Archives of Mechanics • Archiwum Mechaniki Stosowanej • 28, 5-6, pp. 817-825, Warszawa 1976

Shock attenuation in beds of granular solids

N. ZLOCH (DARMSTADT)

ATTENUATION of a shock wave travelling through beds of granular solids has been investigated experimentally and theoretically. For diverse Mach numbers and diverse Reynolds numbers shock attenuation has been measured with transducers in various places along the axis of a tube which is a part of a bundle of very long tubes; this bundle is a simple model of a bed of solids. The experimental results closely correspond to the theoretical results of a numerical computation carried out by means of the method of characteristics and thereby confirm the simplifying assumptions which were made (one-dimensional problem, fully-developed flow in the total flow field behind the shock wave). For small initial Mach numbers ($M_s - 1 \ll 1$) an approximate solution was found, based on the assumption that the first C_1-characteristic nearly coincides with the shock wave. This is analogous to Witham's assumption. This approximate solution can be generalised, so that it can be valid also for stronger shocks, by introducing the factor $G(M_{s0})$. A following experimental investigation with flow through beds of spherical pebbles has shown that the solution mentioned is succesfully applicable to such configurations as well, provided the assumptions made for the computations are not too seriously violated.

Zbadano doświadczalnie i teoretycznie malenie amplitudy fali uderzeniowej wędrującej przez warstwy ośrodków granulowanych. Dla różnych liczb Macha i różnych liczb Reynoldsa spadek amplitudy fali uderzeniowej mierzono czujnikami umieszczonymi w różnych miejscach wzdłuż osi rury będącej częścią wiązki bardzo długich równoległych rur. Wiązka taka stanowi prosty model złóż ciał stałych. Dane doświadczalne ściśle pokrywają się z wynikami obliczeń numerycznych otrzymanych metodą charakterystyk; potwierdzają więc słuszność poczynionych założeń upraszczających (problem jednowymiarowy, w pełni rozwinięty przepływ w całym obszarze przepływu za falą uderzeniową). Dla małych początkowych liczb Macha ($M_s - 1 \ll 1$) znaleziono rozwiązanie przybliżone przy założeniu, że pierwsza charakterystyka C_1 pokrywa się prawie całkowicie z falą uderzeniową. Założenie to jest analogiczne do założenia poczynionego przez Withama. Powyższe rozwiązanie przybliżone może być uogólnione przez wprowadzenie współczynnika $G(M_{s0})$ tak, aby mogło obejmować również silniejsze fale uderzeniowe. Przeprowadzone następnie badania doświadczalne dotyczące przepływów przez złoża kulistych kamyków wykazały, że rozwiązanie to może być z powodzeniem stosowane dla takich konfiguracji pod warunkiem, że założenia poczynione przy obliczeniach nie są zbyt dalece naruszane.

Исследовано экспериментально и теоретически уменьшение амплитуды ударной волны распространяющейся через слои гранулированных сред. Для разных чисел Маха и разных чисел Рейнольдса падение амплитуды ударной волны измерялось датчиками помещенными в разных местах вдоль оси трубы, будучей частью пучка очень длинных параллельных труб. Этот пучок составляет простую модель залежи твердых тел. Экспериментальные данные точно совпадают с результатами численных расчетов полученных методом характеристик, итак подтверждают они справедливость проведенных упрощающих предположений (одномерная задача, вполне развернутое течение в целой области течения за ударной волной). Для малых начальных чисел Маха ($M_s - 1 \ll 1$) найдено приближенное решение при предположении, что первая характеристика C_1 совпадает почти полностью с ударной волной. Это предположение аналогично предположению сделанному Витхемом. Вышеупомянутое приближенное решение, путем введения коэффициента $G(M_{s0})$, может быть обобщено так, чтобы охватило тоже более сильные скачки. Затем проведены экспериментальные исследования, касающиеся течений через залежи сферических камней, и они показали, что это решение может быть с успехом применено для таких конфигураций под условием, что предположения, проведенные при расчетах, слишком далеко не нарушаются.

1. Nomenclature

a sound velocity,
B empirical factor of Bannister,
C characteristic curve,
c_f friction factor for tubes,
D diameter of the tubes,
d diameter of the particle,
F force per unit volume,
f friction factor for beds of solids,
G factor, given by Eq. (4.5),
L length of the bundle of tubes,
M Mach number,
p pressure,
q added heat per unit volume and unit time,
R constant of gas,
Re Reynolds number,
r radius of the tube,
r_p radius of the particle,
S shock wave,
T temperature,
t time,
u velocity,
x space coordinate along the shock tube,
γ ratio of specific heats,
ε ratio of the average free cross section to the total cross section,
η viscosity,
ϱ density.

2. Experimental investigation by the bundle of tubes

FOR the first experiments, a bundle of very thin tubes was installed in the shock tube of the TH Darmstadt [1]. This bundle of tubes was a simple model of a bed of solids. It consisted of many tubes with a very large ratio of length to diameter of the tube, $L/D \simeq 10^3$. The free cross section of the bundle was much smaller than the free cross section of the shock tube (Fig. 1).

As a consequence of this large reduction of the open area, the incident shock wave S_i was reflected nearly completely at the entrance of the bundle. Therefore, the particle velocity in front of the bundle could be neglected and state *4* of the gas was constant for the time Δt_m. This is the time between the arrival of the first incident shock wave S_i and the next wave S_n. The constancy of state *4* was experimentally confirmed by measuring pressure immediately before the bundle. By choosing the large ratio L/D it was intended to keep the zones, where the boundary layers at the entrance and behind the shock grow together, small in comparison to the total flow field.

The measurements were performed with piezo-electric manometers in various places x_i along the axis of one representative tube. The pressure jump p_2/p_1 across the shock wave was used as a direct measure for the strength of the shock.

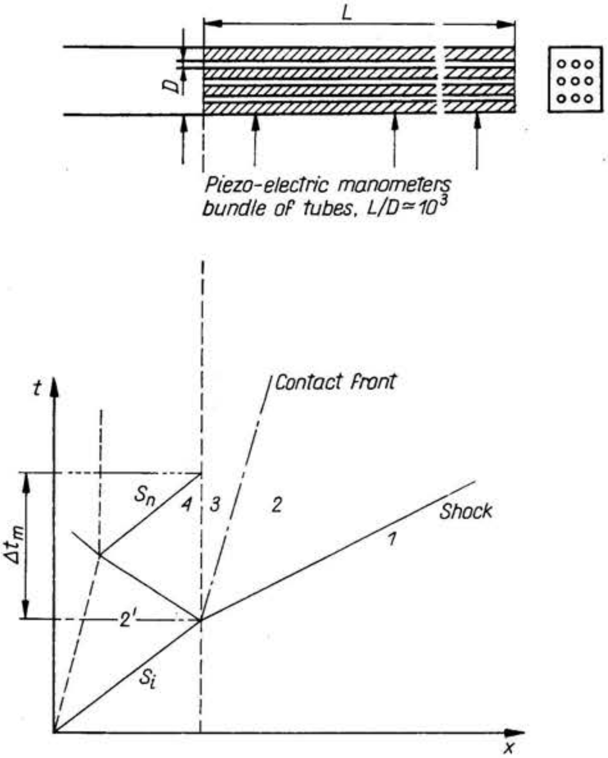

FIG. 1. Test set-up and *x-t*-diagram.

For all measurements the range of the initial Mach number M_{s0} was

$$1.3 < M_{s0} < 2.0$$

and the range of the Reynolds number was

$$2.10^4 < \mathrm{Re} < 4.10^5,$$

where $\mathrm{Re} = \varrho_2 u_2 D/\eta_2$, computed with state *2* immediately behind the initial shock wave M_{s0}. The shock tube was operated as an air/air tube.

However, before presenting the experimental results a few words must be said about the theoretical calculations and predictions.

3. Theoretical investigation by the bundle of tubes

For the theoretical investigation it was assumed that in the total flow field between the entrance of the bundle and shock existed fully-developed turbulent flow. A simple estimate shows that indeed the zones of boundary layer growth were small in comparison to the zone where an essential attenuation of the shock took place. The assumption of fully-developed flow allows the use of cross-section averages for all flow variables [2]. Then, the variables are only functions of the space coordinate x along the axis of the tube and, of course, of the time t. The problem is described by the known differential equations for the balance of mass, momentum and energy [3]. In these equations, the effect of wall shear stress can be taken into account by introducing a volume force, F, into the equations. The added heat per unit volume and unit time can be computed by the Reynolds-analogy.

The system of three differential equations for the three dependent variables p, ϱ and u may be solved by the method of characteristics, taking into account the associated initial and boundary conditions. With the assumption that air is a perfect gas, the system of equations, written in characteristic directions, is

$$(3.1)\quad C_{1/2}:\quad \frac{dx}{dt} = u \pm a:$$

$$dp \pm \varrho a du = \left[(\gamma-1)uF\left(1+\frac{q}{uF}\right) \mp aF\right]dt,$$

$$(3.2)\quad C_3:\quad \frac{dx}{dt} = u:$$

$$dp - a^2 d\varrho = (\gamma-1)uF\left(1+\frac{q}{uF}\right)dt,$$

where

$$(3.3)\quad F = \frac{c_f}{r}\varrho u^2,$$

$$(3.4)\quad q = -\frac{c_f}{r}\varrho u\left[\frac{\gamma}{\gamma-1}\left(\frac{p}{\varrho} - \frac{p_1}{\varrho_1}\right) + \frac{u^2}{2}\right],$$

with

$$\frac{p_1}{\varrho_1} = RT_1 = RT_w = \text{const.}$$

Henceforth, we take an x-t-diagram where the x-coordinate runs from the entrance of the bundle and the coordinate t begins at the time when the incident shock S_i has reached the entrance of the bundle. The calculation starts with the following initial conditions: along the first C_1-characteristic in the bundle the state is that for the flow without effects of friction and heat transfer. This is correct for the sufficiently small time interval Δt.

The boundary condition at the entrance of the bundle is that the gas has undergone quasi-stationary acceleration from the constant state *4* to the actual state *3e* immediately in the entrance. The conditions across the contact front are the known kinematic and dynamic conditions

$$U_{3c} = U_{2c} \quad \text{and} \quad p_{3c} = p_{2c}.$$

The boundary conditions at the shock are the known jump relations for the variables across the shock wave (Fig. 2).

The numerical solution of the characteristic system subject to the associated initial and boundary conditions gives the state of flow in the total x-t-plane. Therefore, the pres-

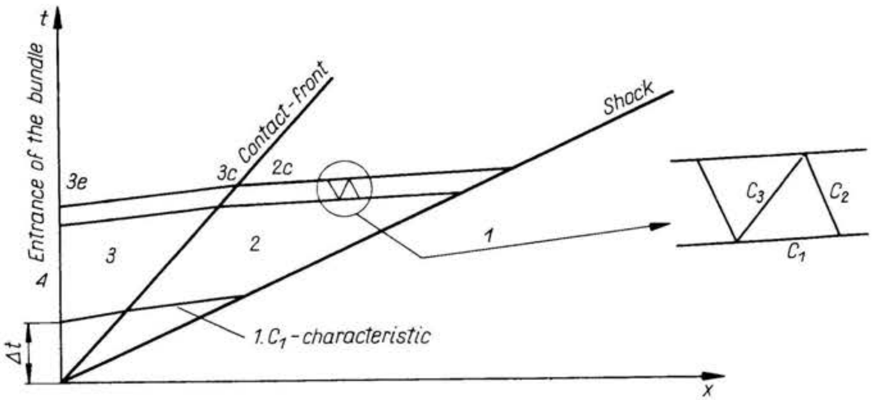

FIG. 2.

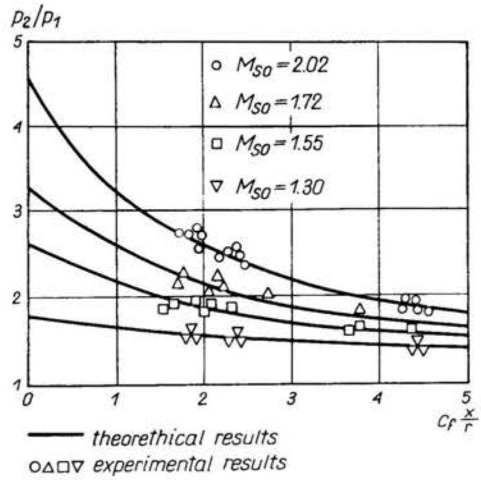

FIG. 3. Comparison of the theoretical results by the characteristic method to the experimental results.

sure jump p_2/p_1 across the shock can be plotted as a function of the dimensionless coordinate $c_f(x/r)$ with the initial Mach number M_{s0} as parameter. Figure 3 shows the theoretical results for the attenuation of shock waves as a result of friction and heat transfer. The experimental results were also recorded in this diagram. They agree closely with the theoretical results and thereby confirm the assumptions which were made for the computations.

4. Approximate solution

In addition, an approximate solution could be found for small initial Mach numbers M_{s0} ($M_{s0}-1 \ll 1$). In this case, the first C_1-characteristic nearly coincides with the shock and therefore Eq. (3.1) is approximately valid along the shock front. This is analogous to WITHAM's assumption [4]. Therefore, the known jump-relations of the variables across the shock can be inserted in Eq. (3.1). Thereby, we get a single differential equation for the Mach number M_s as a function of the dimensionless coordinate $c_f(x/r)$. Using the linearized relations across the shock wave, we get the following simple relation for the attenuation of shock waves with the initial condition $M_s(x=0) = M_{s0}$:

$$\frac{M_s - 1}{M_{s0} - 1} = \frac{1}{1 + \frac{2\gamma}{\gamma - 1}(M_{s0} - 1) c_f \frac{x}{r}}. \tag{4.1}$$

Figure 4 shows that the approximate solution nearly coincides with the exact result by the characteristic method for initial Mach numbers $M_{s0} < 1.3$.

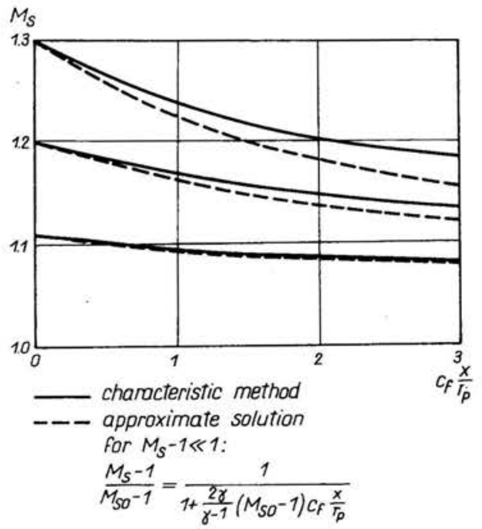

FIG. 4. Comparison of the approximate solution with the result of the characteristic method.

Equation (4.1) can also be written as follows

$$\frac{p_2 - p_1}{p_{20} - p_1} = \frac{1}{1 + \frac{2\gamma}{\gamma - 1}(M_{s0} - 1) c_f \frac{x}{r}}. \tag{4.2}$$

This equation agrees with a relation found empirically by BANNISTER [5] for the attenuation of shock waves in long, small tubes

$$\frac{p_2 - p_1}{p_{20} - p_1} = \frac{1}{1 + Bx}, \tag{4.3}$$

where B was an empirical factor.

Comparison with Eq. (4.2) shows that B must include the friction factor c_f, the radius of the tube r, the ratio of specific heats and the initial condition M_{s0} in the following combination

$$B = \frac{2\gamma}{\gamma - 1}(M_{s0} - 1) c_f \frac{1}{r}. \tag{4.4}$$

This result can be generalised, so that it can be valid also for stronger shocks, by introducing a factor $G(M_{s0})$ which reduces to 1 if $M_{s0} \to 1$. This factor was determined by numerical calculations which showed that it depends only on the Mach number and is practically independent of the ratio of specific heats and other parameters of the problem

$$G(M_{s0}) = \left(1+\sqrt{1-\frac{1}{M_{s0}}}\right)^{-1}. \tag{4.5}$$

By inserting this factor in Eq. (4.2), all the theoretical results practically collapse to one single curve which is shown in the next figure (Fig. 5). The measured points fall closely on to that curve.

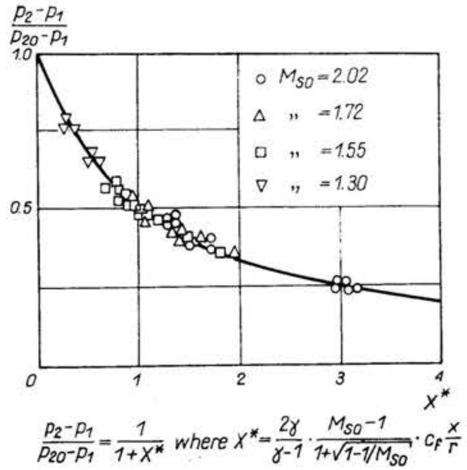

FIG. 5. Generalised approximate solution and the measured points of Fig. 3.

5. Investigation with beds of solids

After this investigation with bundles of tubes, the attenuation of shock waves was also investigated in beds of solids. The corresponding experimental investigation was done by Mr. Büren, a student of Engineering at the THD. For these measurements, the bundle of tubes was replaced by beds of solids in the same shock tube with the same test set-up. The measurements were made with different initial Mach numbers M_{s0} and different ratios of particle diameter, d, to diameter of the shock tube, D. The range of the modified Reynolds number $\mathrm{Re}' = \varepsilon \frac{\varrho \bar{u} d}{\eta}$ was $3.10^4 < \mathrm{Re}' < 5.10^5$. Here, ε is the ratio of the average free cross section in the bed to the total cross section and $\bar{u}$ is the average velocity in the bed of solids.

For modified Reynolds numbers $\mathrm{Re}' > 10^3$, the modified-friction factor f for beds of solids is not a function of the shape of the particle [6]. Therefore, spherical pebbles were taken as particles. For turbulent flow, the friction factor f for beds of solids is [7]

$$f = \frac{r_p}{\varrho \bar{u}^2} \frac{1}{\varepsilon^2} \frac{\Delta p}{\Delta x} = 0.875 \frac{1-\varepsilon}{\varepsilon^3} \qquad (r_p = d/2), \tag{5.1}$$

where $\Delta p/\Delta x$ is the local pressure gradient. Analogously to the investigation with the bundle of tubes, we can define a force per unit volume F_s which takes account of the effect of the pressure drop in the beds of solids

$$F_s = 0.875\frac{1-\varepsilon}{\varepsilon}\frac{1}{r_p}\varrho\bar{u}^2. \tag{5.2}$$

On the other hand, the effect of heat transfer can be neglected for the attenuation of shock waves in beds of solids, because the ratio $q/F_s\bar{u}$ by beds of solids is very small:

$$q/F_s\bar{u} = 0(10^{-2}) \ll 1.$$

By the assumption that the flow in the beds of solids is nearly one-dimensional, we can also compute this problem by the system of Eqs. (3.1) to (3.3) by putting F equal to F_s and $\frac{q}{F_s\bar{u}}$ equal to zero.

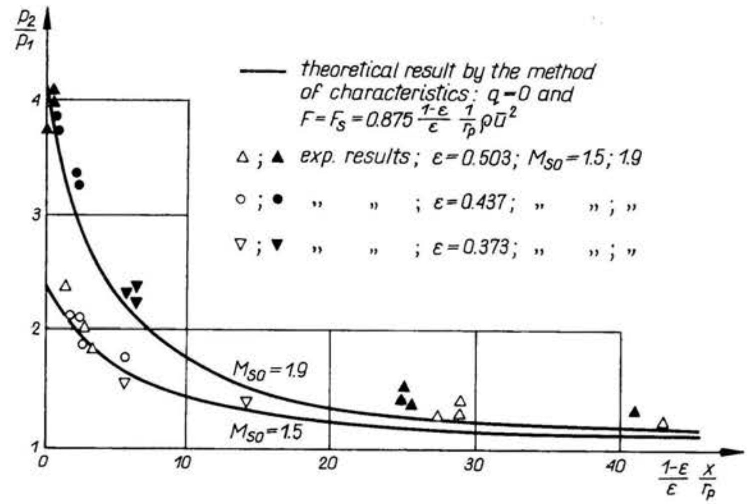

FIG. 6. Attenuation of shock waves in beds of solids.

Figure 6 shows that the solution found is succesfully applicable to beds of solids as well, provided the assumptions made for the computations are not too seriously violated.

References

1. N. ZLOCH, *Abschwächung von Verdichtungsstössen infolge Reibungswirkungen und Wärme-Übergang bei voll ausgebilderer turbulenter Strömung*, Dissertation D 17, Darmstadt, 1974.
2. L. D. LANDAU, B. M. LIFSCHITZ, *Lehrbuch der theoretischen Physik*, Band VI, Hydrodynamik, Akademie Verlag, Berlin 1971.
3. A. H. SHAPIRO, *The dynamics and thermodynamics of compressible fluid flow*, Vol. 1, The Ronald Press Company, New York 1953.
4. G. B. WITHAM, *On the propagation of shock waves through regions of non-uniform area of flow*, J. Fluid Mech., 4, 4, 337–360, 1958.
5. F. K. BANNISTER and S. F. MUCKLOW, *Wave action following sudden release of compressed gas from a cylinder*, Proc. Inst. Engrs., London 1948, p. 159.

6. J. H. PERRY, *Chemical engineers handbook*, Fourth edition, Mc Graw-Hill Book Company, Inc. 1963.
7. R. B. BIRD, W. E. STEWART and E. N. LIGHTFOOT, *Transport phenomena*, John Wiley a. Sons, Inc., New York, London, Sydney 1960.

TECHNISCHE HOCHSCHULE DARMSTADT.

Received October 21, 1975.