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ON THE SIGNIFICATION OF AN ELEMENTARY FORMULA OF SOLID GEOMETRY.

[From the *Philosophical Magazine*, vol. XXX. (1865), pp. 413, 414.]

THE expression for the perpendicular distance of a point (x, y, z) from a line through the origin inclined at the angles (α, β, γ) to the three axes respectively, is

$$\begin{aligned} p^2 = x^2 + y^2 + z^2 - (x\cos\alpha + y\cos\beta + z\cos\gamma)^2 \\ = \quad (y\cos\gamma - z\cos\beta)^2 \\ + (z\cos\alpha - x\cos\gamma)^2 \\ + (x\cos\beta - y\cos\alpha)^2; \end{aligned}$$

and the remark in reference to it is that, if at the given point P we draw, perpendicular to the plane through P and the given line, a distance PK equal to the distance of P from the given line, then the expressions

$$y\cos\gamma - z\cos\beta, \quad z\cos\alpha - x\cos\gamma, \quad x\cos\beta - y\cos\alpha,$$

which enter into the preceding formula, denote respectively the coordinates of the point K referred to P as origin.

If the given line instead of passing through the origin pass through the point x_0, y_0, z_0, then the corresponding expressions are of course

$$(y-y_0)\cos\gamma - (z-z_0)\cos\beta, \quad (z-z_0)\cos\alpha - (x-x_0)\cos\gamma, \quad (x-x_0)\cos\beta - (y-y_0)\cos\gamma,$$

and if we denote the "six coordinates" of the given line, viz.

$$\cos\alpha, \quad \cos\beta, \quad \cos\gamma, \quad y_0\cos\gamma - z_0\cos\beta, \quad z_0\cos\alpha - x_0\cos\gamma, \quad x_0\cos\beta - y_0\cos\gamma,$$

by

$$a\;, \qquad b\;, \qquad c\;, \qquad f \qquad , \qquad g \qquad , \qquad h$$

respectively (so that $af+bg+ch=0$), then the three expressions become

$$cy-bz-f\,,\quad az-cx-g,\quad bx-ay-h$$

respectively.

It is moreover clear that if the point P be moved to P' by an infinitesimal rotation ω about the given line, then P' lies on the line PK at a distance PP', $=\omega PK$, from the point P, and the displacements of P in the directions of the axes are consequently equal to

$$\omega\,(cy-bz-f),\quad \omega\,(az-cx-g),\quad \omega\,(bx-ay-h)$$

respectively, which is a fundamental formula in the theory of the infinitesimal rotations of a solid body.

Cambridge, October 26, 1865.