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Thermally-controlled centrifuge for isotopic separation

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AMONG the various methods proposed to obtain lighter component enrichment in the isotopic separation of uranium, ultracentrifugation is becoming more and more interesting today, as this process becomes a useful alternate method to gaseous diffusion. The ultracentrifuge main gasdynamic features are investigated in the present study. In particular, the field inside the centrifuge has been subdivided into three axial zones: A) an internal central zone, characterized by an essentially axial flow; B) two external zones, near the two caps of the centrifuge; C) two intermediate zones, of a length of the order of the radius. For the analytical solution the linearized Navier-Stokes equations have been considered. The central zone flow is solved by separating the independent variables; the corresponding eigenvalue problem has been solved numerically. A series of eigensolutions which satisfy boundary conditions at the walls of the cylinder has been calculated. An integral method for the superimposition of the above mentioned eigensolutions is proposed in order to satisfy the conditions at the tops for thermally-controlled centrifuges.

Wśród różnych metod proponowanych do otrzymywania lżejszej składowej wzbogacania przy oddzieleniu izotopów uranu stosowanie ultrawirówki staje się dziś coraz bardziej atrakcyjne, podobnie jak w dyfuzji gazów proces ten jest bardzo użytecznym i alternatywnym sposobem. W niniejszej pracy badane są główne cechy gazodynamiczne ultrawirówki. W szczególności pole wewnątrz wirówki podzielono względem jej osi na trzy strefy: A) strefa wewnętrzna, centralna scharakteryzowana przez główny przepływ osiowy; B) dwie strefy zewnętrzne w pobliżu dwóch kołpaków wirówki,; C) dwie strefy pośrednie o długości rzędu promienia. Celem uzyskania rozwiązania analitycznego rozważono zlinearyzowaną postać równań Naviera-Stokesa. Przepływ w strefie środkowej wyznaczono metodą rozdzielenia zmiennych niezależnych, przy czym odpowiedni problem na wartości własne rozwiązano numerycznie. Z rozwiązań własnych skonstruowano szereg spełniający warunki brzegowe na ściankach cylindra. Dla superpozycji wyżej wspomnianych rozwiązań własnych zaproponowano metodę całkową, by móc spełnić warunki na końcach sterowanych termicznie wirówek.

Среди разных методов, предложенных для получения более легкого компонента обогащения при разделении изотопов урана, применение ультрацентрифуги становится сегодня все более и более атракционным, и аналогично как в диффузии газов этот процесс является полезным и альтернативным способом. В настоящей работе иследованы главные газодинамические свойства ультрацентрифуги. В частности поле внутри центрифуги разделено, по отношению к ее оси, на три зоны: А) центральная внутренная зона охарактеризована главным осевым течением: Б) две внешние зоны вблизи двух колпаков центрифуги: В) две промежуточные зоны, с длиной порядка радиуса. С целью получения аналитического решения рассмотрен линеаризованный вид уравнений Навье-Стокса. Течение в центральной зоне определено методом разделения независимых переменных, причем соответствующая задача на собственные значения решена численно. Из собственных решений построен ряд удовлетворяющий граничным условием на стенках цилиндра. Для суперпозиции выше упомянутых собственных решений предложен интегральный метод, чтобы можно было удовлетворить условиям на концах управляемых термически центрифуг.

Symbols

f $\exp\left(-M^2\dfrac{1-x^2}{2}\right)$,

g correction factor,

l half length of the centrifuge,

p pressure perturbation,

$\mathbf{q}$ velocity,
r radial coordinate,
u, v, w adimensional components of the velocity,
x r/R,
y z/R,
z axial coordinate, evaluated from the plane of symmetry of the centrifuge,
K conductivity of the gas,
°K degree Kelvin,
M $\omega R/V$,
P radial factor of the pressure,
R radius of the centrifuge,
T temperature perturbation,
V $[p_0(1)/\varrho_0(1)]^{1/2}$,
X, Y, Z radial factors of the velocity components,
β_n n^{th} eigenvalue,
δ cap zone thickness,
ε $q(0, 0)/V_p$, perturbation parameter,
ζ $\eta/\delta(x)$,
η $\lambda - y$,
ϑ radial factor of the temperature,
λ l/R,
μ viscosity of the gas,
ϱ density perturbation,
φ azimuthal coordinate,
ω angular velocity of the centrifuge,
p_0 rigid rotation dimensional pressure,
ϱ_0 rigid rotation dimensional density,
T_0 average temperature of the gas, in degrees Kelvin,
T_λ adimensional temperature perturbation of the top of the centrifuge,
V_p ωR, peripheral speed,
Re $R^2\omega\varrho_0(1)/\mu$,
Ce $\omega^3 R^4 \varrho_0(1)/KT_0$,
S_e M^2,
R_0 zero point for the axial component of velocity.

1. Introduction

AMONG the various proposed methods for the enrichment of uranium in its lightest isotope U_{92}^{235}, ultra-centrifugation plays a fundamental role. In fact, this process is considered to be the only concrete alternative to gaseous diffusion [1, 2, 3].

Uranium, brought to the gaseous state in the form of UF_6, is centrifugated; consequently, the axial zone is enriched in the lightest component while the peripheral zone is enriched in the heaviest component.

In order to obtain appreciable results, the small difference between the molecular weights of the two isotopes requires very elevated peripheral velocities (over 300 m/sec) and a considerable series of stages.

If a recirculation is generated in the centrifuge a variation in the concentration is obtained in the axial direction as well, [4], in such a way that by extracting from and injecting the fluid in the proper zones, it is possible to increase the efficiency of each stage considerably. Gaseous recirculation can be generated by means of proper asymmetries of a mechanical or thermic type (for example, by bringing the tops to different temperatures).

The practical impossibility of carrying out experimental measurements in the fluid-dynamic field and the requirement of knowing such a field in order to utilize the technique, has greatly encouraged studies of a theoretical nature; however, such studies are only in part able to face the problem in its entirety and complexity.

For this reason the fluid-dynamic field can be studied only at the price of making notable simplifications.

Afterwards, the thermically-controlled centrifuge will be taken into consideration by assuming that the jump in temperature between the tops is such that the recirculation velocities are small as compared to the peripheral velocity. Assuming as the perturbation parameter, ε, the ratio between the velocity of the fluid at the center of the centrifuge $w(0,0)$ and the peripheral velocity ωR and having developed all the quantities in a power series of such a parameter, the equations which govern the phenomenon are of the first order [14, 15]:

$$\nabla\cdot f\mathbf{q} = 0, \tag{1.1}$$

$$-\mathbf{x}\varrho\mathbf{i}+2\mathbf{k}\wedge\mathbf{q}+\frac{1}{M^2 f}\nabla(fp) = \frac{1}{\mathrm{Re} f}\nabla^2\mathbf{q}+\frac{1}{3\,\mathrm{Re} f}\nabla(\nabla\cdot\mathbf{q}), \tag{1.2}$$

$$\nabla^2 T+\frac{\mathrm{Ce}}{M^2}\mathbf{q}\cdot\nabla f = 0, \tag{1.3}$$

$$p = \varrho+T. \tag{1.4}$$

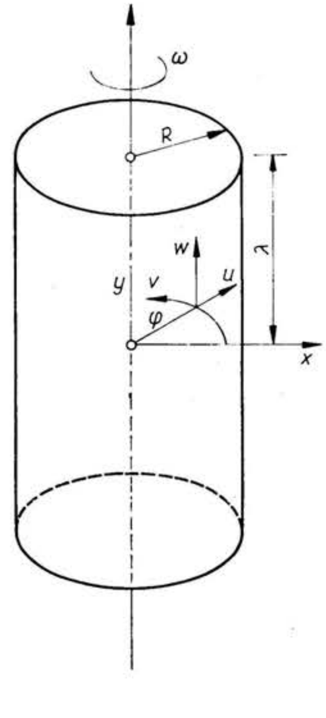

FIG. 1. General view of the centrifuge.

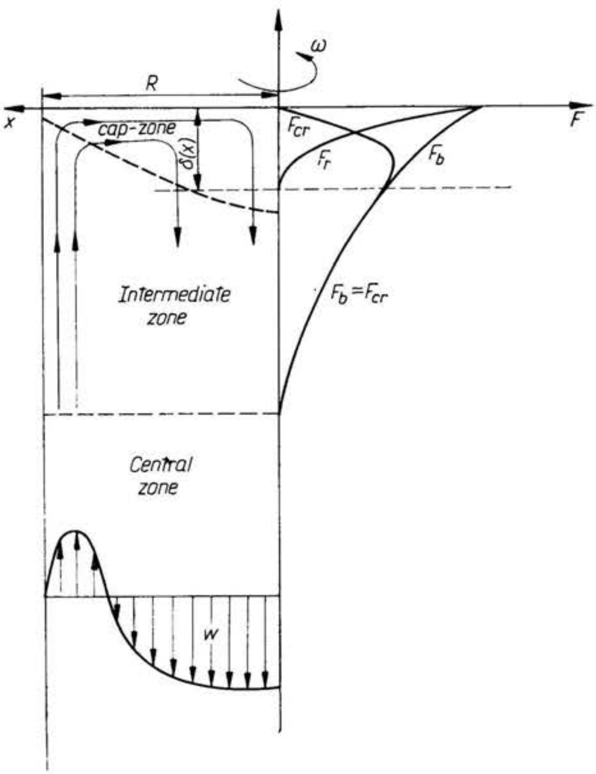

FIG. 2. Behaviour of the radial forces, of the velocity components and of the temperature in the three zones.

In these equations the terms due to the effect of gravity have been neglected since, due to the high rotational speeds at work, they prove to be a few orders of magnitude inferior as compared to those due to centrifugal forces.

The centrifuge (Figs. 1 and 2) can be subdivided into three typical zones in the axial direction.

A — central zone, with a substantially axial motion in which radial forces are absent.

B — cap-zone in which the buoyancy forces $\mathbf{F}_b$ of a thermal nature generate an intense radial motion and, by means of the Coriolis force $\mathbf{F}_{ct}$, an azimuthal motion. This motion in turn generates a radial Coriolis force, $\mathbf{F}_{cr}$, which is contrary to the effect of $\mathbf{F}_b$; the resultant of the radial forces is attenuated in the cap-zone within the thickness $\delta(x)$ $\left(\delta(x) \simeq \pi/\sqrt{\mathrm{Re}\cdot f(x)}\right)$; at the edge of this zone the radial velocity tends towards zero, while the velocities and the temperatures remain different from zero [8, 9, 10, 11].

C — intermediate zone, of a thickness of the order of the radius of the centrifuge, in which each one of the two radial forces (thermic and Coriolis) almost equal and opposite to each other, is attenuated exponentially toward the central zone because of the friction and the thermic conduction; in a parallel way the temperature and the azimuthal velocity are reduced to zero exponentially.

2. Intermediate zone

On the boundary between the intermediate and cap zones radial velocity goes nearly to zero while the azimuthal velocity v remains finite $v_*(x)$.

Thus, after imposing $u = 0$, the azimuthal equation of motion becomes:

$$\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}+\frac{v}{x}\right)+\frac{\partial^2 v}{\partial y^2} = 0 \tag{2.1}$$

with the following boundary conditions:

$$v(x, 0) = v(0, y) = v(1, y) = 0, \tag{2.2}$$

$$v(x, \lambda) = v_*(x). \tag{2.3}$$

By separating the variables through the position $v = V(y)Y(x)$, the following equations are obtained:

$$V(y) = \operatorname{senh}\beta y, \tag{2.4}$$

$$\frac{d}{dx}\left(\frac{dY}{dx}+\frac{Y}{x}\right)+\beta^2 Y = 0. \tag{2.5}$$

The solution of the differential equation (2.5) is the Bessel function $J_1(\beta x)$ of the first kind and of the order of 1; from the wall conditions the acceptable values β_n for β are deduced

$$J_1(\beta_n) = 0,$$

v is obtained by means of a superimposition

$$v = \sum_{1}^{\infty}{}_n\, a_n \operatorname{senh}(\beta_n y)\cdot J_1(\beta_n x), \tag{2.6}$$

where the constants a_n are to be calculated in order to satisfy the condition at the top (2.3). Thus, with the hypothesis of zero radial velocity outside the cap-zone, the azimuthal velocity attenuates exponentially towards the center of the centrifuge

$$v \simeq F_1(x)e^{-\beta_1\eta}+F_2(x)e^{-\beta_2\eta}+\ldots, \tag{2.7}$$

where $\beta_1 = 3.832$ and η is the distance from the top. It can be deduced that v vanishes in a distance of the order of the radius.

Since the coefficients which multiply the terms, where u appears in the azimuthal equilibrium equation and in the energy equation, are very large, (Re and Ce being of the order 10^7) it is not possible to neglect them in the case in which u is not exactly zero.

The complete equations are the following:

$$\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}+\frac{v}{x}\right)+\frac{\partial^2 v}{\partial y^2} = 2\,\mathrm{Re} fu, \tag{2.8}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{1}{x}\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial y^2} = -\mathrm{Ce}\, xfu. \tag{2.9}$$

Furthermore, from the equation of the radial impulse it is deduced that

$$2v - xT \simeq 0 \tag{2.10}$$

since, except near the wall, it results that $\frac{\partial p}{\partial x} = 0$. This conclusion is true for the case of the isolated side wall, while in the case of the wall having a fixed temperature, different from zero, it is no longer possible to neglect $\frac{\partial p}{\partial x}$.

By combining the preceding equations, the following is obtained

$$\frac{\partial^2 T}{\partial x^2}+\frac{1+3ax^2}{1+ax^2}\frac{1}{x}\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial y^2} = 0, \tag{2.11}$$

where $a = \mathrm{Ce}/4\mathrm{Re}$; this equation is solved by separating the variables through the position

$$T(x, y) = \theta(x)\cdot \mathrm{senh}(\beta y), \tag{2.12}$$

where θ satisfies the equation

$$\theta''+\frac{1+3ax^2}{1+ax^2}\frac{1}{x}\theta'+\beta^2\theta = 0. \tag{2.13}$$

Since it results that constant a is of the order of unity, Eq. (2.13) gives for $\theta(x)$ a behaviour of the type similar to that of the Bessel function $J_0(\beta x)$ of the first kind and of the order of zero. The values of β are deduced from the conditions at the limits $T(1, y) = 0$.

Since $\beta_1 = 2.4$ in this case also it is deduced that the temperature and thus, for Eq. (2.10), the azimuthal velocity are attenuated exponentially in the axial direction, in a distance of the order of the radius.

3. Central zone

The central zone has been solved [15] by separating variables through the following positions:

$$\begin{aligned} u &= \beta\sinh\beta y\cdot X(x),\\ v &= \beta\sinh\beta y\cdot Y(x),\\ w &= \cosh\beta y\cdot Z(x),\\ p &= \beta^{-1}\sinh\beta y\cdot P(x),\\ T &= \beta^{-1}\sinh\beta y\cdot\theta(x). \end{aligned} \tag{3.1}$$

Having projected the equation of motion along the axes, the system $(1.1\div1.4)$ becomes:

$$X'+\frac{1}{x}X+\mathrm{Se}\,xX+Z = 0, \tag{3.2}$$

$$\mathrm{Se}^{-1}P' = 2\beta^2 Y-x\theta+\frac{\beta^2}{\mathrm{Re}f}\left(\beta^2 X+\frac{4}{3}\mathrm{Se}\,x(\mathrm{Se}\,xX+Z)-Z'\right), \tag{3.3}$$

$$\frac{d}{dx}\left(Y'+\frac{Y}{x}\right)+\beta^2 Y = 2\,\mathrm{Re}fX, \tag{3.4}$$

$$Z''+\frac{1}{x}Z'+\beta^2 Z = \frac{\mathrm{Re}f}{\mathrm{Se}}P+\frac{1}{3}\beta^2\,\mathrm{Se}\,xX, \tag{3.5}$$

$$\theta''+\frac{1}{x}\theta'+\beta^2\theta+\beta^2\,\mathrm{Ce}fxX = 0, \tag{3.6}$$

the boundary conditions being the following

$$(3.7)\qquad \begin{aligned} X(0) &= Y(0) = Z'(0) = \theta'(0) = 0, \\ X(1) &= Y(1) = Z(1) = \theta'(1) = 0, \end{aligned}$$

the resulting series of eigenvalues β has been evaluated in a previous work [15]. The corresponding eigenfunctions have to be combined in order to satisfy proper conditions at the tops. As an example, the trends of the first eigenfunctions for $w(x)$ are shown in Fig. 3.

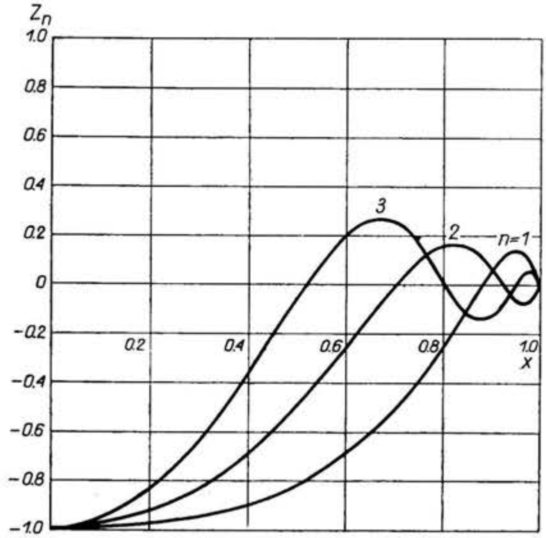

FIG. 3. Radial behaviour of the first three eigenfunctions, relative to the axial velocity, $V_p = 300$ m/sec., $R = 10$ cm.

4. Cap-zone

From the previous observations it can be deduced that the axial velocity is essentially independent of variable y, except in the cap-zone where it is brought to zero and that the resulting radial force, expressed by the term $(2v - xt)$ is essentially zero within the intermediate zone. As a first approximation we could thus assume that the fluid dynamic behaviour in the intermediate zone does not influence the other zones of the centrifuge: in this situation it is possible to connect the cap-zone directly to the central zone.

A criterion which allows to find a valid solution within the cap-zone and which, at the same time, gives a superimposition criterion for the eigensolutions valid in the central zone is suggested.

Let

$$(4.1)\qquad u_c(x, y) = \sum_{1}^{N} {}_n A_n X_n(x) \cdot \beta_n \sinh \beta_n y,$$

(4.1) [cont.]

$$v_c(x,y) = \sum_{1}^{N}{}_n A_n Y_n(x)\beta_n \sinh\beta_n y,$$

$$w_c(x,y) = \sum_{1}^{N}{}_n A_n Z_n(x)\cosh\beta_n y,$$

$$p_c(x,y) = \sum_{1}^{N}{}_n A_n P_n(x)\beta_n^{-1}\sinh\beta_n y,$$

$$T_c(x,y) = \sum_{1}^{N}{}_n A_n \vartheta_n(x)\beta_n^{-1}\sinh\beta_n y$$

be the solution in the central zone and

$$\begin{aligned} u(x,y) &= u_c(x,y) - u_b(x,\eta),\\ v(x,y) &= v_c(x,y) - v_b(x,\eta),\\ w(x,y) &= w_c(x,y) - w_b(x,\eta),\\ p(x,y) &= p_c(x,y) - p_b(x,\eta),\\ T(x,y) &= T_c(x,y) + T_b(x,\eta) \end{aligned} \tag{4.2}$$

be the complete solution within the cap-zone. The second terms of the second members of (4.2) are corrective terms which are zero in the central zone, and make it possible to verify the conditions at the top. In $\eta = \delta$ it results that:

$$u_b \equiv w_b \equiv p_b \equiv 0, \tag{4.3}$$

while v_b and T_b do not turn out to be zero, in order to account for the already observed fact that the azimuthal velocity and the temperature tend to zero exponentially throughout the whole intermediate zone; the link between these last two quantities at $\eta = \delta$ is given by:

$$2v_b + xT_b = 0.$$

Since the equations used up to this point are linearized, they cannot be taken to study the fluid dynamic behaviour near the edge ($x = 1$; $y = \lambda$); in order to account for this and to avoid inconsistencies which may be brought about by the equations in this zone, a $g(\varkappa)$ factor equal to one in the whole field, except near the wall where it is rapidly brought to zero, is introduced.

Having accounted for boundary layer simplifications, the equations in the cap zone become

$$2\delta^2 \,\mathrm{Re}\, f u_b = \frac{\partial^2 v_b}{\partial \zeta^2} g, \tag{4.4}$$

$$\mathrm{Ce}\, x\delta^2 f u_b = \frac{\partial^2 T_b}{\partial \zeta^2} g^2, \tag{4.5}$$

$$\frac{\partial^2 u_b}{\partial \zeta^2} = -\delta^2 \,\mathrm{Re} f (xT_b g^2 + 2v_b g), \tag{4.6}$$

where $\zeta = \eta/\delta(x)$.

The w_b can later be deduced from the equation of continuity. For the quantities u_b, v_b and T_b within the cap-zone polynominal expansions are assumed such that $u_b(x, \delta) = 0$ may result and the first and second derivatives of these quantities become zero at the extreme boundary of this zone

$$
\begin{aligned}
T_b &= D_1 + (D_2 + D_3\zeta)(1-\zeta)^3,\\
v_b &= E_1 + (E_2 + E_3\zeta)(1-\zeta)^3,\\
u_b &= (B_1 + B_2\zeta + B_3\zeta^2)(1-\zeta)^3.
\end{aligned}
\tag{4.7}
$$

The ten quantities δ, B_k, D_k, E_k ($k = 1, 2, 3$) are determined by imposing first the following three conditions at the tops:

$$
\begin{aligned}
u_b(x, 0) &= \sum_1^N{}_n A_n X_n(x)\cdot \beta_n \sinh\beta_n\lambda = u_c(x, \lambda),\\
v_b(x, 0) &= \sum_1^N{}_n A_n Y_n(x)\cdot \beta_n \sinh\beta_n\lambda = v_c(x, \lambda),\\
T_b(x, 0) &= T_\lambda(x) - \sum_1^N{}_n A_n \vartheta_n(x)\frac{\sinh\beta_n\lambda}{\beta_n} = T_\lambda(x) - T_c(x, \lambda).
\end{aligned}
\tag{4.8}
$$

If, as has already been stated, we forego giving a detailed description of the intermediate zone ($\beta_n \simeq 1$) and we utilize only the first eigenvalues ($\beta_n \ll 1$), we obtain

$$
D_1 + D_2 = T_\lambda(x), \quad E_1 + E_2 = 0, \quad B_1 = 0.
\tag{4.9}
$$

Three conditions are then given from the differential Eqs. (4.4), (4.5) and (4.6) applied to the top:

$$
\begin{aligned}
E_2 - E_3 &= 0,\\
D_2 - D_3 &= 0,\\
B_2 - \frac{1}{3}B_3 &= \frac{1}{6}\mathrm{Re}\,\delta^2 f x T_\lambda g^2,
\end{aligned}
\tag{4.10}
$$

one condition by applying (4.6) to the boundary of the cap-zone

$$
xD_1 g + 2E_1 = 0.
\tag{4.11}
$$

Note that Eqs. (4.4) and (4.5) are identically satisfied at $\zeta = 1$ from the polynominal positions assumed.

The last three conditions are obtained by integrating Eqs. (4.4), (4.5) and (4.6) within the cap zone

$$
\begin{aligned}
\mathrm{Re}\,\delta^2 f(3B_2 + B_3) &= 60\,E_2 g,\\
\delta^2 f\,\mathrm{Ce}\,x(3B_2 + B_3) &= 120\,D_2 g^2,\\
10B_2 &= 3\delta^2\,\mathrm{Re} f(xD_2 q^2 + 2E_2 q).
\end{aligned}
\tag{4.12}
$$

From the preceding relationships it results that

$$D_1 = \frac{4\,\mathrm{Re}\,T\lambda}{4\mathrm{Re}+\mathrm{Ce}\,x^2},$$

$$D_2 = D_3 = \frac{\mathrm{Ce}\,x^2 D_1}{4\,\mathrm{Re}},$$

$$E_2 = E_3 = -E_1 = 0.5\,x D_1 g,$$

$$B_1 = 0,$$

$$B_2 = 0.3\,x\,\mathrm{Re}\,\delta^2 f T_\lambda g^2,$$

$$B_3 = 4\,B_2/3,$$

$$\delta^2\,\mathrm{Re} f = \frac{60\sqrt{\mathrm{Re}}}{\sqrt{39(4\,\mathrm{Re}+\mathrm{Ce}\,x^2)}}.$$

Knowing $B_2(x)$ it is possible to calculate the coefficients A_n which appear in Eq. (4.1) from the continuity equation

$$\frac{\partial u_b}{\partial x}+\frac{u_b}{x}+M^2 x u_b = \frac{\partial w_b}{\partial \eta} \tag{4.13}$$

that is, by integrating

$$G'+\frac{G}{x}+M^2 xG+w_b(x,0) = 0 \tag{4.14}$$

with

$$G = \int_0^{\delta(x)} u_b\,d\eta = \frac{13}{180}B_2(x)\,\delta(x) \tag{4.15}$$

assuming that

$$H(x) = -\left(\frac{d}{dx}+\frac{1}{x}+M^2x\right)G(x) \tag{4.16}$$

and keeping in mind that $w_b(x,0) = w_c(x,\lambda)$ from (4.1) the following is obtained:

$$\sum_1^N{}_n A_n \cosh(\beta_n\lambda)Z_n(x) = H(x) \tag{4.17}$$

this relationship has been solved by means of the method of least squares

$$\int_0^1 \left|\sum_1^N{}_n A_n\cosh(\beta_n\lambda)Z_n(x)-H(x)\right|^2 x\,dx = \min. \tag{4.18}$$

5. Results

As an example, the equations set forth have been applied to a centrifuge having a 20 cm diameter and a length of 2 m.

The centrifuge rotates at the peripheral velocity of 300 m/sec with a peripheral pressure of 0.5 atmospheres, an average temperature of 330°K and 20°K difference of temper-

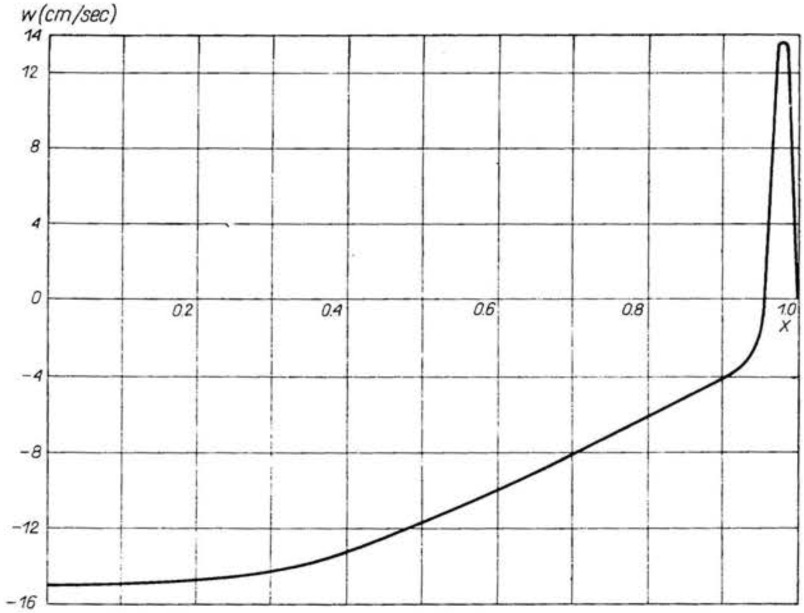

FIG. 4. Axial velocity in an actual case; $V_p = 300$ m/sec, $R = 10$ cm, $\Delta T = \pm 10$ degrees Kelvin.

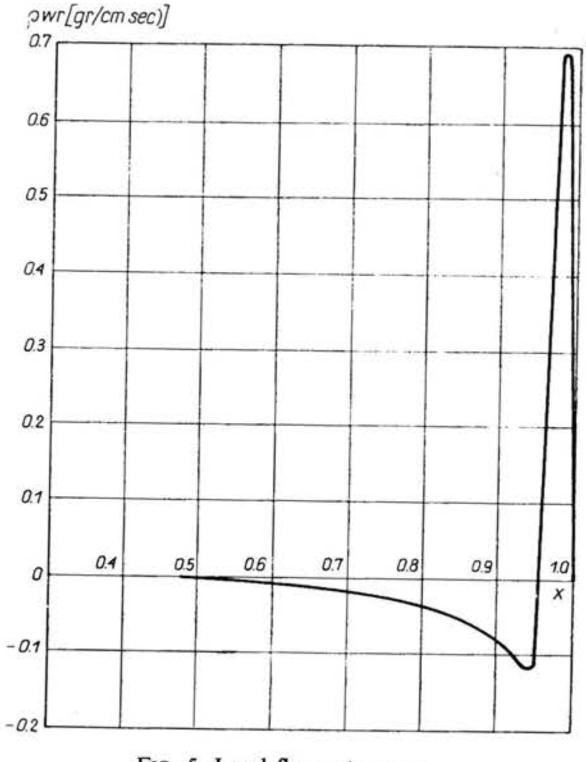

FIG. 5. Local flow rate $r\varrho_0 w$.

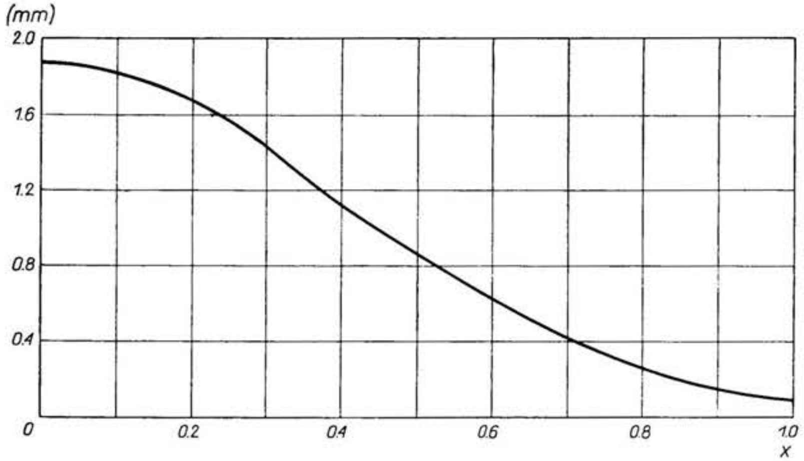

FIG. 6. Thickness of the cap-zone, $\delta(x)$.

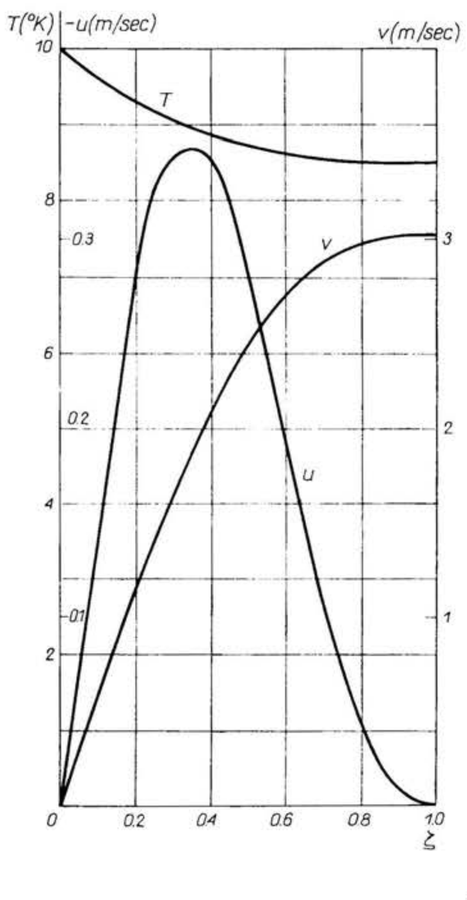

FIG. 7. Temperature and radial and azimuthal components of the velocity in the cap-zone at the station $r = 9.8$ cm.

ature between the two tops; the temperatures have been assumed to be uniform at the tops; it is not difficult, however, to examine cases in which this hypothesis of uniformity does not apply.

In Fig. 3 the radial trends of the first three eigenfunctions relating to the axial velocity are shown. Figure 4 gives the actual trend of the axial velocity; this quantity becomes zero at $r = 9.6$ cm and reaches a maximum in the periphery of 13.5 cm/sec. Figure 5 gives the local flow rate ($r\varrho w$) as a function of the radius; this flow rate has a maximum of 0.7 gr/(sec. cm), corresponding to a total recirculating flow rate of

$$\int_{R_0}^{R} 2\pi r\varrho w dr \simeq 2\,\text{gr/sec.}$$

In Fig. 6 the thickness $\delta(x)$, which varies from 1.88 mm. at the center to 0.1 mm at the periphery, is shown. Finally, Fig. 7 shows the temperature trend and the radial and azimuthal components of the velocity within the cap-zone, at the station $r = 9.8$ cm, the point at which the maximum axial velocity is reached in the central zone.

This study should be considered as the first step towards the complete solution which will take into account the intermediate zone as well.

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