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NOTE ON THE PROJECTION OF THE ELLIPSOID.

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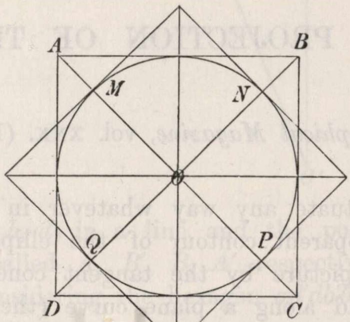
Consider an ellipsoid, situate any way whatever in regard to the eye and the plane of the picture; the apparent contour of the ellipsoid is an ellipse, the intersection of the plane of the picture by the tangent cone having the eye for vertex; this cone touches the ellipsoid along a plane curve (the intersection of the ellipsoid by the polar plane of the eye), which may be called the contour section; and the apparent contour is thus the projection of the contour section. Consider any other plane section; the projection thereof has double contact (real or imaginary) with the projection of the contour section: the common tangents are the intersections with the plane of the picture of the tangent planes of the tangent cone which pass through the pole of the section; or, what is the same thing, they are the tangents to the projection of the contour section, or to the projection of the section, from the point which is the projection of the pole of the section. The projection of the pole lies in the line which is the projection of the diameter conjugate to the plane of the section; and in particular, if the section is central, that is, if the plane thereof passes through the centre of the ellipsoid, then the pole is the point at infinity on the conjugate diameter; whence also if the eye be at an infinite distance, so that the projection is a projection by parallel rays, then the projection of the pole is the point at infinity on the projection of the conjugate diameter; and therefore the common tangents of the projections of the section and the contour section are in this case parallel to the projection of the diameter conjugate to the plane of the section.

Suppose that the plane of the picture is parallel to a principal plane of the ellipsoid, and that the projection is by parallel rays; then if OA, OB, OC are the projections of the semiaxes (OA, OC will be at right angles to each other if the plane parallel to the plane of the picture is that of xz), the projections of the principal sections are the ellipses having for conjugate semidiameters OB, OC; OC, OA; OA, OB

respectively. Hence to the ellipse OB, OC drawing the two tangents which are parallel to OA, to the ellipse OC, OA the two tangents which are parallel to OB, and to the ellipse OA, OB the two tangents which are parallel to OC, we have on each of these ellipses the two points which are the points of contact therewith of the ellipse which is the projection of the contour section, or apparent contour of the ellipsoid; that is, we know six points, and at each of these points the tangent, of the last-mentioned ellipse; and the ellipse in question, or apparent contour of the ellipsoid, can thus be traced by hand accurately enough for ordinary purposes.

In connexion with what precedes, I may notice a convenient construction for the projection of a circle. Suppose that we have given the projection of the circumscribed square $ABCD$; then if we know the projection of one of the points M, N, P, Q, say of the point M, the projections of all the points and lines of the figure can be obtained



graphically by the ruler only with the utmost facility; that is, in the ellipse which is the projection of the circle we have *eight* points, and the tangent at each of them, and the ellipse may then be drawn by hand. And to find the projection of the point M, it is only necessary to remark that in the figure the anharmonic ratio $\dfrac{AM \,.\, OC}{AC \,.\, MO}$ of the points A, M, O, C is $= \frac{1}{2}(\sqrt{2}-1)$; hence the corresponding anharmonic ratio of the projections of the four points is also $= \frac{1}{2}(\sqrt{2}-1)$; and the projections of A, B, C, D, and consequently those of A, C, O, being known, the projection of M is thus also known.

Cambridge, June 15, 1865.