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A note on the influence of energy dissipation on the propagation of elastic-plastic waves

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THE influence of the energy dissipation in an adiabatic "loading-unloading" process on the phenomenon of propagation of second order spatial waves in an elastic-plastic medium at the assumption of small strains is examined. It was demonstrated that in the adiabatic process the velocities of the plastic waves and unloading waves are not greater than the corresponding velocities in the isothermic process under assumption that in the adiabatic process the yield limit is a decreasing function of temperature. As an example illustrating the problem considered the analysis of longitudinal and transverse simple waves in an infinite thin-walled tube made of mild steel is presented. It was shown that the energy dissipation taken into account influences not only on the propagation velocities of simple waves but also involves the change in the wave profile.

W pracy zabadano wpływ dysypacji energii w adiabatycznym procesie "obciążenie-odciążenie" na przebieg zjawiska propagacji przestrzennych fal drugiego rzędu w przypadku małych odkształceń ośrodka sprężysto-plastycznego. Wykazano, że w procesie adiabatycznym prędkości fal plastycznych i odciążenia nie są większe od odpowiednich prędkości w procesie izotermicznym przy założeniu, że w procesie adiabatycznym granica płynięcia jest malejącą funkcją temperatury. W charakterze przykładu dokonano analizy podłużno-poprzecznych fal prostych w półnieskończonej, cienkościennej rurze stalowej. Okazało się, że uwzględnienie dysypacji energii prowadzi nie tylko do zmiany prędkości rozprzestrzeniania się fal prostych, lecz także wywołuje zmianę profilu fali.

В работе исследовано влияние диссипации энергии в адиабатическом процессе „нагрузка-разгрузка" на ход явления распространения пространственных волн второго порядка к случае малых деформаций упруго-пластической среды. Показано, что в адиабатическом процессе скорости пластических волн и волн разгрузки не больше чем соответствующие скорости в изотермическом процессе при предположении, что в адиабатическом процессе предел текучести является убывающей функцией температуры. В характере примера проведен анализ продольно-поперечных простых волн в полубесконечной, тонкостенной стальной трубе. Оказалось, что учет диссипации энергии ведет не только к изменению скорости распространения простых волн, но также вызывает изменение профиля волны.

1. Introduction

THE paper is devoted to the comparative study of the second-order isothermal and adiabatic waves of infinitesimal strain in an elastic-plastic medium, under moderate pressure of order of the usual yield limit. Such studies are of practical interest, since the adiabatic waves (i.e., waves in non-conductors) and isothermal waves are two extreme idealizations of the actual wave process generated by mechanical impact at the surface of an elastic-plastic body in routine experiments.

The simplified equations for non-conductors derived in the paper [1] are used. They are presented in Sect. 2. In Sect. 3 it is shown that the speeds of adiabatic plastic waves and adiabatic unloading waves are smaller than the corresponding speeds of isothermal plastic and unloading waves, provided that the yield limit decreases with the temperature.

The results of the numerical example concerning the adiabatic and isothermal simple waves of combined stress in a thin-walled tube made of mild steel are presented in Sect. 4. In this section the influence of the energy dissipation on wave speeds and wave profile is also discussed.

Notation and abbreviations

$$\mathbf{A}\cdot\mathbf{B} \leftrightarrow A_i B_i \quad \text{or} \quad A_{ij}B_{ij},$$

$$\mathbf{A}\otimes\mathbf{B} \leftrightarrow A_j B_i \quad \text{or} \quad A_{ij}B_{mn}$$

$$\mathbf{A}\mathbf{B} \leftrightarrow A_{ij} B_j \quad \text{or} \quad A_{ijkl}B_{kl}$$

1 metric tensor.

2. Simplified equations for the elastic-plastic non-conductors

To investigate the influence of the energy dissipation on the propagation of three-dimensional elastic-plastic waves we use the simplified equations derived in paper [1]. Those simplified equations are obtained from the general one [2] by disregarding both the change of yield limit with temperature increment caused by the piezocaloric effect, and the thermal expansion resulting from energy dissipation. As it was shown in the paper [1], these two effects at moderate pressure of the order of usual yield limit are of much lesser significance than the change of yield limit with a temperature increment caused by energy dissipation.

For elastic-plastic non-conductors the simplified equations may be written as follows

$$\dot{\boldsymbol{\epsilon}} = \begin{cases} \mathbf{L}^{(p)}\dot{\boldsymbol{\sigma}} & \text{if} \quad f(\boldsymbol{\sigma}, \boldsymbol{\varkappa}, \varkappa_1, \theta) = 0, \ \psi \geqslant 0, \\ \mathbf{L}\dot{\boldsymbol{\sigma}} & \text{if} \quad f < 0 \quad \text{or} \quad f = 0, \ \psi < 0, \end{cases} \tag{2.1}$$

$$\dot{\boldsymbol{\varkappa}} = \begin{cases} \dfrac{1}{h_1}\psi \mathbf{d}^*(\boldsymbol{\varkappa}, \varkappa_1, \theta) & \text{if} \quad f = 0, \ \psi \geqslant 0, \\ \mathbf{0} \quad \text{if} \quad f < 0 & \text{or}\ f = 0, \ \psi < 0, \end{cases} \tag{2.2}$$

$$\dot{\varkappa}_1 = \begin{cases} \dfrac{1}{h_1}\psi d_1^*(\boldsymbol{\varkappa}, \varkappa_1, \theta) & \text{if} \quad f = 0, \ \psi \geqslant 0, \\ 0 \quad \text{if} \quad f < 0 & \text{or}\ f = 0, \ \psi < 0, \end{cases} \tag{2.3}$$

where

$$\mathbf{L}^{(p)} = \mathbf{L} + \frac{1}{h_1}\mathbf{f}_\sigma \otimes \mathbf{f}_\sigma,$$

$$h = -\frac{\partial f}{\partial \boldsymbol{\varkappa}}\cdot \mathbf{d}^* - \frac{\partial f}{\partial \varkappa_1} d_1^* > 0, \quad h_1 = h - m f_\theta > 0,$$

$$m = \frac{1}{\varrho c_\sigma}\left(\mathbf{f}_\sigma \cdot \boldsymbol{\sigma} - \varrho \frac{\partial u^{(s)}}{\partial \boldsymbol{\varkappa}} \cdot \mathbf{d}^* - \varrho \frac{\partial u^{(s)}}{\partial \varkappa_1} d_1^*\right), \quad \psi = \mathbf{f}_\sigma \cdot \dot{\boldsymbol{\sigma}}, \tag{2.4}$$

$$\mathbf{f}_\sigma = \frac{\partial f}{\partial \boldsymbol{\sigma}}, \quad f_\theta = \frac{\partial f}{\partial \theta}.$$

Here $\dot{\boldsymbol{\sigma}}$ and $\dot{\boldsymbol{\epsilon}}$ are the stress rate tensor and the infinitesimal strain rate tensor respectively, $\mathbf{L}$ is the isothermal elastic compliance tensor, f denotes the yield function, ψ is the rate of loading, $\varkappa_{ij} = \varkappa_{ji}$ and $\varkappa_1$ are hardening parameters, h_1 and h are adiabatic and isothermal hardening functions respectively, ϱ is the material density, c_σ denotes the specific heat at constant $\boldsymbol{\sigma}, \boldsymbol{\varkappa}, \varkappa_1, \theta = T - T_0$, T is the thermodynamic temperature, T_0 is the temperature at the reference state where $\boldsymbol{\varkappa}, \varkappa_1, \boldsymbol{\sigma}$ are assumed to vanish, and

$$u^{(s)} = u^{(s)}(\boldsymbol{\varkappa}, \varkappa_1, \theta)$$

is the energy stored during plastic deformation per unit mass. The functions $\mathbf{d}^*$ and d_1^* occurring in the formal definitions (2.2), (2.3) of hardening parameters are assumed to be continuous, whereas the functions $u^{(s)}$ and f are assumed to be continuous and to possess the continuous first derivatives.

Note that the simplified adiabatic flow rules (2.1) differ from the isothermal ones only insofar as the isothermal hardening function h is replaced by the adiabatic one, h_1.

The temperature of elastic-plastic non-conductors may be calculated using the following simplified equation:

$$\dot{\theta} = \begin{cases} \dfrac{m}{h_1}\psi & \text{if} \quad f = 0,\ \psi \geqslant 0, \\ 0 \quad \text{if} \quad f < 0 & \text{or} \quad f = 0,\ \psi < 0. \end{cases} \tag{2.5}$$

The physical meaning of the factor m occurring in Eqs. (2.4), (2.5) may be perceived by considering the heat output during the isothermal cyclic process (in $\boldsymbol{\sigma}$-space) of loading-unloading of a macroelement. From such consideration it follows that [2]

$$m \geqslant 0 \tag{2.6}$$

and therefore, if the yield limit decreases with temperature ($f_\theta > 0$), then $h_1 \leqslant h$.

For the case of the Huber-Mises yield condition

$$f = \sigma_{(i)} - Y(\varkappa_1, \theta) = 0 \tag{2.7}$$

under the additional assumptions

$$d_1^* = Y(\varkappa_1, \theta), \quad \mathbf{d}^* = \mathbf{0}; \quad \varrho u^{(s)} = \pi_1 \varkappa_1, \quad \pi_1 = \text{const} \tag{2.8}$$

the quantities occurring in Eqs. (2.4) take the form

$$\begin{gathered} \mathbf{L}^{(p)} = \mathbf{L} + \frac{9}{4h_1\sigma_{(i)}^2}\mathbf{S}\otimes\mathbf{S}, \\ h = \frac{\partial Y}{\partial \varkappa_1}\sigma_{(i)}, \quad m = \frac{\sigma_{(i)}}{\varrho c_\sigma}(1-\pi_1), \\ h_1 = \left(\frac{\partial Y}{\partial \varkappa_1} + \frac{1-\pi_1}{\varrho c_\sigma}\frac{\partial Y}{\partial \theta}\right)\sigma_{(i)} \end{gathered} \tag{2.9}$$

and Eqs. (2.3), (2.5) become

$$\varkappa_1 = \mathbf{S} \cdot \dot{\boldsymbol{\epsilon}}^p, \quad \theta = \theta_0 + \frac{1-\pi_1}{\varrho c_\sigma}\varkappa_1. \tag{2.10}$$

Here $\mathbf{S} = \boldsymbol{\sigma} - \frac{1}{3}\mathbf{1}\operatorname{tr}\boldsymbol{\sigma}$, $\sigma_{(i)} = \left(\frac{3}{2}\mathbf{S}\cdot\mathbf{S}\right)^{1/2}$, Y is the yield limit in a simple tension, $\dot{\boldsymbol{\epsilon}}^p$ denotes the rate of plastic strain and θ_0 is the initial temperature.

By solving the set of equations $(2.10)_2$ and (2.7) with respect to $\varkappa_1$ and θ, and by substituting the obtained results into h_1, the adiabatic hardening function may be expressed in terms of $\sigma_{(i)}$,

$$h_1 = h_1(\sigma_{(i)}). \tag{2.11}$$

The above function is different from the corresponding function $h = h(\sigma_{(i)})$ which enters the isothermal flow rules associated with Huber-Mises yield.

3. Some properties of the adiabatic three-dimensional elastic-plastic waves

On account of the similarity of the adiabatic and isothermal flow rules all properties of isothermal second order waves in elastic-plastic medium discussed by J. MANDEL in the paper [3] also concern adiabatic waves, provided that small coupling effect mentioned in Sect. 2 are neglected.

Let us first recall some of the most important properties demonstrated by MANDEL. Denote by $\mathbf{n}$ the unit normal to a travelling surface $S(t)$ (wave) in the direction of its propagation. The jump of the acceleration $[\dot{\mathbf{v}}]$ across the wave is determined by the formula

$$(\mathbf{Q}-\varrho\Omega^2\mathbf{1})[\dot{\mathbf{v}}] = 0 \tag{3.1}$$

whereas the wave speed Ω may be calculated from the characteristic equation

$$\det(\mathbf{Q}-\varrho\Omega^2\mathbf{1}) = 0. \tag{3.2}$$

Here, $\mathbf{Q}$ is the so-called acoustic tensor defined by

$$Q_{ij} = \begin{cases} Q_{ij}^{(e)}(\mathbf{n}) = n_k M_{ikmj} n_m & \text{for elastic waves,} \\ Q_{ij}^{(p)}(\mathbf{n}) = Q_{ij}^{(e)} - r_p a_i a_j & \text{for adiabatic plastic waves,} \\ Q_{ij}^{(u)}(\mathbf{n}, \nu) = Q_{ij}^{(e)} - r_u a_i a_j & \text{for adiabatic unloading waves,} \\ Q_{ij}^{(l)}(\mathbf{n}, \nu) = Q_{ij}^{(e)} - r_l a_i a_j & \text{for adiabatic loading waves,} \end{cases} \tag{3.3}$$

$$\mathbf{M} = \mathbf{L}^{-1}, \quad \mathbf{a} = (\mathbf{M}\mathbf{f}_\sigma)\mathbf{n};$$

$$\frac{1}{r_p} = h_1 + M_f^2, \quad M_f^2 = \mathbf{f}_\sigma \cdot \mathbf{M}\mathbf{f}_\sigma, \tag{3.4}$$

$$\frac{1}{r_u} = h_1(1-\nu) + M_f^2, \quad \frac{1}{r_l} = \left(1 - \frac{1}{\nu}\right)h + M_f^2;$$

ν is the loading "index" which is equal to the ratio of the rate of loading behind the wave ($\psi^{(2)}$—cf. Eq. (2.4)) to the rate of loading in front of wave ($\psi^{(1)}$)

$$\nu = \frac{\psi^{(2)}}{\psi^{(1)}}. \tag{3.5}$$

The loading index is non-positive across unloading waves, and non-negative across loading waves.

The most important properties mentioned above are as follows:

1. For the given unit normal $\mathbf{n}$ there exist three speeds

$$\Omega_1^p \geqslant \Omega_2^p \geqslant \Omega_3^p \tag{3.6}$$

of adiabatic plastic waves. The eigenvectors $[\dot{\mathbf{v}}]$ corresponding to the values (3.6) are mutually orthogonal. The speeds Ω_i^p $(i = 1, 2, 3)$ are bounded by respective ordered speeds $\Omega_1 \geqslant \Omega_2 \geqslant \Omega_3$ of elastic waves according to the formula

$$\Omega_3^p \leqslant \Omega_3 \leqslant \Omega_2^p \leqslant \Omega_2 \leqslant \Omega_1^p \leqslant \Omega_1. \tag{3.7}$$

2. For the given unit normal $\mathbf{n}$ and for given loading index $\nu \leqslant 0$ there exist three speeds

$$\Omega_1^u \geqslant \Omega_2^u \geqslant \Omega_3^u \tag{3.8}$$

of adiabatic unloading waves. The eigenvectors $[\dot{v}_i]$ corresponding to the values (3.8) are mutually orthogonal. The speeds of plastic waves Ω_i^p are the lower bounds for Ω_i^u, whereas the speeds of elastic waves are upper bounds for Ω_i^u, i.e.,

$$\Omega_i^p \leqslant \Omega_i^u \leqslant \Omega_i, \quad i = 1, 2, 3. \tag{3.9}$$

3. The unloading waves propagate through the region of homogeneous state ($\boldsymbol{\sigma} = \text{const}$, $\boldsymbol{\varkappa} = \text{const}$, $\varkappa_1 = \text{const}$, $\theta = \text{const}$) with the speeds of elastic waves.

4. The values of speeds of adiabatic loading waves belong to the completion of open intervals

$$(\Omega_i^p, \Omega_i), \quad i = 1, 2, 3$$

on the semi-axis of positive numbers. When the loading index $\nu \geqslant 0$ statisfies the inequalities

$$0 \leqslant \nu \leqslant \frac{h_1}{h_1 + M_f^2} \quad \text{or} \quad \nu > 1$$

then, for given $\mathbf{n}$, there exists three speeds $\Omega_1^L \geqslant \Omega_2^L \geqslant \Omega_3^L$ of adiabatic loading waves. Otherwise, the number of loading waves may be less than three.

Waves of finite strain in elastic-plastic conductors have the similar properties as shown by MANDEL [4] and PIAU [5]. Let us now suppose that the yield limit is the decreasing function of a temperature, i.e.,

$$f_\theta > 0 \text{ so that } h_1 < h. \tag{3.10}$$

The following two additional properties of adiabatic waves may be then justified (see Appendix).

5. For a given $\mathbf{n}$ and for a given thermodynamic state $\boldsymbol{\sigma}$, $\boldsymbol{\varkappa}$, $\varkappa_1$, θ the ordered speeds $\overset{(T)}{\Omega_1^p} \geqslant \overset{(T)}{\Omega_2^p} \geqslant \overset{(T)}{\Omega_3^p}$ of isothermal plastic waves are not less than the corresponding speeds of adiabatic plastic waves

$$\overset{(T)}{\Omega_i^p} \geqslant \Omega_i^p, \quad i = 1, 2, 3. \tag{3.11}$$

6. For a given $\mathbf{n}$, ν and thermodynamical state, the ordered speeds $\overset{(T)}{\Omega_1^u} \geqslant \overset{(T)}{\Omega_2^u} \geqslant \overset{(T)}{\Omega_3^u}$ of isothermal unloading waves are not less than the corresponding speeds of adiabatic unloading waves

$$\overset{(T)}{\Omega_i^u} \geqslant \Omega_i^u, \quad i = 1, 2, 3. \tag{3.12}$$

It may also be shown that under prescribed thermodynamical state and a unit normal **n** the speeds of adiabatic loading waves may be both greater and smaller than the corresponding speeds of isothermal loading waves depending upon the value of the loading index $\nu \geqslant 0$.

Thus, we may conclude that due to a decrease of the yield limit with temperature the energy dissipation results in diminishing the speeds of plastic and unloading waves.

4. The influence of energy dissipation on combined longitudinal and torsional simple waves in a thin-walled tube

1. The simple wave solution is the solution which depends on the material particle $\mathbf{x}$ and time t only through dependence on a single independent variable, say η, on $\mathbf{x}$ and t. Therefore, the possible stress path (stress trajectories) for all particles are the same, and state variables are constant on the travelling surface (simple wave) $\eta = \text{const}$. In the case of elasticplastic non-conductors described by constitutive equations (2.1), simple waves are planes which travel with the speed of plastic waves

$$\eta = \eta\left[\frac{\mathbf{l}(\eta)\cdot\mathbf{x}}{\Omega^P(\eta)} - t\right], \quad \mathbf{l}\cdot\mathbf{l} = 1, \tag{4.1}$$

where Ω^P is equal to one of the plastic wave speeds Ω^P_i $(i = 1, 2, 3)$.

2. In order to estimate quantitatively the influence of the energy dissipation on the propagation of elastic-plastic waves, we have analysed and compared the isothermal and adiabatic one-dimensional simple waves in an isotropic thin-walled semi-infinite tube, adopting the Huber-Mises yield condition (2.7) and assumption (2.8). The waves are generated by combined longitudinal and torsional impact at the end of a tube.

The isothermal waves of that type were originally examined by CLIFTON in his outstanding paper [6] in which the details of the mathematical analysis may be found. In a thin-walled tube there exist only two types of isothermal simple waves: fast waves (speed $\overset{T}{\Omega}{}^P_1$) and slow waves (speed — $\overset{T}{\Omega}{}^P_2 \leqslant \overset{T}{\Omega}{}^P_1$). The corresponding speeds of adiabatic waves are denoted by Ω^P_i $(i = 1, 2)$.

The yield limit Y, isothermal and adiabatic hardening functions $h(\sigma_{(i)})$ and $h_1(\sigma_{(i)})$ [cf. Eqs. (2.7) and (2.9)–(2.11)] are specified by using the following relation between the true stress σ, plastic strain ε^P and temperature θ in simple tension

$$\sigma = c_0(1 - a_0\theta)(\varepsilon^P + b_0)^n. \tag{4.2}$$

For the specific values of the parameters entering (4.2), this relation can describe fairly well Manjoine's data [7] for mild steel in the temperature range 400°C–650°C. The values of interest are [8]

$$a_0 = 1.4\times 10^{-3}\ {}^\circ\mathrm{C}^{-1}, \quad n = 0.2, \quad b_0 = 0.01, \tag{4.3}$$

and for the strain rate of the order of $10^3\ \mathrm{sec}^{-1}$, the value of c_0 is

$$c_0 = 330\ \mathrm{kGmm}^{-2}. \tag{4.4}$$

Using (4.2)–(4.4) and assuming that

$$\pi_1 = 0.1, \quad E = 1.5\times 10^4 \text{ kGmm}^{-2}, \quad \mu = 0.6\times 10^4 \text{ kGmm}^{-2},$$
$$\varrho_0 c_\sigma = 0.55 \text{ kG}^\circ\text{C}^{-1}\text{mm}^{-2}, \quad \theta_0 = 400^\circ\text{C},$$

where E and μ are the Young modulus and shear modulus respectively, the functions h_1 and h are found to be

$$h_1(\sigma_{(i)}) = a_1^* \sigma_{(i)}^{-4} - a_2^* \sigma_{(i)}^2 + a_2^* \sigma_{(i)}^8, \tag{4.5}$$
$$h(\sigma_{(i)}) = a\sigma_{(i)}^{-4},$$
$$a_1^* = 1.30\times 10^{10} \text{ kGmm}^{-10}, \qquad a_2^* = 0.86\times 10^{-2} \text{ mm}^2\text{kG}^{-1},$$
$$a_3^* = 1.44\times 10^{-5} \text{ kG}^{-7}\text{mm}^{-14}, \quad a = 1.29\times 10^{10} \text{ kG}^5\text{mm}^{-10}.$$

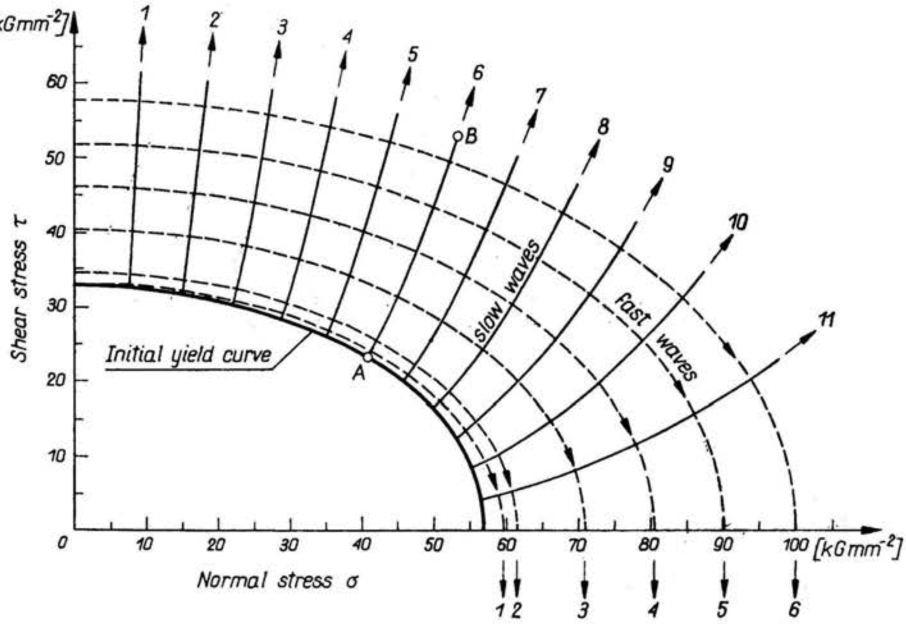
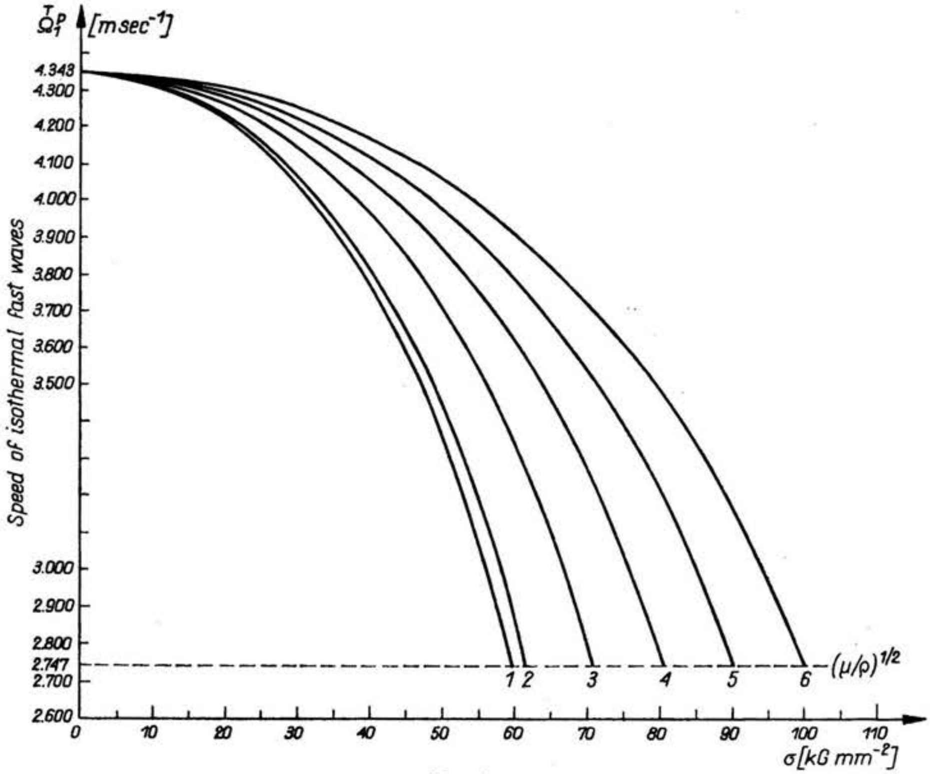


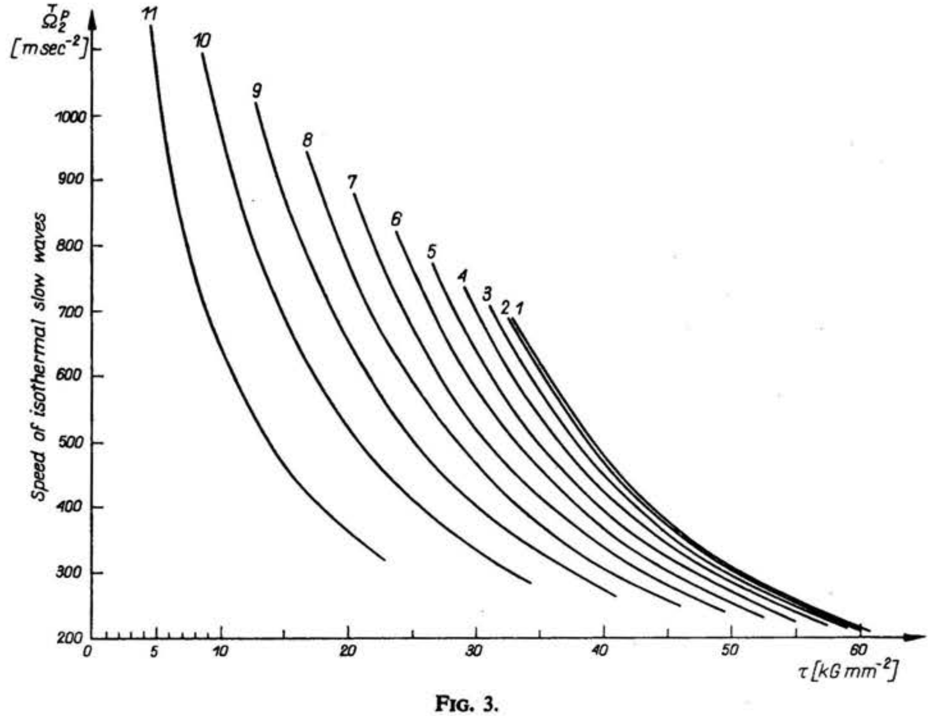
FIG. 1.

The functions (4.5) were used to determine the stress trajectories and speeds of adiabatic and isothermal simple waves in the tube. The stress trajectories are shown in Fig. 1. The adiabatic stress trajectories turned out to be "very close" to the isothermal one, and this is why they cannot be distinguished in the stress scale used in Fig. 1. Since for other ranges of the temperature the yield limit of the mild steel is less temperature-sensitive, one can conclude that the energy dissipation does not influence the stress trajectories for the mild steel. Since the hardening of the mild steel is small at temperature ranging from 400°C to 650°C, all slow wave trajectories intersect the initial yield curve, as it is shown in Fig. 1. The successive trajectories for slow and fast waves are numbered in Fig. 1, and the change in isothermal fast wave speed $\overset{T}{\Omega}{}_1^p$ as the normal stress σ varies along every wave trajectory is shown in Fig. 2. Similarly, the change in the isothermal slow wave speed $\overset{T}{\Omega}{}_2^p$ as shear stress τ varies along every slow wave trajectory is presented in Fig. 3. The speeds of adiabatic



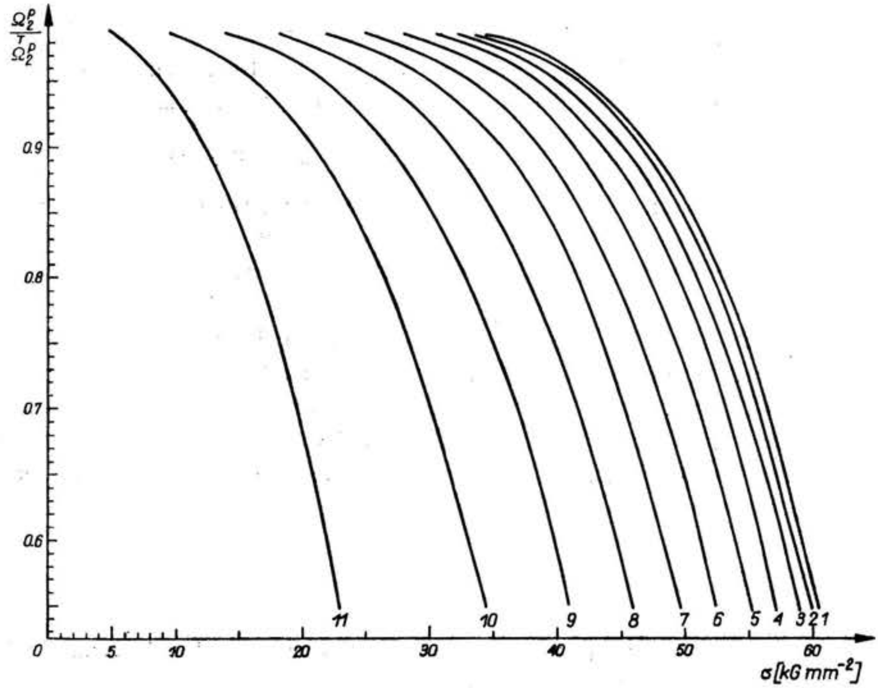
$\overset{T}{\Omega}{}_1^P$ [m sec^{-1}]
Speed of isothermal fast waves
4.348
4.300
4.200
4.100
4.000
3.900
3.800
3.700
3.600
3.500
3.000
2.900
2.800
2.747
2.700
2.600
0
10
20
30
40
50
60
70
80
90
100
110
σ [kG mm^{-2}]
1 2 3 4 5 6
$(\mu/\rho)^{1/2}$

FIG. 2.



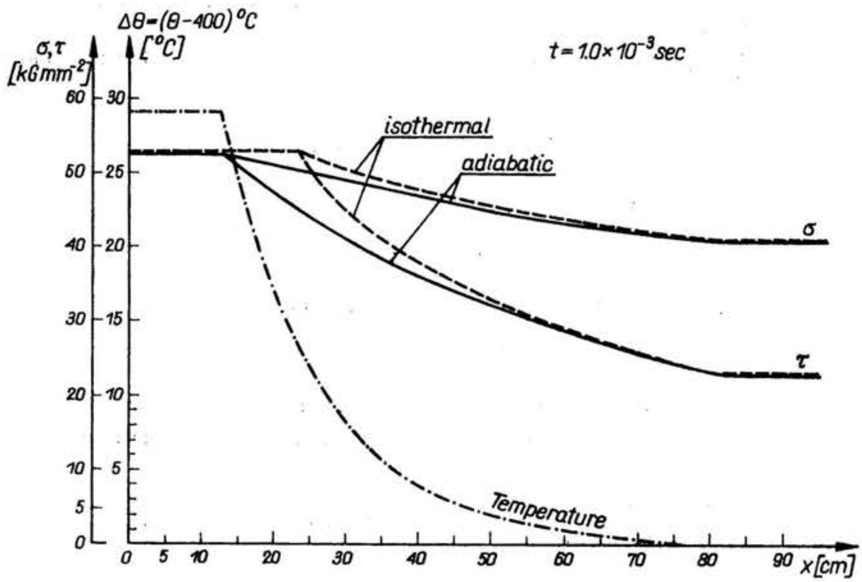
$\overset{T}{\Omega}{}_2^P$ [m sec^{-2}]
Speed of isothermal slow waves
11
10
9
8
7
6
5
4
3
2 1
1000
900
800
700
600
500
400
300
200
0
5
10
20
30
40
50
60
τ [kG mm^{-2}]

FIG. 3.



$\Omega_2^P / \overset{T}{\Omega}{}_2^P$
0.9
0.8
0.7
0.6
0
5
10
20
30
40
50
60
11
10
9
8
7
6
5
4
3
2
1
σ[kG mm^{-2}]

FIG. 4.



σ, τ [kG mm^{-2}]
Δθ=(θ−400) °C [°C]
t=1.0×10^{-3} sec
isothermal
adiabatic
σ
τ
Temperature
x [cm]

FIG. 5.

fast waves Ω_1^p turned out to be again "very close" to the speeds of isothermal fast waves. For given (σ, τ), $\overset{T}{\Omega}{}_1^p$ and Ω_1^p may, therefore, be assumed to be the same with sufficient accuracy. There are, however, essential differences between the speeds of adiabatie slow waves Ω_2^p and the speeds of isothermal slow waves $\overset{T}{\Omega}{}_2^p$, as may be seen from Fig. 4. In this figure the change in the ratio $\Omega_2^p/\overset{T}{\Omega}{}_2^p$ as τ varies along every slow wave trajectory, is presented. It is seen that the speeds of slow waves may be diminished by 45% as a result of energy dissipation. The profiles of the adiabatic stress waves and isothemal stress waves at time $t = 1.0\times10^{-3}$ sec are shown in Fig. 5. The tube was initially prestressed to the level represented by point A in Fig. 1 and then was subjected to step loading at the end $x = 0$ to the level represented by point B in Fig. 1. In Fig. 5 the profile of the temperature wave in the considered non-conductor is also shown. It is seen from Fig. 5 that the energy dissipation results in the reduction of the constant state region (plateau) and causes stress reduction up to about 15% in some cross-sections.

Appendix

Denote by $Q_1^{(e)} \geqslant Q_2^{(e)} \geqslant Q_3^{(e)}$ the eigenvalues of the tensor $\mathbf{Q}^{(e)}$ and assume that the coordinate axes coincide with the principal axes of the tensor $\mathbf{Q}^{(e)}$. From Eq. (3.2) it then follows that the eigenvalues $Q_1^{(p)} \geqslant Q_2^{(p)} \geqslant Q_3^{(p)}$ of the tensor $\mathbf{Q}^{(p)}$ are the roots of the following equation [3]

(A1) $$F(X) = (Q_1^{(e)}-X)(Q_2^{(e)}-X)(Q_3^{(e)}-X)-r_p[(Q_2^{(e)}-X)(Q_3^{(e)}-X)a_1^2 + (Q_3^{(e)}-X)(Q_1^{(e)}-X)a_2^2+(Q_1^{(e)}-X)(Q_2^{(e)}-X)a_3^2] = 0.$$

Denote by $\overset{T}{Q}{}_1^{(p)} \geqslant \overset{T}{Q}{}_2^{(p)} \geqslant \overset{T}{Q}{}_3^{(p)}$ the eigenvalues of the acoustic tensor $\overset{T}{\mathbf{Q}}{}^{(p)}$ for isothermal plastic waves

$$\overset{T}{\mathbf{Q}}{}^{(p)}(\mathbf{n}) = \mathbf{Q}^{(e)} - r_p'\mathbf{a}\otimes\mathbf{a},$$

where (cf. Eq. (3.4))

(A2) $$r_p' = \frac{1}{h+M_f^2} \leqslant r_p.$$

The eigenvalues $\overset{T}{Q}{}_i^{(p)}$ $(i = 1, 2, 3)$ are the roots of the equation

(A3) $$F'(X) = 0.$$

Here $F'(X)$ has the same form as (A1) except for r_p which is replaced by r_p'.

Multiplying (A3) and (A1) by r_p and r_p' respectively, and subtracting the second obtained result from the first one, we get

(A4) $$r_pF'(X)-r_p'F(X) = (r_p-r_p')(Q_1^{(e)}-X)(Q_2^{(e)}-X)(Q_3^{(e)}-X).$$

Since

$$F(Q_i^{(p)}) = 0 \quad (i = 1, 2, 3), \quad r_p - r_p' \geqslant 0 \quad (r_p \geqslant 0),$$

from the property (3.7) and (A4) it follows that

$$F'(Q_1^{(p)}) \geqslant 0, \quad F'(Q_2^{(p)}) \leqslant 0, \quad F'(Q_3^{(p)}) \geqslant 0.$$

Hence

$$Q_i^{(p)} \leqslant \overset{T}{Q}_i^{(p)}, \quad i = 1, 2, 3$$

on account of $F(\infty) < 0$. This proves the property 5.

The proof of the fact that the ordered eigenvalues of the tensor $Q_i^{(u)}$ are less than the corresponding ordered eigenvalues of the acoustic tensor $\overset{T}{\mathbf{Q}}^{(u)}$

$$\overset{T}{\mathbf{Q}}^{(u)} = \mathbf{Q}^{(e)} - r'_u \mathbf{a} \otimes \mathbf{a}, \quad r'_u = \frac{1}{h(1-\nu)+M_f^2} \leqslant r_u$$

for isothermal unloading waves (property 6) may be done in a similar manner. It is sufficient to replace r_p in Eq. (A1) by r_u, and r'_p in Eq. (A3) by r'_u.

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