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ON RUDIO'S INVERSE CENTRO-SURFACE.

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DR F. RUDIO, in an inaugural dissertation "Ueber diejenigen Flächen deren Krümmungsmittelpunktsflächen confokale Flächen zweiten Grades sind," Berlin, 1880, and *Crelle's Journal*, t. XCV., p. 240, has determined the surfaces having for their centro-surface (i.e., the locus of centres of curvature) the aggregate of the confocal quadric surfaces

$$\frac{x^2}{a-\lambda}+\frac{y^2}{b-\lambda}+\frac{z^2}{c-\lambda}=1,$$

$$\frac{x^2}{a-\mu}+\frac{y^2}{b-\mu}+\frac{z^2}{c-\mu}=1,$$

or, what is the same thing, the surfaces orthotomic to the common tangents of these two surfaces. He obtains, as the final result of an elegant analytical investigation, the following formulæ:

$$x=\sqrt{(a-\lambda)}\sqrt{\left(\frac{a-u\,.\,a-v}{a-b\,.\,a-c}\right)},$$

$$y=\sqrt{(b-\lambda)}\sqrt{\left(\frac{b-u\,.\,b-v}{b-c\,.\,b-a}\right)},$$

$$z=\sqrt{(c-\lambda)}\sqrt{\left(\frac{c-u\,.\,c-v}{c-a\,.\,c-b}\right)},$$

$$U=\sqrt{\left(\frac{a-u\,.\,b-u\,.\,c-u}{\lambda-u\,.\,\mu-u}\right)},$$

$$V=\sqrt{\left(\frac{a-v\,.\,b-v\,.\,c-v}{\lambda-v\,.\,\mu-v}\right)},$$

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$$\xi \frac{U-V}{x} = 1 - \frac{b-\lambda \,.\, c-\lambda}{\lambda - u \,.\, \lambda - v} - \frac{a-\mu}{a-u \,.\, a-v} UV,$$

$$\eta \frac{U-V}{y} = 1 - \frac{c-\lambda \,.\, a-\lambda}{\lambda - u \,.\, \lambda - v} - \frac{b-\mu}{b-u \,.\, b-v} UV,$$

$$\zeta \frac{U-V}{z} = 1 - \frac{a-\lambda \,.\, b-\lambda}{\lambda - u \,.\, \lambda - v} - \frac{c-\mu}{c-u \,.\, c-v} UV,$$

(values which are such that $\xi^2 + \eta^2 + \zeta^2 = 1$),

$$\rho = \tfrac{1}{2}\left(\int \frac{du}{U} + \int \frac{dv}{V}\right) + C.$$

And then

$$x' = x + \rho\xi, \quad y' = y + \rho\eta, \quad z' = z + \rho\zeta,$$

viz. the equations give x, y, z, U, V, ξ, η, ζ, each of them as a function of two independent parameters u, v; ρ is a function of u, v and of the arbitrary constant C; hence, giving to C any assumed value, we have x', y', z' each of them a function of the two arbitrary parameters u, v; that is, x', y', z' are the coordinates of a point on a surface, one of a system of parallel surfaces (corresponding to the different values of C) which are the surfaces in question.

Observe that u, v are the elliptic coordinates of the point (x, y, z) on the first of the two confocal surfaces, and that ξ, η, ζ are the cosine-inclinations of one of the tangents from this point to the other confocal surface; so that, if ρ were left arbitrary, the equations $x' = x + \rho\xi$, $y' = y + \rho\eta$, $z' = z + \rho\zeta$ would be the equations of a common tangent of the two confocal surfaces; but ρ, as determined, is the distance of the point x, y, z from the point x', y', z' on the required surface. The expression for ρ involves hyper-elliptic integrals of the first species, which are the same as those which present themselves in the determination of the geodesic lines upon either of the two confocal surfaces.

I have, in quoting these remarkable results, written for greater simplicity a, b, c instead of the author's a^2, b^2, c^2.

Cambridge, Dec. 20, 1886.

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