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On the dynamics of eddy viscosity models for turbulent boundary layers

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IT HAS been recently attempted to extend the valitidy of existing phenomenological models of turbulent boundary layers to the time dependent flows. The present paper is concerned with generalizations and improvements on the mixing length model as recently developed by Cebeci and Smith. In particular, the damping factor is augmented to include dynamic pressure effects, a generalized displacement thickness is derived through a differential equation, etc. The system of differential equations is then integrated numerically in a three-dimensional space. The calculations proceed marching in the direction of the two parabolic variables: downstream distance and time. The theoretical predictions are carefully compared with the only available experimental data, those of Karlsson. Oscillatory flows are examined and the phase advance or delay of various properties and, in particular, the skin friction are estimated.

Ostatnio czynione były próby rozszerzenia poprawności istniejących fenomenologicznych modeli przepływów turbulentnych w warstwach przyściennych na przepływy zależne od czasu. Niniejsza praca dotyczy tych zagadnień, a przede wszystkim uogólnienia i udoskonalenia modelu drogi mieszania Prandtla, które ostatnio intensywnie badali Cebeci i Smith. W szczególności rozszerzono wartości współczynnika tłumienia celem uwzględnienia efektów ciśnienia dynamicznego oraz wyprowadzono z równania różniczkowego uogólnioną wielkość przemieszczenia. Następnie układ równań różniczkowych scałkowano numerycznie w przestrzeni trójwymiarowej. Obliczenia przebiegały krok po kroku w kierunku dwóch zmiennych parabolicznych — odległości wzdłuż linii prądu i czasu. Wyniki teoretyczne porównano skrupulatnie z jedynymi dostępnymi danymi doświadczalnymi Karlssona. Zbadano przepływy oscylujące i oszacowano różne własności wyprzedzenia bądź opóźnienia fazowego, w szczególności tarcia powierzchniowego.

В последнее время были предприняты попытки расширения справедливости существующих феноменологических моделей турбулентных течений в пограничных слоях на течения зависящие от времени. Настоящая работа касается этих вопросов, прежде всего обобщения и усовершенствования модели пути смешивания Прандтля, которая в последнее время интенсивно была исследована Цебечи и Смитом. В частности расширены значения коэффициента затухания с целью учета эффектов динамического давления а также из дифференциального уравнения выведена обобщенная величина перемещения. Затем система дифференциальных уравнений численно проинтегрирована в трехмерном пространстве. Расчеты проводились шаг за шагом в направлении двух параболических переменных — расстояния вдоль линии тока и времени. Теоретические результаты старательно сравнены с единственными доступными экспериментальными данными Карлсона. Исследованы осциллирующие течения и оценкиы разные свойства фазового опережения или запаздывания, в частности поверхностного трения.

1. Introduction

DESPITE the criticism that phenomenological models of turbulence have received, it appears that today such models are the only available tools for engineering estimates of turbulent boundary layers and are extensively used in engineering design. Efforts in developing such heuristic and approximate models for turbulence were initiated quite early by Prandtl, von Kármán and other well known aerodynamicists. In the last few years a few more models have appeared, based on different assumptions and each was suggested as pertinent or remedying a specific drawback of the original idea (see review articles of REYNOLDS (1970), MELLOR and HERRING (1973) and LAUNDER and SPALDING (1974)). It is the opinion of the author that some of these models may eventually prove to be more accurate and perhaps more widely applicable than the early rather simplistic models. However, it is felt though that the ability of the future computers and/or the development of theoretical stochastic approximations will soon render all phenomenological models obsolete. Until then and mainly for practical applications, it is necessary to improve and extend the existing approximate models.

The present paper is an extension of work on the most simple model which is based on Prandtl's original mixing length idea. Despite its simplicity, this model, as later improved by VAN DRIEST (1956), CEBECI (1970) and others, has been shown to predict with very good accuracy a wide variety of flow situations. In fact, in a recent comparative study (see, e.g., BURGGRAF (1974)) it was shown to compete very successfully with many other more sophisticated models.

Most of the boundary layer calculations for laminar or turbulent flow have been confined for decades to steady two-dimensional incompressible flows over geometrically-simple body configurations. The value of such calculations was therefore rather qualitative. The evolution of the modern computer though has permitted the numerical integration of the differential equations for more general geometrical configurations and more complex flow conditions. Today such solutions can be used for design purposes in realistic aerodynamic applications. One of the most interesting areas of aerodynamics that appears to require immediate attention is the area of unsteady viscous flow and viscous-inviscid interaction. Laminar and turbulent unsteady boundary layer calculations have been attempted only in the last few years, as described in a recent review article (see, e.g., TELIONIS (1975)). Most of the existing models of turbulent boundary layers have been extended to unsteady flow with varied success. CEBECI and KELLER (1972) and ABBOTT and CEBECI (1971) have extended the mixing length model. PATEL and NASH (1972), NASH, CARR and SINGLETON (1973) and SINGLETON and NASH (1973) have developed a scheme of unsteady turbulent flow calculations based on the turbulent energy equation. A similar method was developed by SHAMROTH and KRESKOVSKY (1974). DWYER (1973) and later MCCROSKEY and PHILIPPE (1974) worked out solutions with a quasi-steady model. These works were performed almost independently and, as a result, very little comparison of the results of different methods was attempted. Modest efforts to compare the relative success of a certain proposed model with others were reported in MCCROSKEY and PHILIPPE (1974) and SHAMROTH and KRESKOVSKY (1974). Unfortunately there is very little experimental information on the problem. To the knowledge of the author the only works that report on unsteady turbulent layers are those of KARLSSON (1959) and MILLER (1969). The experiments of Karlsson were confined to oscillatory flow over a flat plate whereas those of Miller to heat transfer measurements.

The present paper is a further extension of a model suggested in a previous publication by the author and one of his colleagues (see, e.g., TELIONIS and TSAHALIS (1975)). In this paper the reader will also find a more detailed account of previous publications on the topic. A two layer model for the eddy viscosity is adopted again here. In the inner layer a generalized mixing length that incorporates in the well-known damping factor the effect of unsteadiness is assumed. It is pointed out that the original model as suggested by CEBECI and KELLER (1972) and later used by TELIONIS and TSAHALIS (1975) is incomplete. A correction that accounts for the dynamic effects on the wall shear is proposed. The eddy viscosity in the outer layers is based on the Clauser model but a generalized boundary layer thickness for unsteady flows according to MOORE and OSTRACH (1956) is used. The validity of the present model in the neighborhood of a point of zero wall shear and in regions of partially reversed flow is discussed.

Numerical calculations were performed for flows oscillating over a flat plate. Similar calculations were performed for the first time using the eddy viscosity models of VAN DRIEST (1956), ALBER (1971), KAYS (1971) and CEBECI and KELLER (1972). All the numerical results were checked against the experimental data of KARLSSON (1959).

2. The turbulent boundary-layer equations and a closure assumption

Let u, v and x, y be the averaged velocity components and the coordinates parallel and perpendicular to the wall, respectively. Let $U_e(x, t)$ be the outer flow velocity, t the time, ϱ the density, p the pressure, and u', v' the instantaneous values of the velocity fluctuations. The turbulent boundary-layer equations then read

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\varrho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\partial}{\partial y}\langle u'v\rangle, \tag{2.2}$$

$$-\frac{1}{\varrho}\frac{\partial p}{\partial x}=\frac{\partial U_e}{\partial t}+U_e\frac{\partial U_e}{\partial x}, \tag{2.3}$$

where the symbol $\langle\ \rangle$ represents the ensemble average of the fluctuating quantities at the time t. We assume here that the Reynolds stress may be modeled again, for unsteady flow, via an eddy viscosity model whereby the quantity $\langle u'v'\rangle$ depends linearly on the gradient of the mean velocity

$$\langle u'v'\rangle=\varepsilon\partial u/\partial y. \tag{2.4}$$

The eddy viscosity ε is then defined for an inner and outer region as proposed by CEBECI and SMITH (1968) and later generalized for unsteady flow by CEBECI and KELLER (1972) and TELIONIS and TSAHALIS (1975).

In the inner region it is assumed that

$$\varepsilon_i=\varrho l^2|\partial u/\partial y|, \tag{2.5}$$

where l is Prandtl's mixing length and is here given by

$$l=k_l y[1-\exp(-y/A)]. \tag{2.6}$$

In the above equation k_l is a constant which was empirically estimated to be equal to 0.41 and A is van Driest's damping factor (see, e.g., VAN DRIEST (1956)). In the outer region it is assumed that

$$\varepsilon_0=\varrho k_2 U_e\delta^*\bar{\gamma}, \tag{2.7}$$

where $k_2 = 0.0168$, δ^* is the displacement thickness and $\bar{\gamma}$ is the intermittency factor given by

$$2\bar{\gamma} = 1-\text{erf}\,[5(y/\delta-0.78)]. \tag{2.8}$$

It should be emphasized here that the inner region form is assumed to hold in a layer much thicker than the viscous sublayer. The interface between the inner and outer layer, $y = y_0$, is defined by the equation $\varepsilon_i(y_0) = \varepsilon_0(y_0)$. This arbitrary separation in two regions is not altogether unrealistic since the inner and outer regions so defined, roughly correspond to the regions visually observed by NYCHAS, HERSHEY and BRODKEY (1973).

The formulation up to now follows closely the work of CEBECI and KELLER (1972) and TELIONIS and TSAHALIS (1975). However, in the present paper we would like to re-examine the interpretation of the factors that appear in the above formulas.

Consider first the damping factor A which is traditionally assumed to depend on the wall shear according to the empirical formula

$$A = A^+\nu/u_\tau, \tag{2.9}$$

where $A^+ = 26$, u_τ is the friction velocity, $u_\tau = (\tau_w/\varrho)^{1/2}$ and τ_w is the skin friction, $\tau_w = \mu\partial u/\partial y$ at $y = 0$. Following the suggestion of CEBECI (1970), we assume that A depends on the shear evaluated at the edge of the viscous sublayer rather than the wall. This we find justified on physical grounds since the random oscillations that are damped according to the mechanism of a Stokes layer do not extend all the way to the wall but disappear at the edge of the viscous sublayer.

In the immediate neighborhood of the wall we may approximate the momentum equation by

$$\varrho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y}. \tag{2.10}$$

In the above equation the convective terms have been omitted, an approximation justified for very small distances from the wall. The unsteady term $\partial u/\partial t$, in general is not small and its omission in previous publications is not justified. Integration of Eq. (2.10) and evaluation at $y = y_s$, the edge of the viscous sublayer yields

$$\tau_s = -\varrho\left(\frac{\partial U_e}{\partial t} + U_e\frac{\partial U_e}{\partial x}\right)y_s + \int_0^{y_s} \varrho\frac{\partial u}{\partial t}\,dy + \tau_w, \tag{2.11}$$

where $y_s = \nu y^+/u_\tau$ and $y^+ = 11.8$. In the present analysis therefore the eddy viscosity in the inner layer is assumed in the form

$$\varepsilon_i = \varrho k_1^2 y^2\left[1-\exp\left(-\frac{yu_s}{\nu A^+}\right)\right]^2\left|\frac{\partial u}{\partial y}\right|, \tag{2.12}$$

where $u_s = (\tau_s/\varrho)^{1/2}$. Notice that the dynamic effects are influencing the eddy viscosity through the factor u_s which depends on the outer flow via Eq. (2.11).

In the outer region the dynamic effects are introduced through the time dependence of the outer flow velocity U_e and the displacement thickness δ^*. It has failed the attention of previous investigators that the common definition of the displacement thickness cannot be carried over to unsteady flow. Insteady, we have adopted here the more general definition of MOORE and OSTRACH 1(956). Reducing their general equation to its two-dimensional incompressible but unsteady form we arrived at:

$$\frac{\partial}{\partial x}(\delta^* U_e) - \frac{\partial}{\partial x}\int (U_e - u)\,dy + \frac{\partial \delta^*}{\partial t} = 0. \tag{2.13}$$

Notice that for steady flow the above equation reduces further to the familiar formula for the displacement thickness

$$\delta^* = \int (1 - u/U_e)\,dy. \tag{2.14}$$

3. The method of solution

The differential equations were solved numerically for oscillating outer flows. A steady state solution was generated and used as an initial condition. The calculations were then carried out until a periodic oscillatory motion was achieved. More details on this technique can be found by the reader in previous publications [see, e.g., TSAHALIS and TELIONIS (1974), TELIONIS and TSAHALIS (1975)]. At the origin of the calculations a well-rounded leading edge was assumed and Hiemenz type laminar profiles were generated. Turbulence was assumed to be turned on immediately downstream of the origin. The experimental analogy is a boundary layer that is artificially tripped very near the leading edge. On the wall and at the edge of the boundary layer the boundary conditions are

$$u = v = 0 \quad \text{at} \quad y = 0, \tag{3.1}$$

$$u \to U_e(x, t) \quad \text{at} \quad y \to \infty. \tag{3.2}$$

In the present paper oscillatory flows over a flat plate were considered

$$U_e(x, t) = U_\infty(1 + \alpha e^{i\omega t}). \tag{3.3}$$

To incorporate the proposed refinements in the eddy viscosity formulas some assumptions with regard to the term $\partial u/\partial t$ within the viscous sublayer are necessary. It is assumed that the velocity in the viscous sublayer responds to the imposed outer flow according to the formula

$$u = u_0 + \alpha u_1 \cos(\omega t + \pi/4). \tag{3.4}$$

The phase advance is a familiar property of Stokes flow as well as laminar oscillatory flow (see, e.g., LIGHTHILL (1954)). The functions u_0 and u_1 can be approximated by linear functions of y. Further, laminar calculations (see, e.g., TSAHALIS and TELIONIS (1974)) indicate that $u_0 \simeq u_1$ is a fairly good approximation, if α is the amplitude ratio of the outer flow. Our hypothesis then is that

$$u_0(x, y) = u_1(x, y) = \overline{\frac{\partial u(x, y, t)}{\partial y}}\Big|_{y=0} y. \tag{3.5}$$

The shear at $y = y_s$ from Eq. (2.11) can be written as follows:

$$\tau_s = \frac{\partial p}{\partial x} y_s - \varrho\alpha\omega \cdot \frac{\partial u}{\partial y}\Big|_w \frac{y^2}{2} \sin(\omega\dagger + \pi/4) + \tau_w. \tag{3.6}$$

The numerical integration is performed in a Görtler space according to subroutines developed previously for laminar flow (see, E.G. TELIONIS, TSAHALIS, and WERLE (1973)). New dependent and independent variables are defined

$$\xi = U_\infty x, \qquad \eta = U_\infty (2\xi)^{-1/2} y, \qquad \tau = t, \tag{3.7}$$

$$F = u/U_\infty, \qquad V = \frac{(2\xi)^{1/2}}{U_\infty} v + \eta(\beta - 1)F, \tag{3.8}$$

where $\beta = (2\xi/U_e)\,\partial U_e/\partial\xi$ and hence $\beta = 0$ for a flat plate. The continuity and momentum equations in terms of the new variables take the form

$$2\xi \frac{\partial F}{\partial \xi} + F + \frac{U_\infty}{U_e} \frac{\partial V}{\partial \eta} = 0, \tag{3.9}$$

$$\frac{\partial^2 F}{\partial \eta^2} + a_1 \frac{\partial F}{\partial \eta} + a_2 F + a_3 + a_4 \frac{\partial F}{\partial \xi} + a_5 \frac{\partial F}{\partial \tau} = 0, \tag{3.10}$$

where

$$a_1 = \frac{1}{\tilde{\varepsilon}} \left(-V + \frac{\partial \tilde{\varepsilon}}{\partial \eta} \right), \tag{3.11}$$

$$a_2 = -\frac{1}{\tilde{\varepsilon}} \frac{2\xi}{U_\infty^2 U_e} \frac{\partial U_e}{\partial t}, \tag{3.12}$$

$$a_3 = \frac{1}{\tilde{\varepsilon}} \frac{2\xi}{U_\infty^2 U_e} \frac{\partial U_e}{\partial t}, \tag{3.13}$$

$$a_4 = -\frac{2\xi}{\tilde{\varepsilon}} \frac{U_e}{U_\infty} F, \tag{3.14}$$

$$a_5 = -\frac{2\xi}{\tilde{\varepsilon} U_\infty^2}, \tag{3.15}$$

where $\tilde{\varepsilon}$ is the dimensionless total viscosity given by

$$\tilde{\varepsilon} = 1 + \frac{\varepsilon}{\mu \varrho}. \tag{3.16}$$

In previous publications of the author and his colleagues the reader will find detailed descriptions of the numerical integration of parabolic equations like Eqs. (3.9) and (3.10) in a three-dimensional space.

4. Numerical results

The present author and his colleagues have earlier compared the mixing length method with other unsteady turbulent boundary layer approximate methods of calculation (see, e.g., TSAHALIS and TELIONIS (1975)). It was shown then that there is reasonable agreement in predicting wall quantities like the skin friction and its phase advance. In the same paper the method was tested against the experimental data of KARLSSON (1959). In the present paper we introduced refinements in calculating the shear at the edge of the viscous sublayer and the displacement thickness as described in the previous section. We then performed calculations using the present model of the eddy viscosity for some characteristic cases of the experiments of KARLSSON (1959). The same calculations were repeated using the eddy viscosity model in its most simple form (see, e.g., VAN DRIEST (1956)), as well as the models of KAYS (1971), ALBER (1971) and CEBECI and KELLER (1972). The last model, practically unaltered, is the one used in our previous publication. All of the theoretical results presented in this paper therefore were derived by our numerical scheme using the above mentioned models as well as the one presently proposed. For brevity and clarity we have marked the figures with the name of the first author of the publication in which the respective model was originally proposed. Namely: VAN DRIEST for the most simple quasi-steady model of VAN DRIEST (1956), KAYS and ALBER for the models of KAYS (1971) and ALBER (1971) and CEBECI for the model of CEBECI and KELLER (1972). The reader should be cautioned to the fact that the result presented in the figures have not been reported previously in the above-mentioned papers. The eddy viscosity formula in terms of our transformed variables and according to the above models becomes

VAN DRIEST:

$$\varepsilon_t = 1 + R_e^{1/2}\frac{U_e}{U_\infty}k_i^2\left\{1-\exp\left[-\frac{(2\xi R_e)^{1/4}}{A^+}\left(\frac{U_e}{U_\infty}\frac{\partial F}{\partial\eta}\bigg|_w\right)^{1/2}\eta\right]\right\}^2(2\xi)^{1/2}\eta^2\frac{\partial F}{\partial\eta}, \tag{4.1}$$

where $A^+ = 26$.

CEBECI:

Same as Eq. (4.1) but with

$$A^+ = 26\left[1-11.8\frac{(2\xi)^{3/4}}{R_e^{1/4}}\left(U_eU_\infty\frac{\partial F}{\partial\eta}\bigg|_w\right)^{-3/2}\left(\frac{\partial U_e}{\partial t}+U_e\frac{\partial U_e}{\partial x}\right)\right]. \tag{4.2}$$

KAYS:

Same as Eq. (4.1) but with

$$A^+ = 26 + f_1(p^+), \tag{4.3}$$

where

$$p^+ = \frac{(2\xi)^{1/2}}{R_e}U_eU_\infty\left(\frac{\partial F}{\partial\eta}\bigg|_w\right)^{-3/2}\left(\frac{\partial U_e}{\partial t}+U_e\frac{\partial U_e}{\partial x}\right), \tag{4.4}$$

$$\begin{aligned} f_1(p^+) &= 1133p^+ \quad &\text{if}\quad p^+ \leqslant 0.012,\\ f_1(p^+) &= 2133p^+ - 12 \quad &\text{if}\quad p^+ \geqslant 0.012. \end{aligned} \tag{4.5}$$

ALBER:

$$\varepsilon_i = 1 + 0.018\,[2\xi R_e(U_e\eta/U_\infty)^2]^{1/2} \cdot\left\{1-\exp\left[-R_e^{1/4}(2\xi)^{3/4}\left(\frac{\eta}{U_\infty}\right)^{3/2}\frac{1}{26}\left(-\frac{\partial U_e}{\partial t}-U_e\frac{\partial U_e}{\partial x}\right)^{1/2}\right\}^2. \tag{4.6}$$

One of the most characteristic features of the flow is the response of the velocity profiles to the fluctuations of the outer stream. For an outer flow that fluctuates harmonically according to Eq. (3.3), we have calculated the fluctuating parts of the velocity within the boundary layer. Values of the dimensionless velocity component, F, were stored for a whole period, the average, $\bar{F}$, and subsequently the fluctuating part, $F-\bar{F}$, were calculated.

The fluctuating part is a periodic function of time but due to the non-linear character of the equations, it is not necessarily a harmonic function. To demonstrate clearly its character and compare with KARLSSON'S (1959) measurements, it was resolved into two components, one in phase with the outer flow, u_{in}, and one at a phase of 90° with the outer flow

$$u_{\text{in}}(x, y) = \frac{2^{1/2}\overline{[F(x, y, -t) - \bar{F}(x, y, t)]\cos\omega t}}{(\overline{\cos^2\omega t})^{1/2}}, \tag{4.7}$$

$$u_{\text{out}}(x, y) = \frac{2^{1/2}\overline{[F(x, y, t) - \bar{F}(x, y, t)]\cos(\omega t + \pi/2)}}{(\overline{\cos^2\omega t})^{1/2}}. \tag{4.8}$$

In Figs. 1 to 3 we have plotted the functions u_{in} and u_{out} for $\xi = 1.00$ and for various frequencies and amplitudes. Figure 1 represents the lowest frequency $f = \omega/2\pi = 2$ Herz

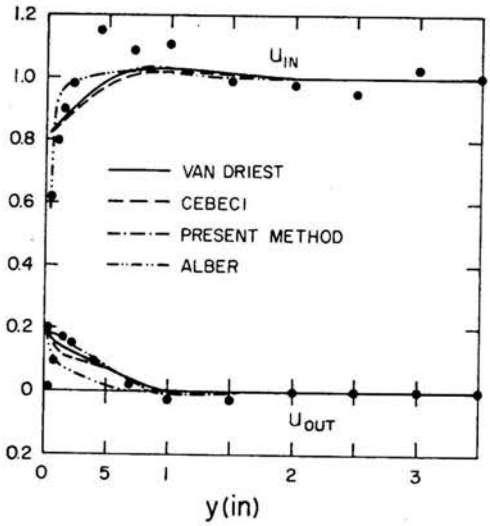


FIG. 1. In-phase and out-of-phase velocity profiles for $\alpha = 0.147$ and $f = 2$ Herz. In all figures circles represent the experimental data of KARLSSON (1959).

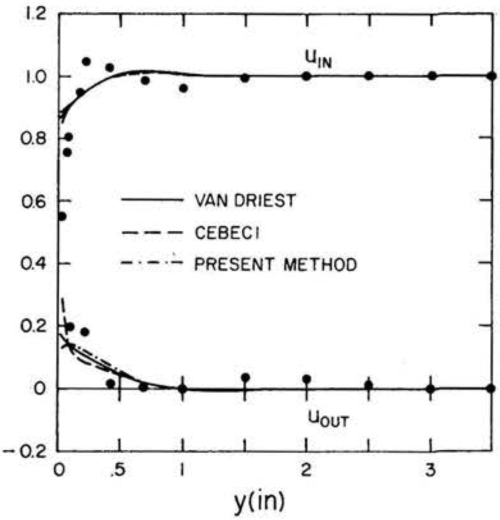


FIG. 2. In-phase and out-of-phase velocity profiles for $\alpha = 0.136$ and $f = 4$ Herz.

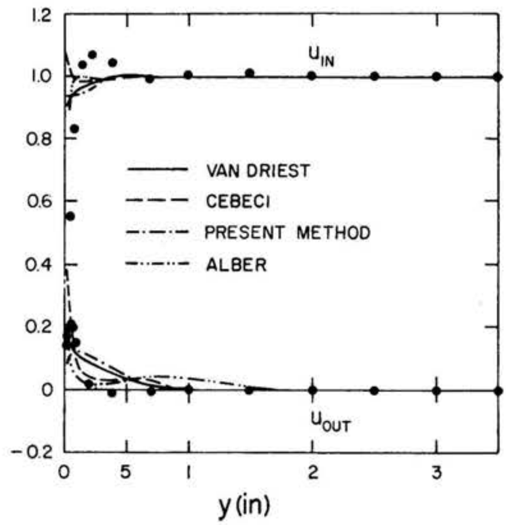


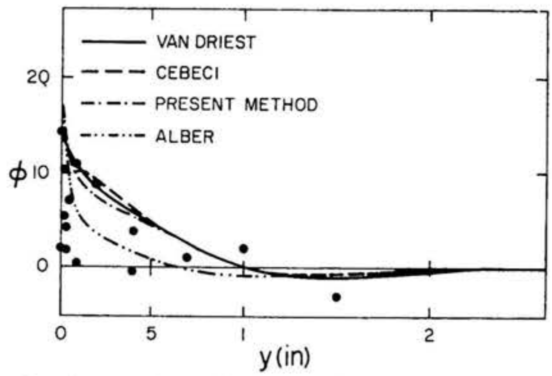
FIG. 3. In-phase and out-of-phase velocity profiles for $\alpha = 0.127$ and $f = 7.65$ Herz.

for an amplitude ratio $\alpha = 0.147$. In this figure it appears that Alber's model is most successful in predicting the function u_{in} but seems to be failing in predicting u_{out} properly. All methods fail to predict the velocity overshoot and a very mild improvement is shown with the present method as compared to our previous calculations. In Fig. 2 results are shown for the frequency $f = \omega/2\pi = 4$ Herz. All models appear to predict, at least qualitatively, a smaller thickness of the unsteady part of the velocity profile but still fail to predict a peculiar growth of the function u_{out} at distances $1'' < y < 3''$. This phenomenon is absent from similar experimental or theoretical results of laminar flows (see, e.g., TELIONIS (1975)). Figure 3 shows the same functions for a frequency of $f = 7.65$ Herz. A careful study of the experimental data indicates that the function u_{out} seems to turn sharply downward and tends to zero as the wall is approached. There is no theoretical justification for this phenomenon but the trend seems to be definite if one observes carefully the data for a whole spectrum of frequencies which KARLSSON (1959) has covered experimentally. The present method appears to be the only method that shows, qualitatively at least, the same trend.

The experimental data of KARLSSON (1959) were previously used (see, e.g., TELIONIS and TSAHALIS (1975)) in order to estimate the phase angle

$$\phi = \arctan (u_{out}/u_{in}). \tag{4.9}$$

The experimental points in the figures that follow are not reported by KARLSSON (1959) as such but were calculated from his data on u_{in} and u_{out}. In Fig. 4 the experimental points for $f = 2$ are very dispersed and no definite conclusions can be drawn. Figures 5 and 6 present the theoretical and experimental results for the frequency $f = 4$ Herz and two different amplitude ratios, $\alpha = 0.136$ and $\alpha = 0.062$, respectively. The dispersion of the experimental results is again unacceptable. In the outer part of the boundary layer the experimental data seem to be more reliable, yet they contradict each other by showing a phase advance and a phase delay for the two amplitudes considered, respectively. In Fig. 7 it appears that the experimental data are more uniformly ordered and, unfortunately,



IG. 4. The phase angle profile for $\alpha = 0.147$ and $f = 2$ Herz.

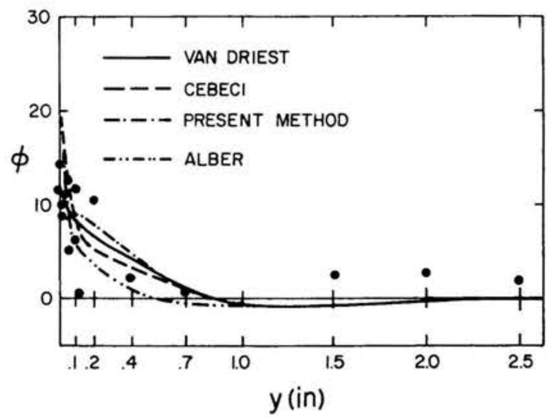


FIG. 5. The phase angle profile for $\alpha = 0.136$ and $f = 4$ Herz.

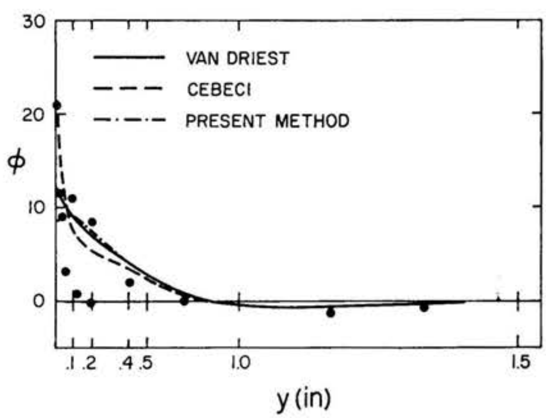


FIG. 6. The phase angle profile for $\alpha = 0.062$ ad $f = 4$ Herz.

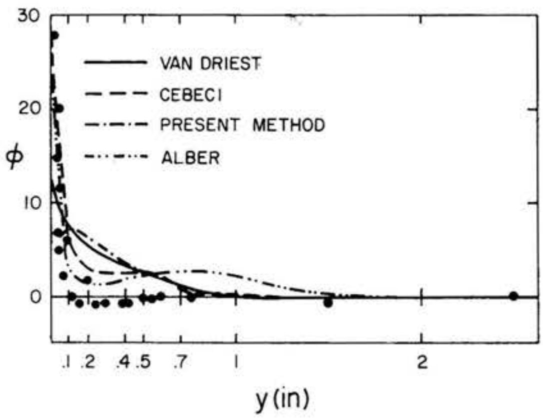


FIG. 7. The phase angle profile for $\alpha = 0.127$ and $f = 7.65$ Herz.

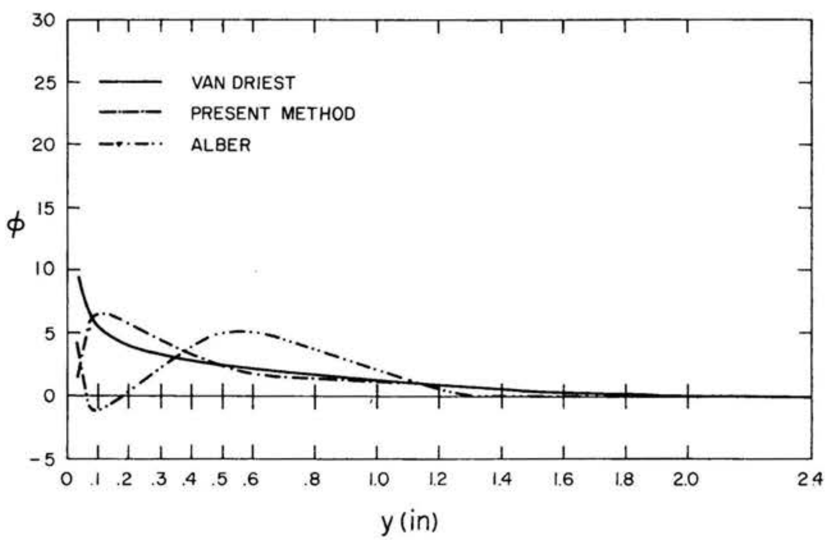


FIG. 8. The phase angle profile for $\alpha = 0.127$ and $f = 20$ Herz.

very poorly predicted by all the available methods. It seems that perhaps ALBER's (1971) model is the most successful. In Fig. 8 we have expanded the abscissa in order to show in more detail the behavior of the various models for $f = 20$. This was the largest frequency for which we were able to carry out calculations and unfortunately there is no available experimental data in the neighborhood of this frequency. ALBER's (1971) method seems now to retain its character, and in fact exagerates it to the point that it seems rather unlikely that the flow would follow such a behavior.

In Figs. 9 and 10 we have plotted the response of the derivative $\partial F/\partial \eta$ which is proportional to the wall shear. In both figures a harmonic function in phase with the oscillations of the outer flow is shown. The three tested methods do not seem to predict any large variations for $f = 2$ Herz (see Fig. 9). A small phase advance and the asymmetry of the

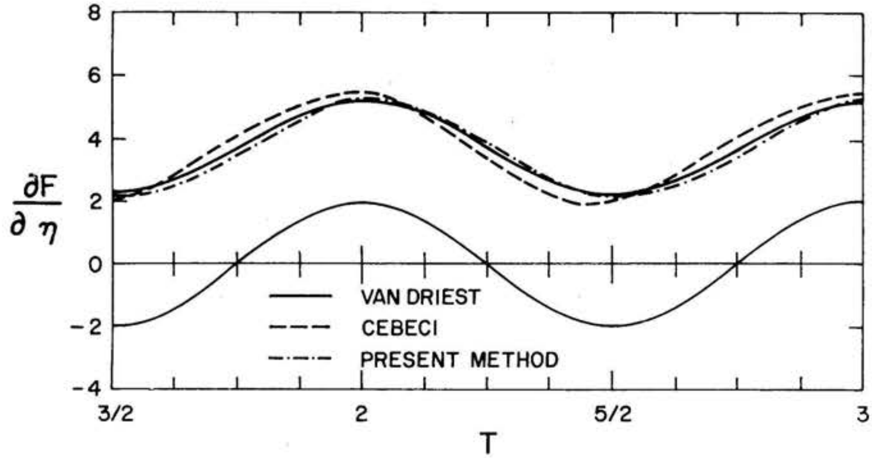


FIG. 9. The derivative $\partial F/\partial\eta$ as a function of time for $\alpha = 0.147$ and $f = 2$ Herz.

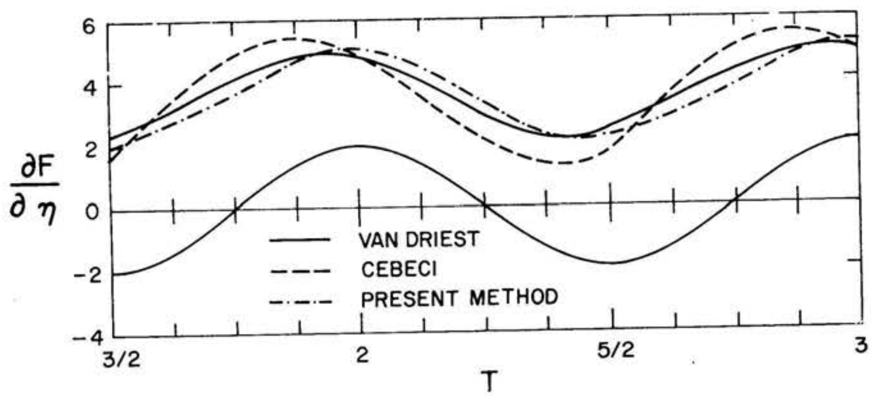


FIG. 10. The derivative $\partial F/\partial\eta$ as a function of time for $\alpha = 0.127$ and $f = 7.65$ Herz.

periodic functions are clearly detectable. In Fig. 10 and for $f = 7.65$, the differences are more pronounced. The Cebeci-Keller model appears to predict much larger phase advances. The experimental evidence in this case is rather inconclusive. This may be due to the inability of hot wire anemometry to measure velocities in the immediate vicinity of the wall.

5. Conclusions and recommendations

In the present paper we have collected the most well-known methods that are based on refinements of the mixing length concept and performed calculations for oscillatory flows over a flat plate. A few improvements on the Cebeci-Smith model were also included and the results were reported as "present method". The improvements consist of a correction in calculating the shear at the edge of viscous sublayer and a new definition of the displacement thickness. The two quantities appear in the inner and the outer model of the eddy viscosity, respectively. For the inner model a few assumptions had to be made with regard to the fluctuations of velocity within the laminar sublayer. Normally a larger number of calculations would have been performed, to indicate what values of the constants involved would provide better matching with the experimental results. This has not yet been attempted. Of the two refinements the first appeared to play a much more important role.

Calculations were performed only for a flat plate. The flow therefore never approached the neighborhood of zero shear. The present model though can be used without any alterations in order to integrate through a point of zero skin friction and into a region of partially-reversed flow. This has been done before with the Cebeci-Keller model (see, e.g., TELIONIS and TSAHALIS (1975)).

The reader may have noticed that there is an abundance of theories and predictions but very little experimental information available. This fact is disheartening especially since all the phenomenological models rely on experiment in order to estimate some of their arbitrary constants. All the efforts up to now, icluding the present, were confined in providing extensions of the existing models, based on theoretical arguments. As a result all the arbitrary constants or functions involved were carried over from comparisons with steady flows. At this point it is felt that more experimental data and perhaps more accurate ones, are badly needed. Certainly, more data for flat plate flow can be useful since none of the methods of calculation have been proved to be successful even in this oversimplified case. Then, of course, it will be necessary to have some experimental information on flows with mild or strong pressure gradients, flows with separation and flows that would involve regions of partially reversed flow.

6. Acknowledgment

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References

1. D. E. ABBOTT and T. CEBECI, in *Fluid dynamics of unsteady, three dimensional and separated flows*, ed. F. J. Marshall, 202, 1971.
2. I. E. ALBER, AIAA Paper No. 71-203, 1971.
3. O. R. BURGGRAF, Ohio University Engineering Report, 1974.
4. T. CEBECI and A. M. O. SMITH, in *Computation of turbulent boundary layers*, AFOSR-IFP-Stanford Conference, 1, 346, 1968.
5. T. CEBECI, AIAA Journal, 8, 2152, 1970.
6. T. CEBECI and H. B. KELLER, in *Recent research on unstedy boundary layers*, ed. E. A. Eichelbrenner, 2, 1072, 1972.
7. W. J. McCROSKEY and J. J. PHILIPPE, AIAA Journal, 13, 71, 1975.
8. E. R. VAN DRIEST, Journal of Aeronautical Sciences, 23, 1007, 1956.

9. H. A. DWYER, AIAA Journal, 11, 773, 1973.
10. S. K. F. KARLSSON, Journal of Fluid Mechanics, 5, 622, 1959.
11. W. M. KAYS, ASME Paper No. 71-HF-44, 1971.
12. B. E. LAUNDER and D. B. SPALDING, Computer Methods in Applied Mechanics and Engineering, 3, 269, 1974.
13. M. J. LIGHTHILL, Proceedings of the Royal Society, 224 A, 1, 1954.
14. G. MELLOR and H. J. HERRING, AIAA Journal, 11, 590, 1973.
15. J. A. MILLER, ASME Paper No. 69-67-34, 1969.
16. F. K. MOORE and S. OSTRACH, Journal of Aeronautical Sciences, 24, 77, 1957.
17. J. F. NASH, L. W. CARR and R. SINGLETON, AIAA Journal, 13, 167, 1975.
18. S. G. NYCHAS, H. C. HERSHEY and R. S. BRODKEY, Journal of Fluid Mechanics, 61, 513, 1973.
19. V. C. PATEL and J. F. NASH, in *Recent research on unsteady boundary layers*, ed. E. A. Eichelbrenner, 1, 1106, 1972.
20. W. C. REYNOLDS, Stanford University, Report No. MD-27, 1970.
21. S. J. SHAMROTH and J. P. KRESKOVSKY, NASA CR-132425, 1974.
22. R. E. SINGLETON and J. F. NASH, Proceedings of AIAA Computational Fluid Dynamics Conference, 84, 1973.
23. D. P. TELIONIS, in Unsteady Aerodynamics, ed. R. Kinney, 1, 155, 1975.
24. D. P. TELIONIS, D. Th. TSAHALIS and M. J. WERLE, Physics of Fluids, 7, 968, 1973.
25. D. P. TELIONIS and D. Th. TSAHALIS, AIAA Paper No. 75-27, 1975, also to appear in the AIAA Journal.
26. D. Th. TSAHALIS and D. P. TELIONIS, AIAA Journal, 12, 1469, 1974.

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