73.

NOTE ON A NINE SCHOOLGIRLS PROBLEM.

[*Messenger of Mathematics*, XXII. (1893), pp. 159, 160.]

THIS is a parallel question to the well-known one of fifteen schoolgirls extended to the supposition of their walking for *one* week, three and three together, so that in any the same *day* no two, and at the end of the *week* no three, taking four walks a day, shall have walked more than once together.

Let us understand by the development of the array

$$\begin{matrix} a, & b, & c, \\ d, & e, & f, \\ g, & h, & k, \end{matrix}$$

the four arrangements

$$\begin{matrix} (abc, & def, & ghk), \\ (adg, & beh, & cfk), \\ (aek, & bfg, & cdh), \\ (afh, & bdk, & ceg), \end{matrix}$$

(corresponding, in fact, to the four sets of three lines through the nine inflexions of a cubic).

If we suppose the nine girls to walk out four times a day, the same two never being together more than once in the same day, and that at the week's end each has been associated with every pair of the remaining eight, the above will serve to represent one day's walks. To find the other six, I first form the three following pairs of subsidiary arrays, by circular motion performed successively on the three columns of the primitive array, namely

$$\begin{matrix} g, & b, & c, \\ a, & e, & f, \\ d, & h, & k, \end{matrix} \qquad \begin{matrix} d, & b, & c, \\ g, & e, & f, \\ a, & h, & k, \end{matrix}$$

$$\begin{matrix} a, & h, & c, \\ d, & b, & f, \\ g, & e, & k, \end{matrix} \qquad \begin{matrix} a, & e, & c, \\ d, & h, & f, \\ g, & b, & k, \end{matrix}$$

$$\begin{matrix} a, & b, & k, \\ d, & e, & c, \\ g, & h, & f, \end{matrix} \qquad \begin{matrix} a, & b, & f, \\ d, & e, & k, \\ g, & h, & c. \end{matrix}$$

Then making any similarly placed line (I have taken the first) in each of the above six groups circulate in one direction as regards the three on the left, and in the opposite direction as regards the three on the right, we obtain six new arrays: these together with the original one furnish the following table:

$$\begin{array}{ccc} a, & b, & c, \\ d, & e, & f, \\ g, & h, & k, \end{array}$$

$$\begin{array}{ccccccc} c, & g, & b, & \qquad & b, & c, & d, \\ a, & e, & f, & & g, & e, & f, \\ d, & h, & k, & & a, & h, & k, \\ c, & a, & h, & & e, & c, & a, \\ d, & b, & f, & & d, & h, & f, \\ g, & e, & k, & & g, & b, & k, \\ k, & a, & b, & & b, & f, & a, \\ d, & e, & c, & & d, & e, & k, \\ g, & h, & f, & & g, & h, & c. \end{array}$$

When the seven arrays in the above table are developed according to the rule previously given, the triads thus arising will be found to be all distinct or, which is the same thing, will comprise among them the whole of the eighty-four ternary combinations of the nine symbols. We have therefore in this table a solution of the proposed problem.

Of course the general problem, when n is any odd multiple of 3, is to construct sets of $\frac{1}{2}(n-1)$ synthemes, each containing $\frac{1}{3}n$ triads with no element in common, and to distribute the whole number of triads into $(n-2)$ such sets.

This problem I solved very many years ago, but I believe have nowhere published, for the case where n is any power of 3, by a method of compound rhythmical displacement strictly analogous to (but of course more intricate than) the one here exhibited.