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ANALYTICAL GEOMETRICAL NOTE ON THE CONIC.

[From the *Messenger of Mathematics*, vol. XV. (1886), p. 192.]

TAKE (X, Y, Z) the coordinates of a point on the conic $yz + zx + xy = 0$, so that $YZ + ZX + XY = 0$; clearly (Y, Z, X) and (Z, X, Y) are the coordinates of two other points on the same conic; I say that the three points are the vertices of a triangle circumscribed about the conic

$$x^2 + y^2 + z^2 - 2yz - 2zx - 2xy = 0.$$

In fact, the equation of one of the sides is

$$\begin{vmatrix} x, & y, & z \\ X, & Y, & Z \\ Y, & Z, & X \end{vmatrix} = 0,$$

say this is $AX + BY + CZ = 0$, where $A, B, C = XY - Z^2, YZ - X^2, XZ - Y^2$; and the condition in order that this side may touch the conic

$$x^2 + y^2 + z^2 - 2yz - 2zx - 2xy = 0$$

is

$$BC + CA + AB = 0.$$

But we have

$$\begin{aligned} BC + CA + AB = Y^2Z^2 + Z^2X^2 + X^2Y^2 - X(Y^3 + Z^3) - Y(Z^3 + X^3) - Z(X^3 + Y^3) \\ + X^2YZ + XY^2Z + XYZ^2 \\ = (YZ + ZX + XY)(-X^2 - Y^2 - Z^2 + YZ + ZX + XY) = 0; \end{aligned}$$

and similarly for the other two sides. The point (X, Y, Z) is an arbitrary point on the conic $yz + zx + xy = 0$; and we thus see that we have a singly infinite series of triangles each inscribed in this conic and circumscribed about the conic

$$x^2 + y^2 + z^2 - 2yz - 2zx - 2xy = 0.$$