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Demixing of spherical particles suspended in a laminar microduct flow(*)

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THE DEMIXTURE, i.e. the lateral migration of little spheres in a plane Hagen–Poiseuille flow, has been investigated. The characteristic dimensions of the particles have been varied around 10 μm. The depth of the duct was 100 μm. Two different optical measuring methods leading to closely corresponding results have been used. Initially homogeneously distributed particles separate into two parallel sheets at about one and three quarters of the duct depth. The particle concentrations have been determined by laser-doppler-velocimetry and by a photographical trace-measuring-method. The experimental results have been compared with theoretical calculations by Ho and LEAL [14].

Rozważono demiksturę, tj. poprzeczną migrację małych kulek w płaskim przepływie Hagena–Poiseuille'a. Charakterystyczne wymiary kulek wahały się wokół 10 μm. Zastosowano dwie różne metody pomiarów optycznych prowadzące do bardzo podobnych wyników. Kulki, które były początkowo rozmieszczone równomiernie, rozdzieliły się na dwie równoległe warstwy odległe od siebie o 7/4 głębokości przewodu. Koncentracje cząsteczek określono metodą laserowo-dopplerowską jak również metodą pomiarów fotograficznych. Wyniki doświadczeń porównano z obliczeniami teoretycznymi Ho i LEALA [14].

Рассмотрена демикстура, т.е. поперечная миграция малых шариков в плоском течении Хагена–Пуазейля. Характеристические размеры шариков колебались вокруг 10 мкм. Применены два разных метода оптических измерений, приводящие к очень аналогичным результатам. Шарики, которые вначале распределены равномерно, разделились на два параллельных слоя, удаленные друг от друга на 7/4 глубины провода. Концентрации частиц определены лазерно-доплеровским методом, как тоже методом фотографических измерений. Результаты экспериментов сравнены с теоретическими расчетами Хо и Леаля [14].

1. Introduction

THE STARTING point for the investigation of this problem was the statement of the physicians that corpuscle-free regions exist in blood flows. The basic work originates from FARAEUS and LINDQVIST (1931) [1]. A summary of the elder literature was published by TAYLOR (1955) [2].

To understand the fundamental phenomenon it seemed to be more adequate to analyse less complicated systems than blood for example. The first experimental works in hydrodynamics were published by SEGRÈ and SILBERBERG (1962) [3] for Poiseuille flows and by HALOW and WILLS (1970) [4] for Couette systems. Segrè and Silberberg examined the behaviour of a suspension flow consisting of polymethacrylate spheres in media of equal density and different viscosities in a tube.

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They found that the particle concentration in the wall- and center region decreases with increasing distance from the intake for tube Reynolds numbers between 3 and 120. The concentration reaches a maximum at $z = 0.6$ ($z = Z/R$, where Z is the distance from the center line of the tube, and R is the tube radius). Halow and Wills found maximal concentrations at $0.43 \leqslant z \leqslant 0.5$ for the Couette flow where z is the nondimensional distance from the center.

The positions of maximal particle concentration may be considered to be equilibrium positions. The quoted measurements as well as our own ones were performed at such low particle concentrations that interactions between single particles could be neglected. The works by MCMAHON (1975) [5], VADAS *et al.* (1976) [6] and KOWALEWSKI (1979) [7] show that plug flows, where the particles move in plugs in the middle of the tube with a distinct flattening of the profile and a considerable slip velocity (i.e. the difference between particle velocity and velocity of the undisturbed profile at the same position) will be clearly observed only at particle concentrations over 5%.
Large density differences between particles and fluid which cause buoyancy forces shall not be taken into consideration. Reports concerned with the influence of density differences on the equilibrium positions have been published by BAUCKHAGE (1975) [8], JEFFREY and PEARSON (1965) [9] and REPETTI and LEONHARD (1966) [10]. This report is concerned with the behaviour of the equilibrium positions of a suspension flow, made of distilled water and synthetic spheres, in a plane Poiseuille flow at very low Reynolds numbers. The process of reaching the equilibrium positions of the spheres and their exact location have been investigated. The particles of our measurements have been nearly neutrally buoyant. The particle concentration was about 0.5 ppm by volume. Diffusion may be neglected with the used particle dimensions of about 10 μm in diameter.

2. Method of particle concentration measurement

Although direct gauging of the velocity profile has been impossible because of the duct dimensions, the assumption of a plane Poiseuille flow is justified. The first reason is that the duct Reynolds numbers Re (Re $= u_m d/\nu$ where d is the depth of the duct, and ν is the kinematic viscosity of the suspension) was chosen between 19 and 23. Secondly the width to depth ratio of the square cross-section amounted to 100: 1 so that half way between the side-walls, one has a plane velocity profile. The requirement for the center velocity u_m for plane Poiseuille flows $u_m = 3\,u/2$ (u is the average velocity) has been confirmed by simultaneous measurement of the flow rate and the maximal particle velocity. The particles moved with the velocity of the undisturbed profile at the same position minus a minimal slip velocity. The slip magnitude has been quite well described by Simha's equation (2.1) which is also used for the correction of our measurements:

$$u_s = 2u_m K^2/3, \tag{2.1}$$

where u_s is the slip velocity, K is the relative particle diameter $K = a/d$, a is the particle diameter, and d is the duct depth.

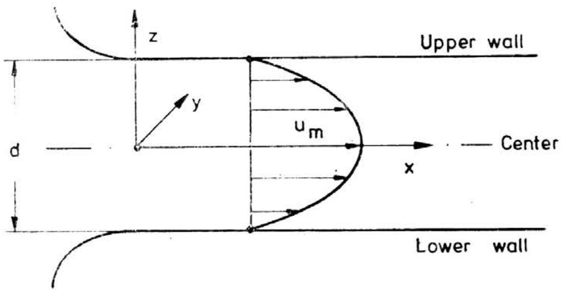

FIG. 1. Sketch of the duct and the coordinate system.

Deviations only occur for large relative particle diameters. The values of K which are used in this work are less than 0.171 and therefore guarantee the applicability of Eq. (2.1).

Knowing the existence of the Hagen–Poiseuille profile, the value of the maximal flow velocity and the velocities of the single particles, it is possible to deduce the particle position in the duct. This relationship is represented by

$$u(z) = u_m\left(1-(2z/d)^2\right) \quad \text{in} \quad -d/2 \leqslant z \leqslant d/2, \tag{2.2}$$

where $u(z)$ is the velocity at position z measured from the center of the duct. Because of the symmetry of the flow velocity profile it is acceptable to assume that also the particles are (in a statistical sense) symmetrically distributed relative to the center plane $z = 0$.

So the number of particles crossing a probe volume per unit time in the region $z \leqslant d/2$ becomes

$$N(z) = 2D\int_0^z n(z)u(z)dz \qquad (0 \leqslant z \leqslant d/2), \tag{2.3}$$

where D is the width of probe volume and $n(z)$ is the particle concentration in number per unit volume.

By differentiating $N(z)$, $n(z)$ may be obtained. For the practical evaluation the depth d of the duct was divided into 60 equidistant sections. Then, with respect to the above mentioned symmetry, the particle number N_i observed in a certain velocity region $u+\Delta u$ becomes

$$N_i = -2Dn_i\int_{z_i}^{z_i+1} u(z)dz, \qquad i = 1, 2, ..., 30. \tag{2.4}$$

Here it is assumed that the function n_i remains nearly constant within a single interval. As we are not interested in the absolute value of the density distribution, the number N_i of observed particles will be normalized by dividing it with the total number of observed particles. With this value N_i^* we get

$$n_i^* = N_i^*/2D\int_{z_i}^{z_i+1} u(z)dz, \tag{2.5}$$

n_i^* is larger than 1 when the particles become more concentrated than in the original suspension, and smaller when a dilution takes place.

3. Experimental apparatus

3.1. Flow system and duct

The flow system, which is identical for both measuring methods, consists of a suspension reservoir, the duct, and a collecting reservoir. The maximal flow velocity was adjusted by positioning the suspension reservoir at the appropriate height. Figure 2 shows the arrangement of the flow system in an explosion view.

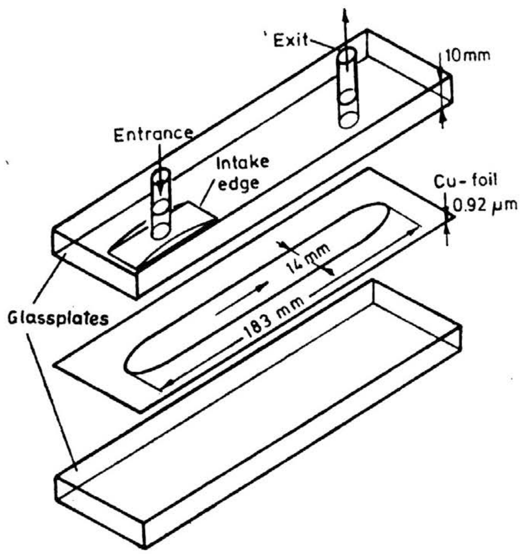

FIG. 2. Explosion view of the duct.

A thin copper foil (92 μm) is mounted between two glass plates each 10 mm thick. The suspension flows in a cut out part of the foil which forms the duct. The intake edge is smoothly rounded to avoid generation of turbulence. The glass plates are held together by a metal frame. A mixture made of destilled water and styrene/divinylbenzene or similar polyvinyltoluene spheres with diameters between 2.02 μm and 25.9 μm (15.7 μm) has been used as a suspension. Because of the low particle concentration, the viscosity of the suspension which is given by EINSTEIN [11] is nearly the same as the viscosity of pure water:

$$\mu_s = \mu(1+5V_s/2), \tag{3.1}$$

where V_s is the solid concentration per volume unit, μ is the dynamic viscosity of the fluid, and μ_s is the dynamic viscosity of the suspension.

3.2. Laser-Doppler-velocimetry

The laser-Doppler-velocimetry is an appropriate method for the considered problem because the tracer particles generally used for an indirect measuring of a flow profile here become the real objects of interest. However, it is important to have such a low particle concentration that only one particle is present in the measuring volume at a time. Otherwise evaluations of the signal may become difficult.

In Fig. 3 the path for the cross beam difference method is shown [12]. A 45° linear polarised He-Ne laser beam is divided by the Wollaston prism W 1 into two partial beams

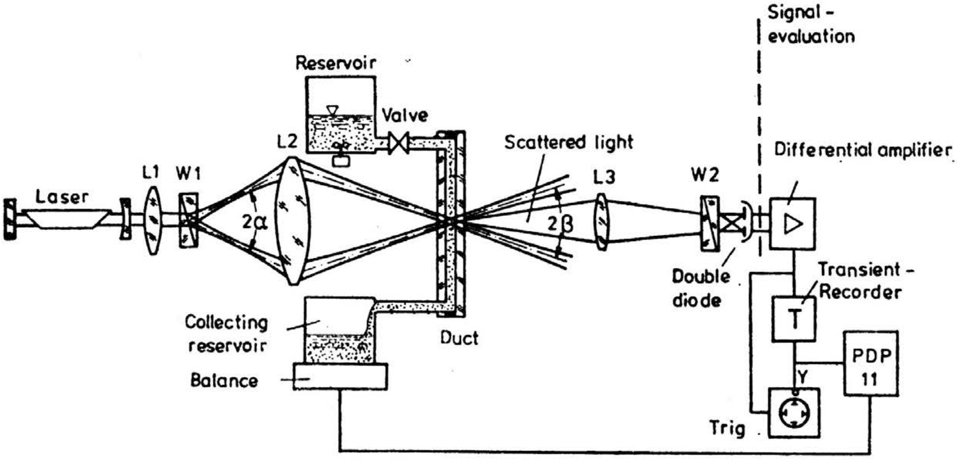

FIG. 3. Duct (in vertical position) with laser-Doppler-velocimeter in cross-beam difference technique and evaluation electronics.

of the polarisation 0° and 90°. These two beams are focussed to the measuring volume by the lens L 2. The lens is adjusted in such a way that the focal points of the laser beams coincide with the crossing point. The interference fringes which are generated in the crossing volume can be made visible only with the aid of a polarisation filter because of the orthogonal polarisation of the two laser beams. Particles, which cross the interference pattern perpendicularly, scatter light of a Doppler-frequency which is directly proportional to the particle velocity. The scattered light is focussed on the sensitive spots of a double--photodiode by the lens L 3. Between lens L 3 and the diode a second Wollaston prism W 2 is placed which divides the scattered light in two beams polarised perpendicular to each other. Therefore the same light intensity falls onto each system of the double diode. After amplifying the difference of the two diode outputs for the Doppler signals which have a 180° phase difference, a high reduction of noise is obtained. Signals of one particle are called bursts. The difference signal is amplified once more and transmitted via band-pass filter and transient recorder to further processing into a computer (PDP 11). The computer calculates the burst frequency and therefrom particle velocity and position. In addition to that the temperature and flow rate are regularly measured by computer controlled sets.

3.3. Trace-measuring-method

The principle of this method is based on the illumination of the flow with a strong light source for a precisely known time. The scattered light of the particles which pass the illuminated flow region expose lines on a film whose length is proportional to the particle velocity. Figure 4 shows the experimental apparatus.

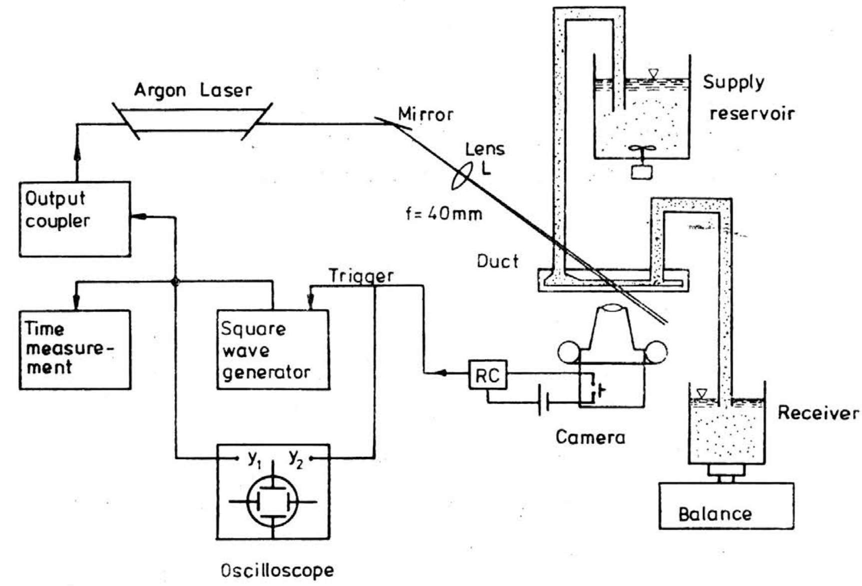

FIG. 4. Arrangement for the trace-measuring method.

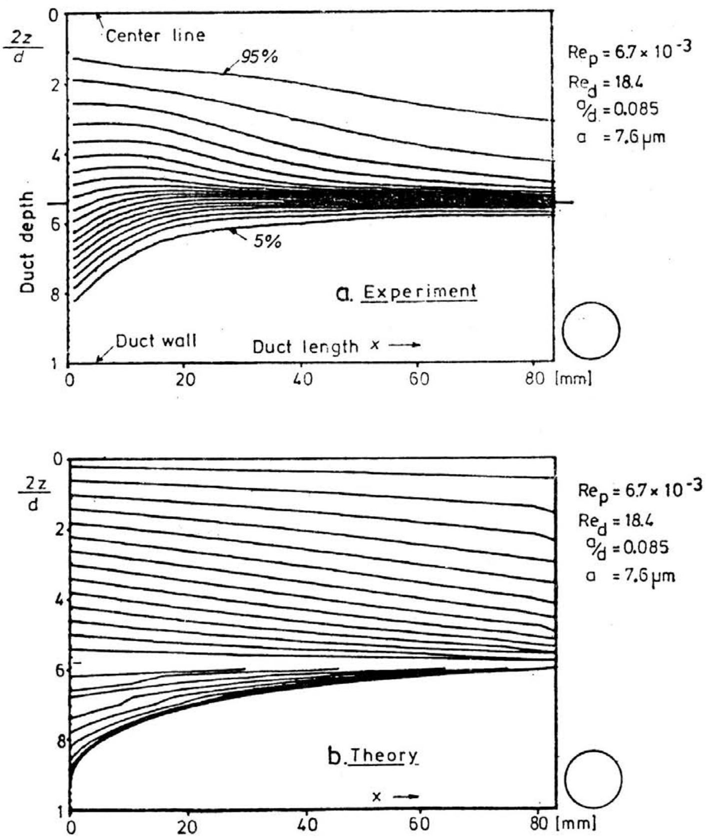

FIG. 5. Probability distributions of particle concentrations in 5% steps. The curves also represent particle--traces in a plane duct of depth d.

The flow system is completely identical to the one described in Sect. 3.1. An Argon-laser as a strong light source (wave length 514 nm) supplies the required light intensity in pulses a few milliseconds long. The lens *L* diverges the beam in such a way that the whole area which is covered by the viewing field of the film camera is illuminated. As soon as the frame stops, a synchronizing contact installed inside the camera is closed. At this moment a trigger pulse is sent into a square wave generator over a RC-filter section with very short delay time. The laser output coupler is initiated with the voltage pulse, the adjustable length of which conforms to the desired exposing time. Then the laser delivers a pulse of light of the same length. The particle velocities are determined by measuring the trace lengths taking into consideration the exposing time.

4. Theory of particle migration

The flow problem in a gap-like duct with a few neutrally buoyant particles is governed by Eqs. (4.1) and (4.2).

Navier Stokes equation:

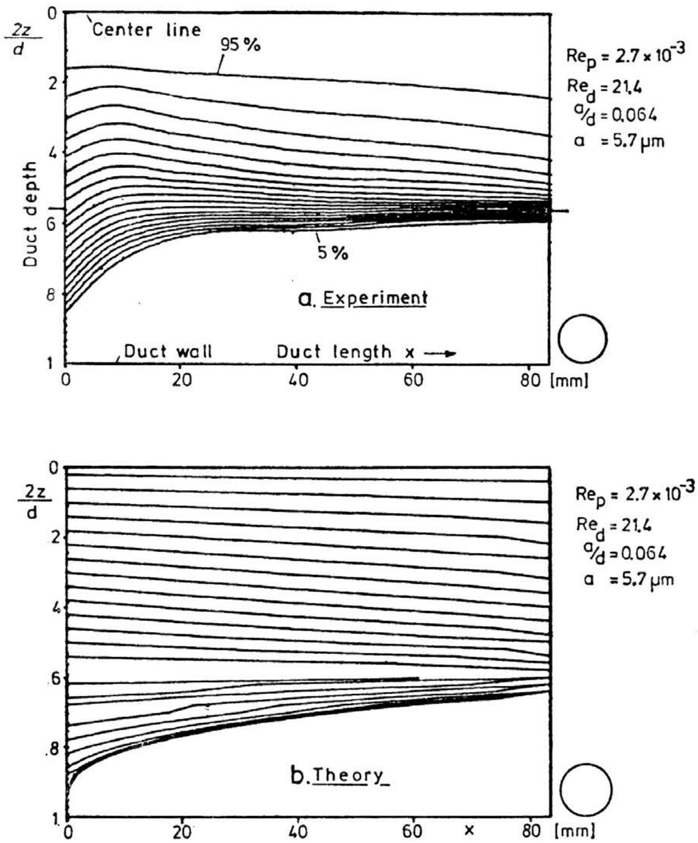

FIG. 6. Probability distributions of particle concentrations in 5% steps. The curves also represent particle-traces in a plane duct of depth *d*.

(4.1) $$\mu\nabla^2\mathbf{u}-\nabla p = \varrho\mathbf{u}\nabla\mathbf{u},$$

where p is the pressure, μ is the viscosity, and ϱ is the density.

Continuity equation for incompressible fluids:

(4.2) $$\nabla\mathbf{u} = 0.$$

The boundary conditions result from the demand that the fluid should adhere not only to the walls but also to the particles.

In a report by BRETHERTON (1961) [13] it has already been shown that no lateral force can be achieved without inertial terms. With the presumption of very low Reynolds numbers ($\mathrm{Re}_d \to 0$) and very small relative particle diameters ($K \to 0$) Ho and LEAL (1974) [14] succeeded for the first time to find a fairly explicit solution of the problem taking the inertial terms into account.

The particle velocity in flow direction u_x (4.2), which results from the Stokes equation (without inertial terms) for low Reynolds numbers, is an expansion in K.

(4.3) $$u_x = u_m\left(1-(2z/d)^2\right)-4u_m K^2/3-40u_m K^3 K_d\left(1-z/dV9+0(K^4)\right),$$

where K_d is the functional which depends on z (Ho and Leal).

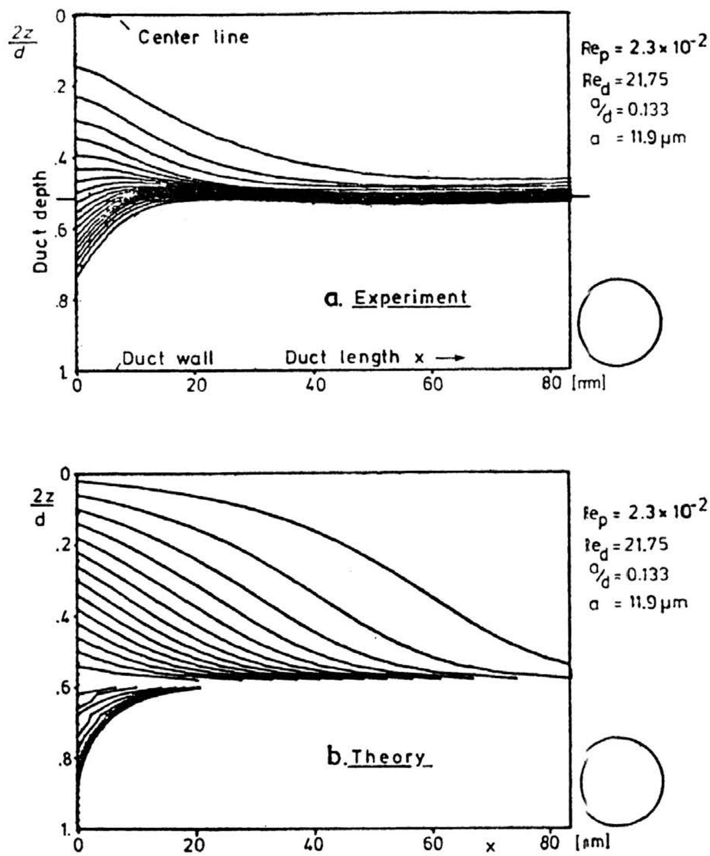

FIG . 7. Probability distributions of particle concentrations in 5% steps. Th curves also represent particle -traces in a plane duct of depth d.

Taking the inertial terms into account, a lateral force F_1 (4.4) perpendicular to the undisturbed flow can be calculated in a first approximation:

$$F_1 = \varrho u_m^2 a^2 K^2 G(z), \tag{4.4}$$

where a is the particle diameter, and $G(z)$ is the functional which depends on z (details in [14]).

It is possible to calculate particle trajectories from Eq. (4.4). This has been done for 20 different particle positions downstream of the intake edge using the parameters of our measurements. In Figs. 5b to 9b the results for five different particle diameters are shown. The particle sizes on the right hand side of the diagrams are drawn true to scale. The equilibrium position is always at $z = 0.6$ independently of the particle diameter.

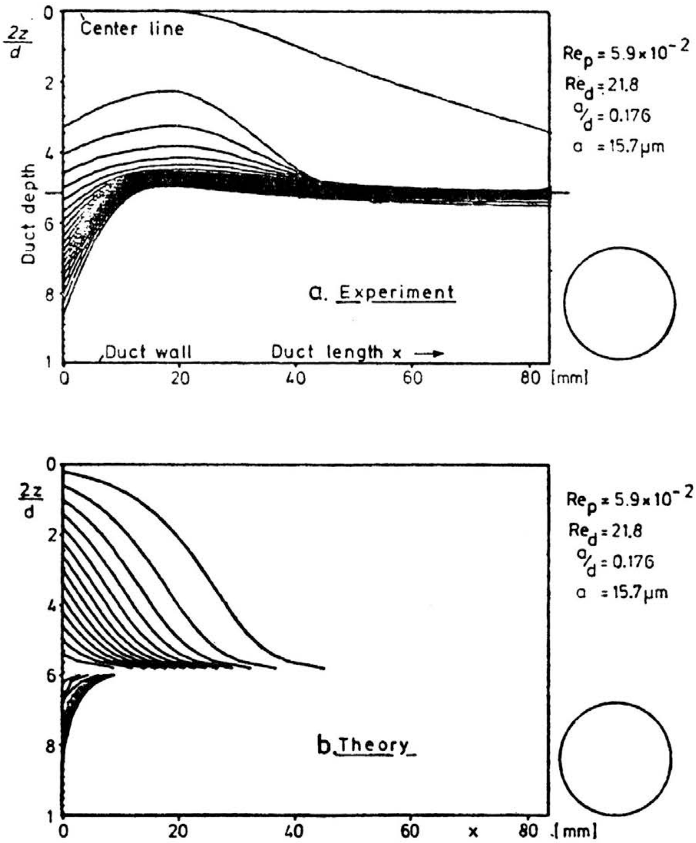

FIG. 8. Probability distributions of particle concentrations in 5% steps. The curves also represent particle-traces in a plane duct of depth d.

5. Experimental results

5.1. Distributions of particle concentrations

The main parameters of our experiments are the duct Reynolds number Re_d and the particle diameter a. Up to ten individual measurements at different distances x from the

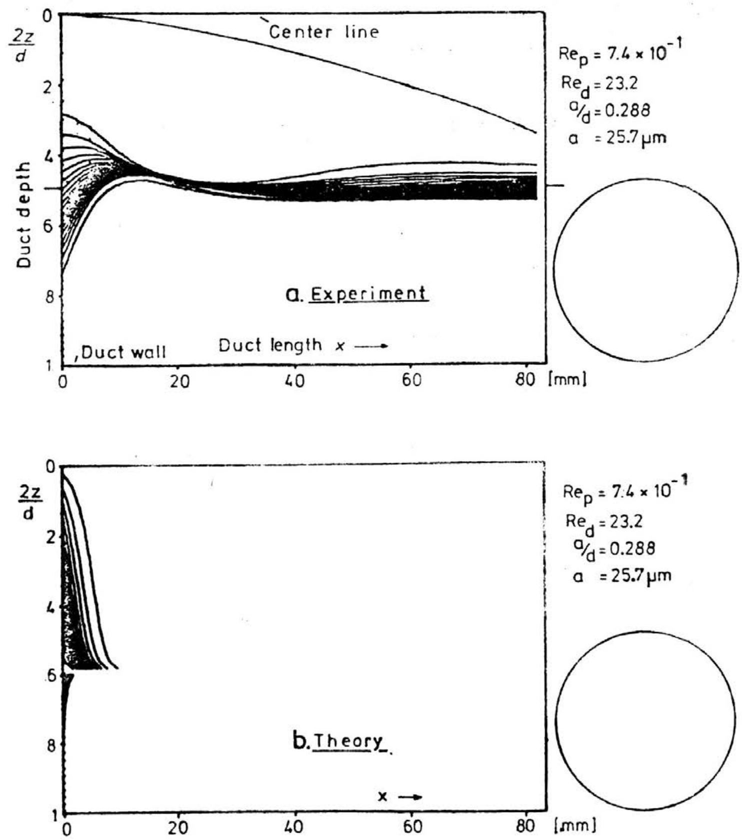

FIG. 9. Probability distributions of particle concentrations in 5% steps. The curves also represent particle--traces in a plane duct of depth d.

intake edge have been carried out for each of the five particle diameters. The maximum velocity and the duct Reynolds numbers for a series of measurements have been kept nearly constant. In addition, the particle Reynolds number Re is shown in the following diagrams. ($Re_p = u_p a/\nu$, where u_p is the slip velocity, and ν is the kinematic viscosity.)

In Figs. 5a to 9a the measured concentration distributions for each particle diameter are shown. In the x-direction the flow length is marked in mm, in the z-direction the distance from the duct center which has been standardized by the duct depth. Up to the first line 5% of all particles have been found. Each of the following lines signifies a further increment of 5%. The individual measurements have been interpolated by cubical spline functions.

It is striking that the particles are more and more concentrated in distinct equilibrium positions with increasing distance from the entering edge; with growing diameter they do it increasingly faster. Concentrations up to the thirteenfold of the normalized value have been reached in individual intervals. A further remarkable fact is that the particles coming from the wall area have reached the equilibrium positions faster than those of the center region.

Figures 5b to 9b may be correspondingly compared with Figures 5a to 9a. The small particles especially show up in correspondence. The deviation of the behaviour of the larger particles shows yet more clearly that the theory is only applicable for the marginal case $\mathrm{Re}_p \to 0$ and $K \to 0$.

5.2. Equilibrium positions

The dependence of the equilibrium position from Re_p, which is closer to the duct center with increasing particle diameter, is shown in Fig. 10. The results of the laser-Doppler-velocimetry and the trace-measuring-method correspond well.

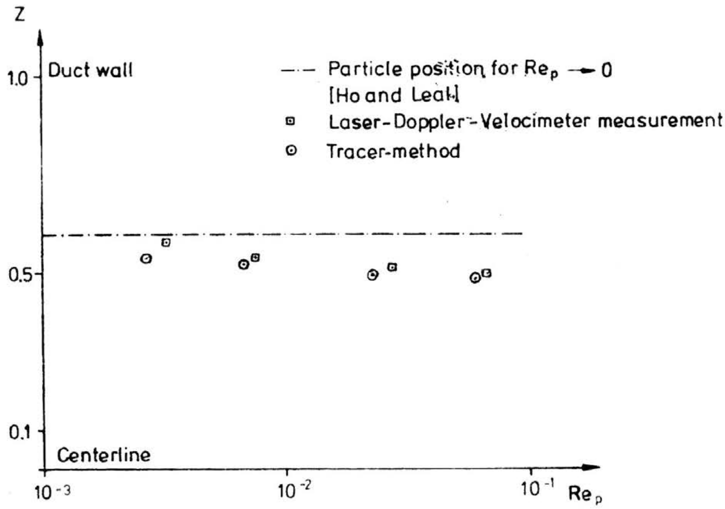

FIG. 10. Equilibrium-position as a function of particle Reynolds-number.

The systematical deviation of about 5% can be explained by different methods of the determination of the maximal velocity. The experimental data for decreasing Re approaches the theoretical limit for $\mathrm{Re}_p \to 0$ and $K \to 0$ at $z = 0.6$.

The displacement of the equilibrium positions of the larger particles toward the duct center may be explained with a simple geometrical argument. When the particle diamater is made 4/10 of the duct depth, the sphere would touch the wall, if it was situated at the theoretically calculated equilibrium position for very small particles. With further increase of the diameter the center point of the sphere would be shifted towards the centre of the duct with simultaneous contact to the wall. The extreme case would be the sphere diameter being equal to the depth of the duct, hence the equilibrium position would have to be at the center of the duct. Since the equilibrium position is the result of interactions between the wall, particle and velocity profile, the displacement already begins at quite small sphere diameters.

6. Summary

Using two independent measuring methods, the velocity distributions of small neutrally buoyant spherical particles dispersed in distilled water have been measured in a plane Poiseuille flow depending on the particle diameter and flow path length. From these velocity distributions, concentration distributions have been calculated. These have been compared with the theoretical results by Ho and Leal. The small particles (2.02 μm–7.6 μm) have been found to correspond well to their theory. Deviations with larger particles may be explained such that the assumptions $\mathrm{Re} \to 0$ and $K \to 0$ neccessary for the theory can only be fulfilled unperfectly.

The equilibrium positions determined from the concentration distributions are at about $2\ z/d = 0.5$, i.e. in the middle between the wall and the center of the duct, for the larger particles while the theoretically calculated value of $2\ z/d = 0.6$ for the smaller particles has been approximated.

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