841.

ON A DIFFERENTIAL OPERATOR.

[From the *Messenger of Mathematics*, vol. XIV. (1885), pp. 190, 191.]

WRITE $X = 1 + bx + cx^2 + \ldots, = (1-\alpha x)(1-\beta x)(1-\gamma x)\ldots$; then by Capt. MacMahon's theorem, any non-unitary function of the roots $\alpha, \beta, \gamma, \ldots$ is reduced to zero by the operation

$$\Delta, = \partial_b + b\partial_c + c\partial_d + \ldots ;$$

for instance, if

$$(2), = \Sigma\alpha^2 = b^2 - 2c,$$

we have

$$\Delta(b^2 - 2c) = 2b + b(-2), = 0.$$

We have

$$\Delta X = x + bx^2 + cx^3 + \ldots = xX;$$

and writing, moreover, $X', = b + 2cx + 3dx^2 + \&c.$, for the derived function of X, then

$$\Delta X' = 1 + 2bx + 3cx^2 + \ldots = (xX)'.$$

We can hence shew that $\Delta\left(\frac{X'}{X} - b\right) = 0$; the value is, in fact,

$$\frac{\Delta X'}{X} - \frac{X'\Delta X}{X^2} - \Delta b, \text{ that is, } \frac{(xX)'}{X} - \frac{X'xX}{X^2} - 1,$$

which is

$$= \frac{X + xX'}{X} - \frac{xX'}{X} - 1, = 0.$$

This is right, for $\frac{X'}{X}$ is a sum of non-unitary symmetric functions of the roots; in fact,

$$\frac{X'}{X} = \Sigma\frac{-\alpha}{1-\alpha x} = -(1) - (2)x - (3)x^2 - \&c.,$$

or since $b = -(1)$, this is

$$\frac{X'}{X} - b = -(2)x - (3)x^2 - \&c.,$$

a sum of non-unitary functions of the roots.