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Numerical calculation of free convective flow around cylinders near rigid walls

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FREE convective flow around cylinders, which are positioned near walls, is investigated by a finite difference method. Streamline isothermal and the overall heatflux are calculated for different geometries. A thermal coanda effect, known already from experimental observations, is confirmed.

Stosując metodę różnic skończonych, zbadano swobodny konwekcyjny przepływ wokół walców usytuowanych w pobliżu ścian. Izotermiczne linie prądu i całkowity strumień ciepła policzono dla różnych parametrów geometrycznych. Potwierdzono termiczny efekt coanda, znany już z obserwacji eksperymentalnych.

Применяя метод конечных разностей исследовано свободное конвекционное течение около цилиндров расположенных вблизи стенок. Изотермические линии тока и полный поток тепла рассчитаны для разных геометрических параметров. Подтвержден термический эффект, «соанда» известный уже из экспериментальных наблюдений.

1. Introduction

THE OBJECT of the following article is the free convective flow around cylinders which are positioned near walls. The investigations were mainly aimed at clarifying certain limits of hot wire anemometry and finding perhaps new methods of application, for instance, in two-phase flow fields.

REIMANN (1972) performed experiments in order to study the influence of the distance between a heated cylinder and a neighbouring wall as well as the influence of the angle of inclination of the wall. During the experiments special attention was given to the development of the plume rising from the heated cylinder and the inclination of the convective jet towards inclined walls nearby the cylinder. It appeared desirable to describe some of the observed phenomena by means of a mathematical theory.

At present a theoretical approach to this problem seems feasible either analytically by applying similarity solutions or by using direct numerical methods. The influence of the wall restricts the first method to a larger wall distance. In the present investigation a direct numerical method is therefore preferred in order to consider all possible situations.

2. The physical problem and its mathematical formulation

In a viscous medium a heated cylinder is positioned near a wall, which is inclined at the angle α to horizontal direction (see Fig. 1).

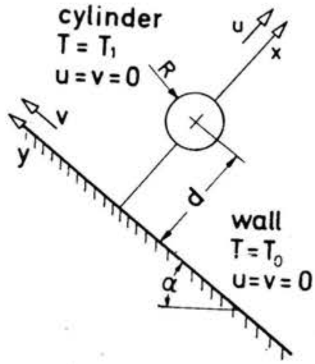

FIG. 1. Principal sketch.

At first, there is neither a motion nor a temperature gradient in the medium. At a certain time t_0 the temperature of the cylinder is raised to the temperature T_1 and kept at this level for all later times. The velocity and the temperature fields, which are generated by the temperature difference between the heated cylinder and the fluid — that is by buoyancy forces — are required for all later times. For technical applications the total heat transition at the cylinder and the wall is of special interest. This can be calculated from the temperature field. For a mathematical formulation of the process one starts with the transport equations for momentum, heat and mass. It is usually assumed that the density is constant in all terms of the transport equations except in the buoyancy term. Here, the usual linear relation between density ϱ and temperature T is used. This approach is known as Boussinesq approximation. As the process is considered to be two-dimensional, the velocity components are suitably reduced to a stream function. The pressure can be eliminated from the equations of momentum by introducing the vorticity function ψ. Hereon, it is useful to introduce dimensionless quantities by the following relations:

$$(x', y') = \frac{1}{R}(x, y), \quad \text{coordinates;} \quad t' = \frac{\nu}{R^2} t, \quad \text{time;}$$

$$\psi' = \frac{1}{\nu}\psi, \quad \text{stream function;} \quad T' = \frac{T-T_0}{T_1-T_0}, \quad \text{temperature.}$$

Here, R means the radius of the cylinder, T_1 and T_0 the temperatures of the cylinder and the wall respectively, and ν the kinematic viscosity. Under these aspects the free convective flow around a cylinder near a rigid wall can be described by an initial boundary value problem in the following form:

transport equations:

$$\frac{\partial \Omega}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \Omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \Omega}{\partial y} - \Delta\Omega = \mathrm{Gr}\left(\cos\alpha\frac{\partial T}{\partial y} - \sin\alpha\frac{\partial T}{\partial x}\right), \quad \text{vorticity,}$$

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} - \frac{1}{\mathrm{Pr}}\Delta T = 0, \quad \text{heat,}$$

$$-\Delta\psi = \Omega, \tag{2.1}$$

initial conditions:

$$t = 0: \quad T(x, y) = 0, \quad \psi(x, y) = 0;$$

boundary conditions:

$$t > 0: \quad \text{cyl.:} \quad T = 1, \quad \psi = 0, \quad \frac{\partial \psi}{\partial n} = 0;$$

$$\text{wall:} \quad T = 0, \quad \psi = f(t), \quad \frac{\partial \psi}{\partial n} = 0.$$

3. The numerical procedure

For a numerical treatment of the problem a finite difference method is employed. In order to simplify and to limit the numerical procedure a bipolar coordinate system ξ, η is introduced (see MILNE-THOMSON (1968), p. 176). These coordinate can be readily adapted to the geometrical configuration of the problem as can be seen from Fig. 2.

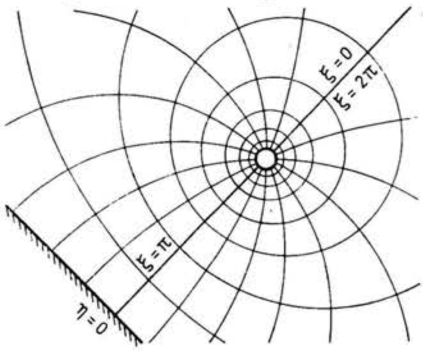

FIG. 2. Bipolar coordinates.

Such a procedure gives, in addition, the advantage that the physical problem originally posed in a doubly-connected half-infinitely extended domain is transformed to a finite simply connected rectangular domain by the mapping mechanism. The cylinder and the wall are mapped on two opposite sides of the rectangular domain. The other two sides are given by the mapping of a line of intersection along the positive y axis between the wall and the cylinder in the x, y-plane (see Fig. 3).

The initial boundary value problem can now be stated as follows:

transport equations

$$F\frac{\partial \Omega}{\partial t} + \frac{\partial \psi}{\partial \eta}\frac{\partial \Omega}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \Omega}{\partial \eta} - \Delta\Omega = \mathrm{Gr}\, F A, \quad \text{vorticity,} \tag{2.2}$$

$$F\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial \eta}\frac{\partial T}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial T}{\partial \eta} - \frac{1}{\mathrm{Pr}}\Delta T = 0, \quad \text{heat,}$$

$$-\Delta\psi = F\Omega,$$

$$F = \left(\frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial y} - \frac{\partial \eta}{\partial x}\frac{\partial \xi}{\partial y}\right)^{-1},$$

$$A = \left(\frac{\partial \xi}{\partial y}\cos\alpha - \frac{\partial \xi}{\partial x}\sin\alpha\right)\frac{\partial T}{\partial \xi} + \left(\frac{\partial \xi}{\partial x}\cos\alpha + \frac{\partial \xi}{\partial y}\sin\alpha\frac{\partial T}{\partial \eta}\right),$$

$$\Delta = \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2};$$

(2.2) initial conditions:
[cont]

$$t = 0, \quad T(\xi, \eta) = \psi(\xi, \eta) = 0;$$

boundary conditions:

$$t > 0, \quad T(\xi, 0) = \psi(\xi, 0) = \frac{\partial \psi}{\partial \eta}(\xi, 0) = 0,$$

$$T(\xi, \eta_1) = 0, \psi(\xi, \eta_1) = f(t), \frac{\partial \psi}{\partial \eta} \quad (\xi, \eta_1) = 0.$$

The boundary conditions for $\xi = 0$ and $\xi = \xi_1$ are fixed by postulating a periodic analytical continuation of the functions of state, that is,

$$T(0, \eta) = T(\xi_1, \eta), \quad \frac{\partial T}{\partial \xi}(0, \eta) = \frac{\partial T}{\partial \xi}(\xi_1, \eta), \quad \text{etc.},$$

$$\psi(0, \eta) = \psi(\xi_1, \eta), \quad \frac{\partial \psi}{\partial \xi}(0, \eta) = \frac{\partial \psi}{\partial \xi}(\xi_1, \eta), \quad \text{etc}.$$

The further procedure is now traced out.

An implicit finite difference method of the Crank-Nicolson type is chosen for the numerical calculations. The convection terms in the transport equations are linearized in such a way that the velocities in these terms are taken from the preceding time or iteration step. Here, $\frac{\partial \psi}{\partial n}$ is the derivative of the stream function in direction of the normal to the surface and $f(t)$ is a function of the time t only. By introducing dimensionless quantities two characteristic parameters appear in the differential equations, the Grashof number Gr and the Prandtl number Pr. The Grashof number gives a criterion for the intensity of the velocity and the deformation of the temperature field due to convection, the Prandtl number correlates the thickness of the boundary layers of the velocity and the temperature (for further details see SESTERHENN (1974)).

The solution of the resulting linear algebraic equations are gained blockwise on lines ξ = const and η = const starting with the temperature field and continuing with the

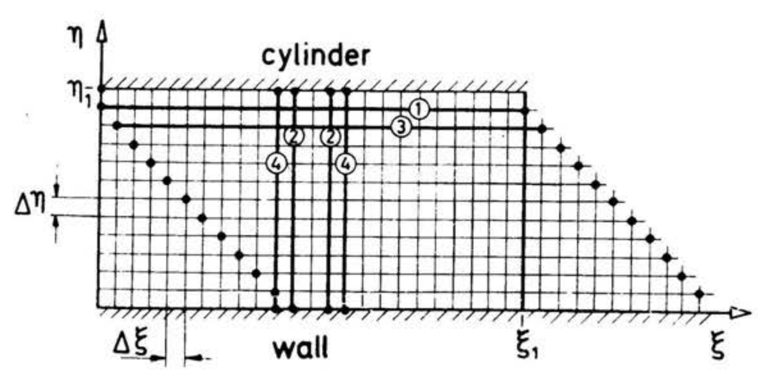

FIG. 3. Domain in the ξ, η-plane.

vorticity and stream function. Each block is then treated by a Gauss elimination process. Finally, an alternating direction line overrelaxation procedure (ADLOR) is applied to the system of blocks (for details see SESTERHENN (1974), FRICK (1975)). In the practical numerical processing a connection of the state functions on lines $\eta = \xi$ and $\eta = \xi + \xi_1$ has proved to be more advantageous as compared to a connection on lines $\xi = 0$ and $\xi = \xi_1$. The reason for this is to obtain an analytical (or numerical smooth) continuation of the unknown functions by using the differential equations on these lines (see Fig. 3).

The state variables are calculated in a double iteration procedure. At first the temperature is iterated and then, in an additional iteration cycle, the vorticity and the stream function are gained. A special problem of the numerical procedure is the fact that the boundary values of the vorticity function are not known *a priori*. These have to be rather calculated by iteration using the stream function and vorticity field of the preceding step. In literature a number of relations between the vorticity and the stream function on the boundary and in the neighbouring grid points are known. ROACHE (1972) has given a summary of such relations. In the studied case the relation

$$\frac{F_0 K^2}{3L^2}\Omega_0 + \frac{F_1 K^2}{6L^2}\Omega_1 = \psi_0 - \psi_1, \quad K = \frac{1}{\Delta\xi}, \quad L = \frac{1}{\Delta\eta}$$

which was first presented by THOM and APELT (1961) has proved to be the most reliable one with respect to the convergence of the iteration procedures. It should be mentioned that this relation represents the first terms of a Taylor series of the stream function with respect to a point on the boundary with the index 0.

A proof of the stability and convergence of the finite difference method in an analytical way is not given. However, the quality of the solution was investigated numerically by a variation of the steps in the time and spatial coordinates. Furtheron, the numerical method was scrutinized by comparing the calculated heat transition on a cylinder in a infinitely extended fluid with a curve of experimental data which were summarized by MCADAMS (1954). The comparison can be seen in Fig. 4. The solid line represents the experimental values, the circles are calculated values of the Nusselt number where Nu is defined as:

$$\mathrm{Nu} = -\frac{1}{2\pi}\int_0^{2\pi} \frac{\partial T}{\partial n}\bigg|_{T=1} d\varphi.$$

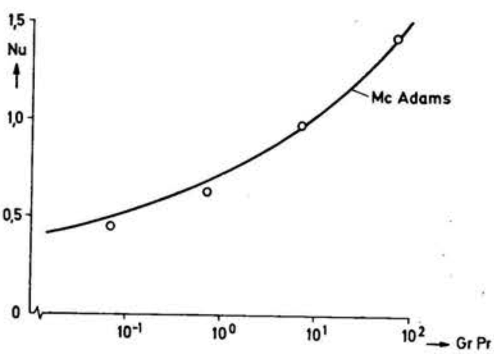

FIG. 4. Curve of heat transfer for the convective flow around a cylinder,
— experimental values, ∘ calculated values.

4. Interpretation of the results

In this chapter some characteristic results of the calculations are discussed. Figure 5 shows isothermals and streamlines of a rising hot plume which was separated from a hot cylinder yet is still fed with hot fluid through a convective jet.

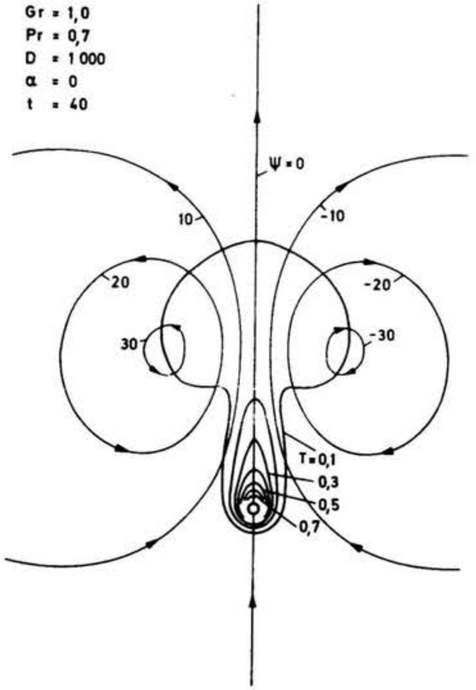

FIG. 5. Isothermals and stream-lines of a rising plume at a fixed time.

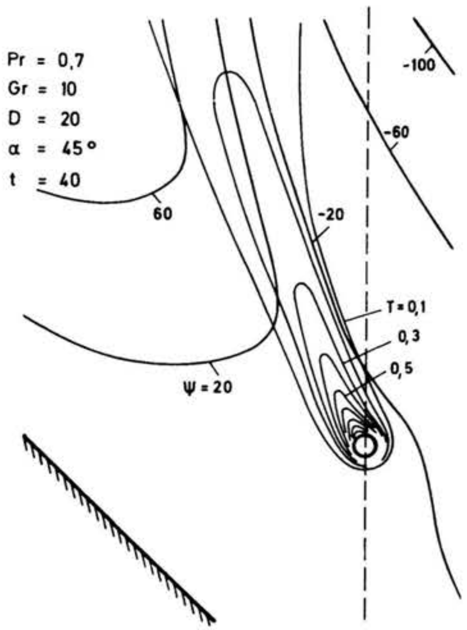

FIG. 6. Isothermals and stream-lines of a convective jet near an inclined wall.

The value of the Grashof number is 1, a comparatively small value, which results in a slow development of the plume. Therefore, a long computing time is necessary for a comparatively coarse grid. It should be mentioned that the rising plume never reaches a steady state; rather the numerical procedure breaks down before the size of the plume has become comparable with the mesh size of the grid. This naturally happens with increasing distance between the plume and the cylinder because the mesh size for bipolar coordinates is not constant. However, the stream function and the temperature in the rising jet near the cylinder have reached a steady state before the procedure fails. Thus a series of questions concerning the heat transfer at the cylinder can be answered. This will be discussed later.

The behaviour of the flow near the inclined walls is demonstrated by Fig. 6.

As shown by the experiments of REIMANN (1974) the inclined wall near heated cylinders induces a deviation of the convective jet from the vertical line according to RIEMANN (1972). The physical interpretation for this is that in a first approximation it can be assumed that the amount of entrained fluid is the same on both sides of the jet. However, due to the inhibition of the wall on one side the velocity of the fluid streaming towards the jet is, in the average, higher on the near wall side. This leads to a lower static pressure and the jet inclines towards the wall.

Apart from these predominant qualitative results the prediction of the heat transfer function at the cylinder is of major interest. In Fig. 7 this function, which is identical with the previously defined Nusselt number, is plotted versus the time for various Grashof numbers.

In order to find out the characteristic behaviour with respect to the history of the heat transfer, the influence of the wall was neglected by setting the angle of inclination of the wall $\alpha = 0$ and the dimensionless distance between cylinder and wall $D = 10^3$. The shape of the curves is directly related to the experimental observations. At first the heat transition decreases as a thermal boundary layer builds up around the heated cylinder. When the separation process between the fluid lump and the cylinder starts the Nusselt number increases sharply and over-shoots a steady state value. That is finally reached after a regeneration of the thermal boundary layer. It is clear that this phenomenon has a stronger influence at higher Grashof numbers. The envelope of all convective curves is of course given by the state of pure heat conduction which is represented by the lowest curve with Gr = 0 in Fig. 7.

The influence of a neighbouring wall on the heat transition from the cylinder can be seen in Fig. 8.

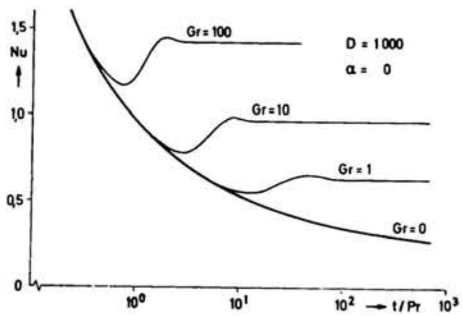

FIG. 7. History of the heat transfer from a heated cylinder.

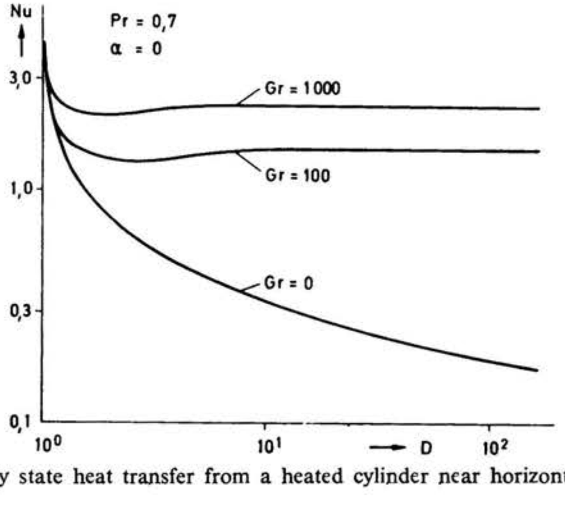

FIG. 8. Steady state heat transfer from a heated cylinder near horizontal walls.

The values of the Nusselt number are steady state values in the sense of the previous explanations concerning Fig. 5. In Fig. 8 the Nusselt number is plotted versus the distance D between the cylinder and the wall for different values of the Grashof numbers.

The wall itself is horizontally positioned. Considering the shape of the curves two different ranges can be distinguished. If the heated cylinder is close to the wall the heat from the cylinder is predominantly transferred by conduction.

It follows that the heat transfer decreases with increasing distance between cylinder and wall. This results from the fact that on one side the convective flow is still inhibited by the wall and on the other side the heat transfer by conduction decreases.

When the distance between cylinder and wall increases the rate of the heat transfer by convection becomes predominant. Finally, an undisturbed convection is possible and the Nusselt number becomes independent of the wall distance.

It should be mentioned that the generation of such a plot is timeconsuming and requires a long computing time. This is the reason why a similar investigation for different angles of inclination of the wall has not yet been performed.

References

1. H. FRICK, *Freie Konvektion in zylindrischen Geometrien mit inneren Wärmequellen*, Diplomarbeit Universität Karlsruhe 1975.
2. L. M. MILNE-THOMSON, *Theorethical hydrodynamics*, Mc Millan, London 1968.
3. J. RIEMANN, *Freie Konvektionsströmung kleiner Grashof-Zahl, verursacht durch waagrechte Zylinder, unter dem Einfluss einer ebenen Wand*, Dissertation Universität Karlsruhe 1972.
4. P. J. ROACHE, *Computational fluid dynamics*, Hermosa Publishers, Albuquerque 1972.
5. A. SESTERHENN, *Numerische Berechnung der freien Konvektion um beheizte Zylinder*, Dissertation Universität Karlsruhe 1974.

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