \frac{\sigma^2}{2E} + 2\int_0^a \frac{K^2}{\Delta A E}\, da.$$

Assuming further that the microcracks placed in the individual unit cells are far enough one from another so that there is no interaction between them, we have a simple formula for the stress intensity factor

(3.2) $$K = \sigma\sqrt{\pi a},$$

which brings the energy equation (3.1) into the following form

(3.3) $$\tilde{\pi} = \frac{\sigma^2}{2E}\left(1 + \frac{2\pi a^2}{\Delta A}\right).$$

The next step consists in introducing a specific form of the damage law (2.4). In the case of the perfectly brittle material, the K_c-function should not depend on the microcrack length. A change in the internal energy due to the microcrack growth is entirely stored into the surface energy and no microplastic zone appears at the microcrack tip. The microcrack growth will occur if the stress intensity factor K reaches a threshold value K_c = constant,

(3.4) $$\sigma\sqrt{\pi a} = K_c.$$

It can be shown that this equation is equivalent to the familiar Griffith criterion for the macrocrack growth. The equations (3.3) and (3.4), when substituted into (2.2) enable us, in this simplest case, to arrive at a closed analytical form of the constitutive law looked for

(3.5) $$\tilde{\varepsilon} = \frac{1}{1+\varkappa}\left(\tilde{\sigma} + \varkappa\frac{1}{\tilde{\sigma}^3}\right),$$

where $\varkappa = \dfrac{2\pi a_0^2}{\Delta A}$ is a material constant which, for the physical reasons, should be smaller than 0.1, [15].

Some numerical results are shown in Fig. 4. The material exhibits elastic behavior up to the point where K reaches its critical value and the microcrack starts to move. Then the material behavior becomes nonlinear. It can be seen that for physically reasonable values of the parameter $\varkappa$, the curves $\tilde{\sigma}-\tilde{\varepsilon}$ go backwards. It should be understood that in a strain-controlled test the material would have manifested unstable behavior as a sudden jump in stresses down to the point from where the microcrack keeps on growing in the stable way. The first stable evolution of the microcrack would be observed for $\varkappa = 0.33$ which is not acceptable, as no microcrack interaction is allowed. Apparently, it is a consequence of such a simple damage law assumed.

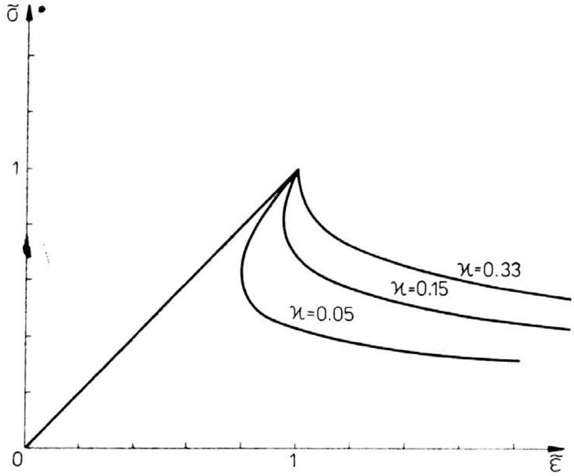

FIG. 4.

The energy required for the microcrack growth in real brittle materials is larger than the pure surface energy needed to create the new surfaces since the process of microcrack evolution is accompanied by certain inelastic, energy-consuming phenomena occuring in front of the microcrack tip, such as: microplastic deformations observed even in very brittle materials, nucleation of microvoids, etc. These phenomena depend on the history of deformation, and thus on the microcrack growth. Once K_c, (2.4) is meant to be a global measure for the microcrack evolution, it should therefore depend upon the microcrack length. This conclusion is experimentally well supported for various materials [14]. Contrary to the case of idealized perfectly brittle materials, the stress intensity factor is no more constant but depends now on the microcrack size and it is larger for larger microcracks. Due to the formation of microvoids around the microcrack tip, the critical stress intensity factor K_c takes initially the lower values, while it is an increasing function of Δa during the stable evolution of the microcrack. On the other hand, it does not seem reasonable to assume that the microcrack resistance will increase infinitely with the increase in the microcrack length, the more so as the process of microcrack growth manifests a stationary character in its advanced stage. After a sufficient growth in the microcrack length, an asymptotic behavior of the curve $K_c(\Delta a)$, Fig. 3, may therefore be expected. We propose the following exponential form dependent on the initial microcrack length a_0 for the material function $K_c(\Delta a)$, Fig. 3:

$$K_c = K_{cu} - (K_{cu} - K_{cl}) e^{-\eta \frac{\Delta a}{a_0}}, \tag{3.6}$$

where K_{cu}, K_{cl}, η are material constants.

It should be emphasized that (3.6) is not characteristic of all the rock-like materials undergoing internal damage. It is for instance conceivable to deal with a periodic form of the function K_c as well. This would correspond to the microcracks growth in a periodically inhomogeneous material in which the microcracks move through the grains or along their boundaries. It is needless to say that we consider exclusively such materials whose resistance to the microcrack growth can be approximated by the specific function (3.6). Nevertheless, any other material function K_c might easily be incorporated if required.

The damage law is readily obtained as

$$\sigma\sqrt{\pi a} = K_{cu} - (K_{cu} - K_{cl})e^{-\eta\frac{\Delta a}{a_0}}. \tag{3.7}$$

An explicit $\sigma - \varepsilon$ relationship in a closed form is not derivable in this case, thus we suggest an incremental formulation of the basic equations as already discussed in Sect. 2. Combining (2.10) with the energy equation (3.3) and the proposed damage law (3.7) after some effort we find that

$$d\tilde{\varepsilon} = \frac{1}{1+\varkappa}\left(1 + \varkappa\tilde{a}^2 + 4\varkappa\tilde{a}^2 \frac{\gamma - (\gamma-1)e^{\eta(1-\tilde{a})}}{(\gamma-1)e^{\eta(1-\tilde{a})}(2\eta\tilde{a}+1) - \gamma}\right) d\tilde{\sigma}, \tag{3.8}$$

where the tildas over ε, σ, a denote the dimensionless variables as defined in (2.11); $\varkappa$, η and $\gamma = \dfrac{K_{cu}}{K_{cl}} \geqslant 1$ are material constants.

A numerical verification of the constitutive equation in loading (3.8) is presented in Fig. 5. It is worth noting that the curves depicted in Fig. 5 show a qualitative agreement

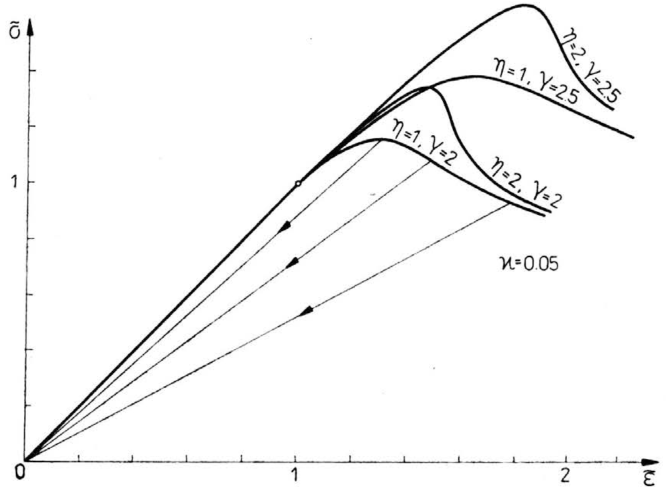

FIG. 5.

with those plotted on the basis of experimental data, Fig. 1. All the characteristic features, that is: linear, hardening and softening portions of the experimental curves as well as their asymptotic behavior at the advanced stage of straining are recorded in the theoretical curves of Fig. 5. In unloading, the Young modulus varies on every step of the process, depending on the actual value of the microcrack length. However, all deformation paths tend to the origin 0 and no permanent strain remains after unloading. There will also be no recovery inside the damaged material during unloading, what means that the damage process is considered to be dissipative since the structural changes are irreversible. It is motivated by an experimental fact that the microcrack faces are quite sharp and irregular

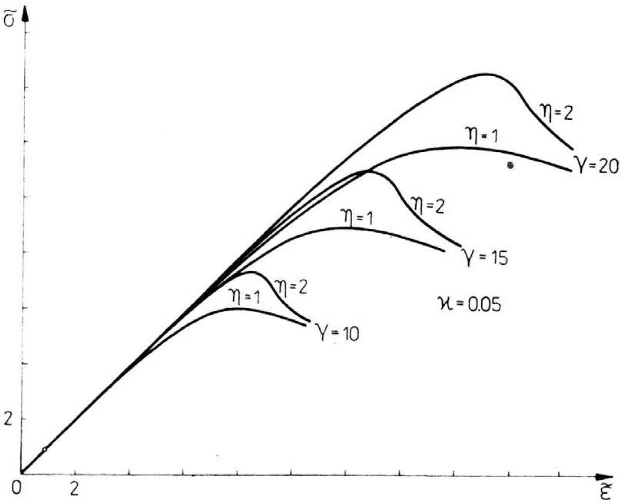

FIG. 6.

and for this reason they never close perfectly after unloading. Figure 6 is meant to give an impression how large the nonlinear part of the $\tilde{\sigma}$—$\tilde{\varepsilon}$ curves can be due to variation of the material parameter $\gamma = K_{cu}/K_{cl}$.

3.2. Microcrack interaction

Actually, the microcracks appearing in the material during the process of loading are not isolated one from another, but they necessarily have an influence on the adjoining microcracks. Therefore, the microcracks interaction should not be neglected if the proposed model is to be realistic. In order to get familiar with the effect of microcrack interaction on the material behavior in the macroscale, we consider the case of a pair of collinear microcracks inside the unit cell, Fig. 7. From the geometry of the problem the ratio a_0/c_0 should be smaller than 1/4, where a_0 denotes some initial microcrack length and the

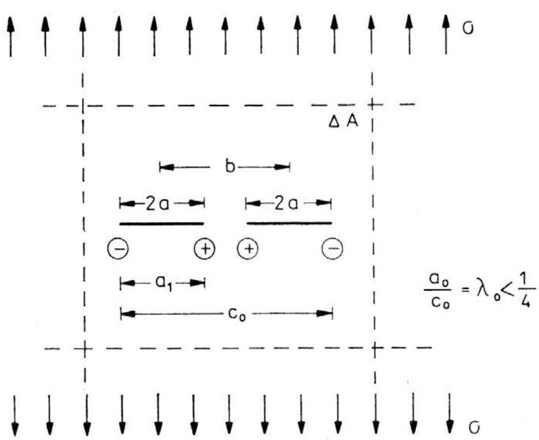

FIG. 7.

distance c_0 is fixed. We assume that these two microcracks affect each other but there is no interaction between the unit cells in question. It is intuitively obvious that the dangerous zone for such an elementary microcrack system is the surrounding of the tip denoted by the plus sign. Calculations of the stress intensity factor K for this array of the microcracks confirms that K^+ is always larger than K^-. This significantly complicates the analysis in its computational aspect since the middle points distance b varies with an increasing a. The stress intensity factor which includes intrinsically the interaction between microcracks follows to be, [15]

$$K = \sigma\sqrt{\pi a}f(\lambda), \tag{3.9}$$

with

$$f(\lambda) = \frac{b_2^2\frac{E_1(l_2)}{E_2(l_2)} - a_2^2}{a_2\sqrt{b_2^2-a_2^2}} = 1+\frac{1}{2}\lambda^2+\frac{1}{2}\lambda^3+\frac{11}{8}\lambda^4+\ldots, \quad \text{(cf.[17])}, \tag{3.10}$$

where $a_2 = \frac{1}{2}(b-a)$, $b_2 = \frac{1}{2}c_0$, $l_2 = \sqrt{1-\frac{a_2^2}{b_2^2}}$ and $E_1(l_2)$, $E_2(l_2)$ are the elliptic integrals of the first and second order, respectively.

The parameter λ is given by

$$\lambda = \frac{a}{b} = \frac{a_1}{2b} = \frac{a_1}{2(c_0-a_1)}. \tag{3.11}$$

The complementary energy density for the solid containing infinite number of such unit cells can be expressed as

$$\tilde{\pi} = \frac{\sigma^2}{2E}+2\int_0^{2a}\frac{K^{+2}(a_1)}{\Delta AE}\,da_1 = \frac{\sigma^2}{2E}+\int_0^{2a}\frac{\sigma^2\pi a_1}{\Delta AE}f^2(\lambda)\,da_1. \tag{3.12}$$

The damage law takes the following form

$$\sigma\sqrt{\pi a}f(\lambda) = K_{cu}-(K_{cu}-K_{cl})e^{-\eta\frac{\Delta a}{a_0}}. \tag{3.13}$$

This is the set of basic equations for the problem at hand. The explicit algebraic form of the obtained incremental relationship is too lengthy to be presented here. The corresponding diagram is shown in Fig. 8. Up to the point A there is a linear material behavior, then comes a slightly nonlinear (hardening) part AB — apparently due to small changes in the internal energy limited by the geometry of the microcrack array — finally the stresses drop to the softening part of the curve. This unstable microcrack growth observed as a sudden jump in the stresses BC in the strain-controlled test means physically that the microcracks have presumably joined each other forming a single microcrack which grows further on, already in the stable manner.

Consider now an infinite row of equal, collinear microcracks, Fig. 9. It is assumed that there is one microcrack in the unit cell and each unit cell influences the adjoining ones. The damage law takes the following closed form

$$K = \sigma\sqrt{b\,\mathrm{tg}\frac{\pi a}{b}} = K_{cu}-(K_{cu}-K_{cl})e^{-\eta\frac{\Delta a}{a_0}}, \tag{3.14}$$

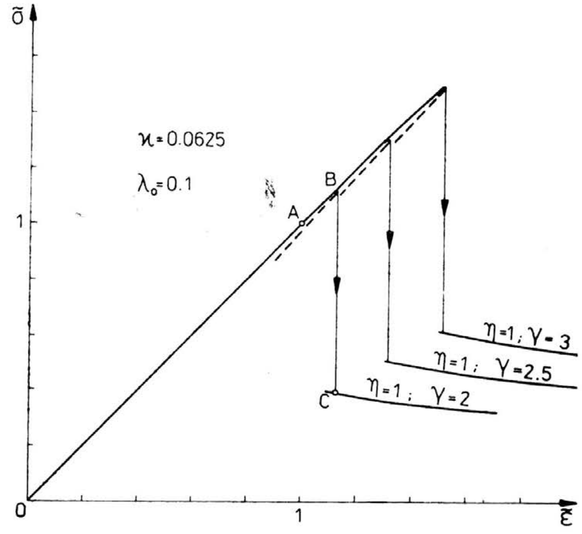

FIG. 8.

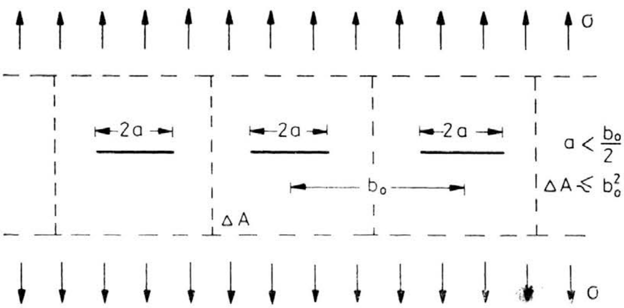

FIG. 9.

since the corresponding stress intensi.. factor K is exact in this case, [16]. The density of complementary energy contained in the microcracked body is

$$\tilde{\pi} = \frac{\sigma^2}{2E} - \frac{2\sigma^2 b^2}{E\Delta A\pi} \ln\left[\cos\left(\frac{\pi a}{b}\right)\right]. \tag{3.15}$$

The incremental formula obtained for the constitutive law reads

$$d\tilde{\varepsilon} = \left\{1 - \frac{2\varkappa}{\pi^2\lambda_0^2}\ln[\cos(\pi\lambda_0)]\right\}\left\{1 - \frac{2\varkappa}{\pi^2\lambda_0^2}\ln[\cos(\pi\lambda_0\tilde{a})] + \frac{4\varkappa}{\pi\lambda_0}\cdot\frac{\sin^2(\pi\lambda_0\tilde{a})\,[\gamma-(\gamma-1)\,e^{\eta(1-\tilde{a})}]}{\eta(\gamma-1)\,e^{\eta(1-\tilde{a})}\sin(2\pi\lambda_0\tilde{a}) - \pi\lambda_0[\gamma-(\gamma-1)\,e^{\eta(1-\tilde{a})}]}\right\}d\tilde{\sigma}, \tag{3.16}$$

while in view of (3.14) the stress is related to the microcrack length as follows

$$\sigma = [\gamma-(\gamma-1)\,e^{\eta(1-\tilde{a})}]\sqrt{\frac{\mathrm{tg}(\pi\lambda_0)}{\mathrm{tg}(\pi\lambda_0\tilde{a})}}, \tag{3.17}$$

$\varkappa$, γ, η are the known material constants defined when arriving at the formulae (3.5), (3.6), (3.8), respectively, whereas λ_0 is

(3.18)
$$\lambda_0 = \frac{a_0}{b_0} \leqslant \sqrt{\frac{\varkappa}{2\pi}}.$$

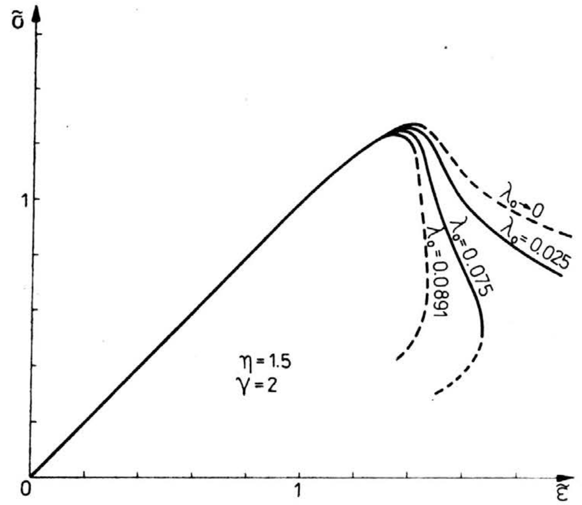

FIG. 10.

Once Eqs. (3.16) and (3.17) are established it is fairly straightforward to plot the $\tilde{\sigma}-\tilde{\varepsilon}$ curves looked for, Fig. 10. The two dashed lines bound an admissible region for the possible material behaviors which do not violate the geometrical requirements marked in Fig. 9.

3.3. Change of material density

It is known that, in general, the internal damage induces the reduction in the material density. The present constitutive model is capable to account for this phenomenon in

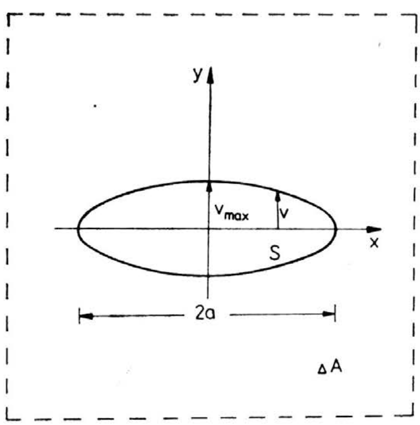

FIG. 11.

quite a simple way. What we intend to show refers to the linear theory, thus it should only be understood as a first approximation to the problem in question.

Assume that the microcracks take the elliptical form after opening in the process of loading, Fig. 11. Denote by ϱ^0, ϱ^D the density of the undamaged and damaged material, respectively. The ϱ^D over ϱ^0 ratio for the unit thickness reads

$$\varrho = \frac{\varrho^D}{\varrho^0} = \frac{\Delta A}{\Delta A + S}, \tag{3.19}$$

where S can be calculated by taking advantage of the formula for the microcrack opening displacement v_{max}, [17]

$$S = \pi a v_{max} = \frac{2\pi}{E} a^2 \sigma. \tag{3.20}$$

Therefore:

$$\tilde{\varrho} = \frac{1}{1 + \frac{2\pi}{\Delta A E} \sigma a^2}, \tag{3.21}$$

or in the dimensionless form

$$\tilde{\varrho} = \frac{1}{1 + \varkappa \chi \tilde{a}^2 \tilde{\sigma}}, \tag{3.22}$$

where $\chi = \frac{\varepsilon_0}{1+\varkappa}$ is a new parameter with ε_0 being the strain at which the first microcrack starts growing. The specific value of ε_0 is recognizable from the experimental curves, Fig. 1. Illustrative curves computed from (3.22) with help of (3.7) and (3.8) are depicted in Fig. 12, whereas $\tilde{\varrho}$ versus $\tilde{\varepsilon}$ ones are shown in Fig. 13.

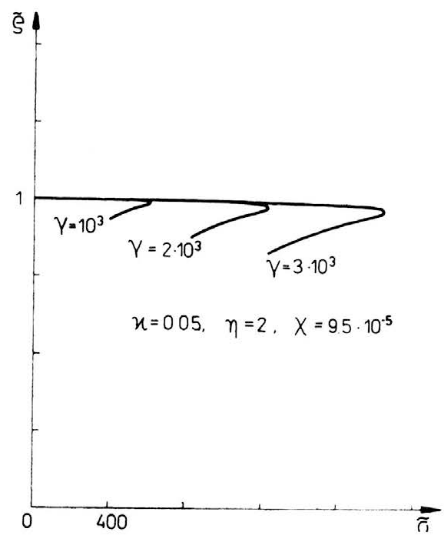

FIG. 12.

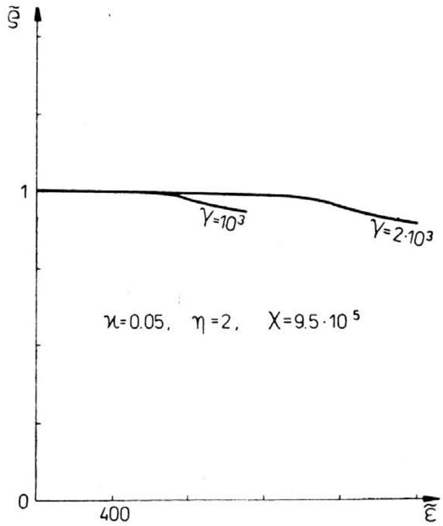

FIG. 13.

4. Summary and conclusions

In this communication a one-dimensional model has been proposed for the concrete-like materials undergoing irreversible changes of their internal structure due to progressive microcracking.

Though much attention has been paid to analyze the behavior of individual microcrack, the developed theory deals in fact with damaged continuum that contains the multitude of microcracks to be continuously smeared throughout the material volume. This has been accomplished by introducing a notion of a damaged unit cell whose behavior is considered to be characteristic of the overall behavior of the material.

The basic assumption of the present model concerns the form of the so-called damage law relating the increments in the stress $d\sigma$ to the microcrack growth da. Motivated by the microscopic observations of the growth of isolated microcrack in brittle materials, a functional dependence between the proper stress intensity factor at microcrack tip and the microcrack length has been postulated leading to an implicit relationship between the increments da and $d\sigma$.

Let us note the encouraging capability of the established analytical model based on the simple damage law in predicting all the response trends observed in the direct tensile tests for concrete.

However, it should be emphasized that we are aware of some important experimental facts which are still to be accounted for. To this end the diverse modes of the internal damage in compression as well as the occurrence of the damage-related residual strain in unloading should be mentioned.

An extended model incorporating these effects as well as its possible generalization for multiaxial stress states are the topics of current studies and will be reported in a subsequent paper.

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