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Dependence of fracture phenomena upon the evolution of constitutive structure of solids (*)

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THIS PAPER aims at describing the dependence of ductile fracture phenomena in tensile test upon the evolution of constitutive features of the material of the specimen investigated. An analysis of different effects on fracture for particular materials has been given and cooperative phenomena are investigated. Two cooperative phenomena, namely the plastic deformation process (together with localization effect) and internal imperfections (generated by nucleation, growth and coalescence of voids) are assumed as most important for a proper description of the final stage of the flow process in tensile test. A simple elastic-viscoplastic model of a material with internal imperfections is proposed. The model describes the postcritical phenomena in dissipative solids observed experimentally as well as the mechanism of fracture. The main role in this model is played by the evolution equation for the imperfection parameter interpreted as the volume fraction of voids. The determination of the material functions in the evolution equations is based on available metallurgical observations for carbon spheroidized steels. A criterion of fracture is proposed. The criterion describes the dependence of fracture phenomenon upon the evolution of the constitutive structure. This criterion is sensitive to the particular choice of the yield condition as well as to the evolution equation proposed for the void volume fraction parameter. Basing on the experimental results for particular materials, the dependence of the fracture criterion on rate sensitivity and thermal effects has also been investigated. An initial-boundary-value problem describing the axi-symmetrical, tensile test is discussed.

Celem pracy jest opis zależności zjawisk zniszczenia ciągliwego w próbie rozciągania od ewolucji właściwości konstytutywnych materiału badanej próbki. Zanalizowano wpływ różnych efektów na zniszczenie dla poszczególnych materiałów oraz zbadano zjawiska współdziałające. Dwa zjawiska, mianowicie proces plastycznych deformacji (łącznie z efektem lokalizacji) oraz powstawanie wewnętrznych imperfekcji (wywołane nukleacją, wzrostem i łączeniem mikropustek) przyjęto jako najbardziej ważne dla prawidłowego opisu końcowego stadium procesu płynięcia w próbie rozciągania. Zaproponowano prosty model sprężysto-lepkoplastycznego materiału z wewnętrznymi imperfekcjami. Model ten opisuje pokrytyczne zjawiska obserwowane eksperymentalnie w ciałach dysypatywnych jak również mechanizm zniszczenia. Główną rolę w zaproponowanym modelu odgrywa równanie ewolucji dla parametru imperfekcji wewnętrznej (interpretowanego jako objętościowy udział pustek). Określenie funkcji materiałowych w równaniu ewolucji przeprowadzono przy pełnym wykorzystaniu dostępnych rezultatów obserwacji metalurgicznych dla sferoidalnej stali węglowej. Zaproponowano kryterium zniszczenia, które opisuje zależność zjawisk zniszczenia od ewolucji struktury konstytutywnej materiału. Kryterium to jest wrażliwe zarówno na wybór warunku uplastycznienia jak również na przyjęte równanie ewolucji dla parametru objętościowego udziału pustek. Wykorzystując rezultaty badań doświadczalnych dla poszczególnych materiałów zbadano zależność kryterium zniszczenia od efektów wrażliwości na prędkość deformacji i wpływ temperatury. Przedyskutowano problem początkowo-brzegowy opisujący osiowosymetryczne deformacje w próbie rozciągania.

Целью работы является описание зависимости явлений тягучего разрушения в испытании растяжения от эволюции определяющих свойств материала исследуемого образца. Приведен анализ влияния разных эффектов на разрушение для отдельных материалов, а также исследованы взаимодействующие явления. Два явления, именно процесс пластических деформаций (совместно с эффектом локализации) и возникновение внутренних имперфекций (вызванное нуклеацией, ростом и соединением микропустот), приняты как наиболее важные для правильного описания конечной стадии процесса

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течения в испытании растяжения. Предложена простая модель упруго-вязкопластического материала с внутренними имперфекциями. Эта модель описывает докритические явления наблюдаемые экспериментально в диссипативных телах, как тоже механизм разрушения. Главную роль в предложенной модели играет уравнение эволюции для параметра внутренней имперфекции (интерпретированного как объемное участие пустот). Определение материальных функций в уравнении эволюции проведено при полном использовании доступных результатов металлургических наблюдений для сфероидальной углеродистой стали. Предложен критерий разрушения, который описывает зависимость явлений разрушения от эволюции определяющей структуры материала. Этот критерий чувствителен так на подбор условия перехода в пластическое состояние, как и на принятое уравнение зволюции для параметра объемного участия пустот. Используя результаты экспериментальных исследований для отдельных материалов, исследована зависимость критерия разрушения от эффектов чувствительности на скорость деформаций и влияние температуры. Обсуждена начально-краевая задача описывающая осесимметрические деформации в испытании растяжения.

1. Introduction

METALLURGICAL experimental observations have shown the dependence of fracture phenomena in tensile test upon the evolution of all constitutive features of the material of the specimen investigated. Fracture is a final stage of the inelastic flow process and is influenced by many cooperative phenomena such as plastic deformations, the localization of plastic deformations, internal imperfections induced by nucleation, growth and coalescence of voids, strain rate sensitivity, thermal effects, etc.

It is our conjecture that the final stage of the necking process depends on the evolution of all constitutive features of the material as well as on the boundary conditions and the shape of the body considered.

In the paper an analysis of different effects on ductile fracture for particular materials has been given and cooperative phenomena are investigated. Of course, it would be unrealistic to include in the description all effects observed experimentally. Constitutive modelling is understood as a reasonable choice of effects which are most important for the explanation of the phenomenon described.

Experimental investigations (cf. J. I. BLUHM and R. J. MORRISSEY [3] and J. GURLAND and J. R. FISHER [8]) suggest that two cooperative phenomena, namely the plastic deformation process (together with localization effect) and internal imperfections (generated by nucleation, growth and coalescence of voids), play the most important role in the proper explanation of the final stage of the flow process in tensile test. The synergetic effect which gives a very sound increase of the results induced by these two cooperative phenomena is also observed. This synergetic effect is especially pronounced at a final stage of the flow process, that is when the fracture mechanism takes place.

In Sect. 2 a simple elastic-viscoplastic model of a material with internal imperfections is proposed. The model is developed within the rate type constitutive structure with internal state variables (1). The crucial idea in this description is the very efficient interpretation of the internal state variable ξ as the void volume fraction parameter. It permits to base all considerations on good physical foundations and to use available experimental observations.

(1) The advantages of the rate-type formulation are sound when the constitutive structure is applied to the solution of the initial-boundary-value problems. For more detailed considerations see Ref. [16].

The model proposed describes the postcritical phenomena in dissipative solids observed experimentally and satisfies the requirement that, during the deformation process in which the effective strain rate is equal to the static value assumed, the response of a material becomes elastic-plastic.

In Sect. 3 the theory of ductile fracture based on the evolution of internal imperfections is developed. The evolution equation for the void volume fraction parameter ξ is modified in such a way as to introduce some important features of the fracture phenomenon directly to the model. A criterion of fracture is postulated. The criterion describes the dependence of the fracture phenomenon upon the evolution of the constitutive structure. Basing on experimental data for perticular materials, the influence of different effects on the fracture criterion is investigated. The discussion is focussed mainly on the strain rate sensitivity and thermal effects.

In Sect. 4 the discussion of the initial-boundary-value problem is given.

Section 5 comments upon the results obtained.

2. Constitutive model

2.1. Physical and experimental motivation

In previous papers of the author [17, 18] the analysis of postcritical behaviour of a specimen during tensile test was given. Based on careful examination of the nucleation, growth and coalescence of voids and the accumulation of damage and the subsequent necking process as well as taking advantage of the physical suggestions, a proper evolution for the scalar measure of the imperfection parameter was postulated.

In this paper the same interpretation of the imperfection internal state variable ξ as the void volume fraction (or porosity) parameter is assumed.

Let us consider a tensile deformation process for a 0.44% C steel specimen and control the load as a function of porosity or void volume fraction parameter (cf. Fig. 1). On the

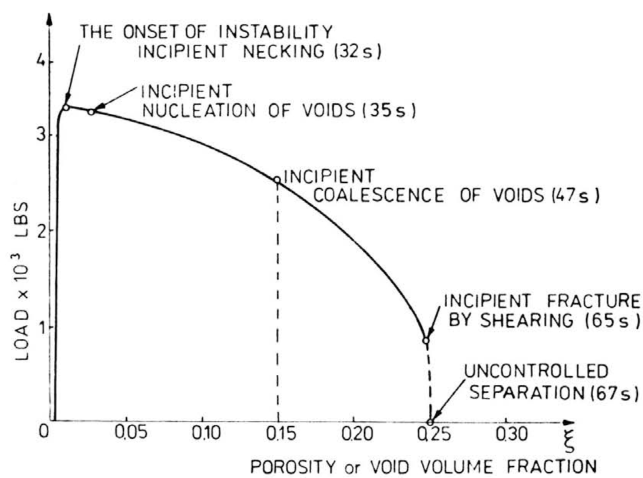

FIG. 1. Load as a function of porosity in a tensile deformation process for 0.44% C steel.

load-porosity trajectory which represents the deformation process we can recognize several important stages. First, the maximum load is achieved and this point is supposed to represent the onset of instability by necking (²). When the tensile process is going on, the load starts to decay as a result of the nucleation and growth of voids. Careful metallurgical investigations have shown that very near to the peak load point the incipient nucleation of voids is observed (³). The voids nucleate and grow and when the porosity is about 15% the inicipient coalescence of microvoids is observed. Starting from that point the mechanism of fracture occurs. Compatibility of deformation then causes gross linkage of the central voids to form large macroscopically visible internal cracks which grow as the final shear lips are formed. In this stage of the tensile process in the neck region the solid material is under a long range shear field and begins to shear in a gross manner, for 0.44% C steel it takes place at the porosity of about 25%. Finally the specimen separates into two pieces. This final stage of the tensile deformation process is characterized by the very small changes of plastic deformations, in other words the difference between the plastic deformation corresponding to the incipient coalescence of voids and the plastic deformation corresponding to the incipient fracture (uncontrolled separation) is very small. This is why we can say that the mechanism of fracture is of catastrophic nature.

Figure 2 (taken from D. R. CURRAN, L. SEAMAN and D. A. SHOCKEY [4]) shows four stages of the tensile deformation process of the specimen of oxygen-free high-conductivity copper.

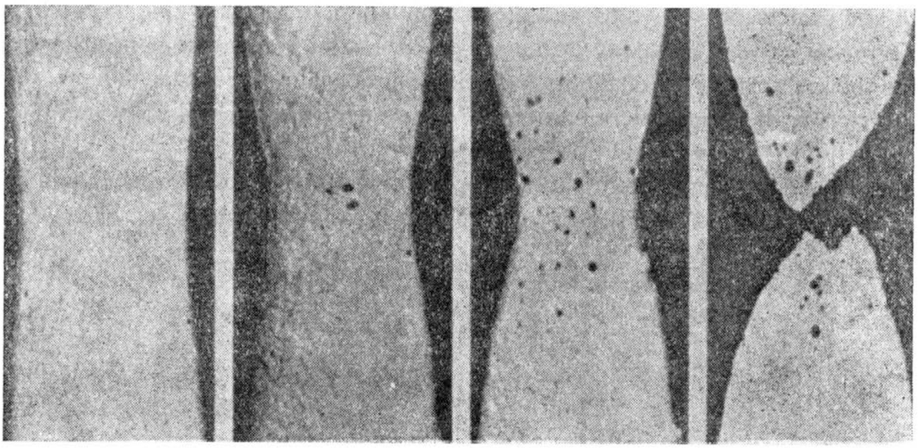

FIG. 2. Four stages of a tensile deformation process of the specimen of oxygen-free, high-conductivity copper. After D. R. CURRAN, L. SEAMAN and D. A. SHOCKEY [4].

(²) A second basic mode of localization is the phenomenon of plastic instability in the direction of pure shear. A necessary condition for the localization of plastic deformation in the form of the plastic shear band is a maximum true flow stress criterion. For recent investigations in this subject see J. R. RICE [24], A. NEEDLEMAN and J. R. RICE [12] and M. SAJE, J. PAN and A. NEEDLEMAN [23]; cf. also the review paper by R. J. ASARO [2].

(³) Basing on experimental observations it has been suggested that microvoids nucleated both by the cracking of the second phase particles and by their decohesion, cf. G. LE ROY, J. D. EMBURY, G. EDWARD and M. F. ASHBY [11], J. GURLAND [7] and J. GURLAND and J. R. FISHER [8].

The suggestion that fracture is a catastrophe has been justified by experimental observations for spheroidized plain carbon steel (1080) performed by R. GARBER, I. M. BERNSTEIN and A. W. THOMPSON [6]. They measured the area density of voids, N_A, at the neck center of an unfractured specimen as a function of plastic strain, cf. Fig. 3, and the void area fraction, A_A as a function of plastic strain, cf. Fig. 4. The rather rapid increase in both measured quantities at a plastic strain near the fracture strain is visible.

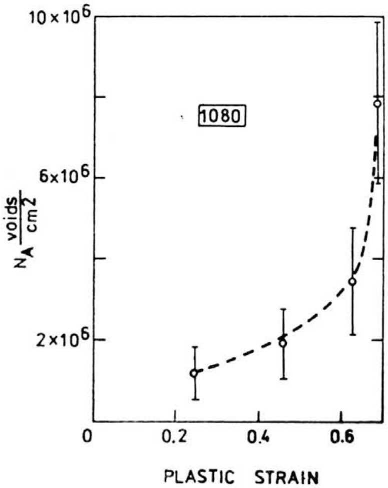

FIG. 3. The areal density of voids N_A as a function of plastic strain for 1080 steel. After R. GARBER, I. M. BERNSTEIN and A. W. THOMPSON [6].

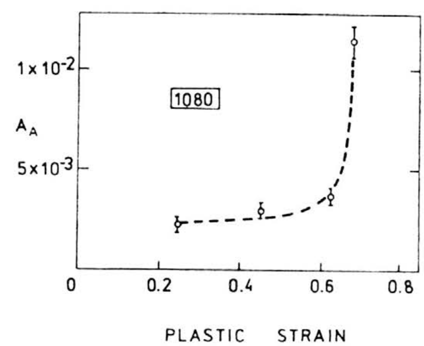

FIG. 4. The void area fraction A_A as a function of plastic strain for 1080 steel. After R. GARBER, I. H. BERNSTEIN and A. W. THOMPSON [6].

A similar conclusion can be drawn from the metallurgical investigations for spheroidized plain carbon steel (1045) performed by A. S. ARGON and J. IM [1]. Figure 5 shows the fraction of separated particles as a function of the equivalent plastic strain; dots are taken from experimental observations and the curve is obtained from the theoretical predictions, cf. Ref. [19].

It has been proved (cf. Refs. [17, 18]) that the efficient interpretation of the internal state variable ξ as the void volume fraction parameter permits to base all considerations on good physical foundations and to use available experimental observations.

2.2. Constitutive assumptions

For our purposes here it is sufficient to assume a simple elastic-viscoplastic model of a material with internal imperfections as it has been proposed by the author in Refs. [16, 17, 18]:

$$\frac{1}{2G}\left[\overset{\nabla}{\sigma} - \frac{\nu}{1+\nu}\,\mathrm{tr}\overset{\nabla}{\sigma}\mathbf{I}\right] = \mathbf{D} - \frac{\gamma}{\varphi}\left\langle \Phi\left[\frac{f(J_1, J_2', J_3', \xi)}{\hat{\varkappa}(\xi)} - 1\right]\right\rangle \partial_{\sigma} f,$$

$$\text{(2.1)}\qquad \dot{\xi} = \frac{h}{1-\xi}\,\mathrm{tr}(\boldsymbol{\sigma}\mathbf{D}^p) + (1-\xi)\Xi\,\mathrm{tr}\,\mathbf{D}^p,$$

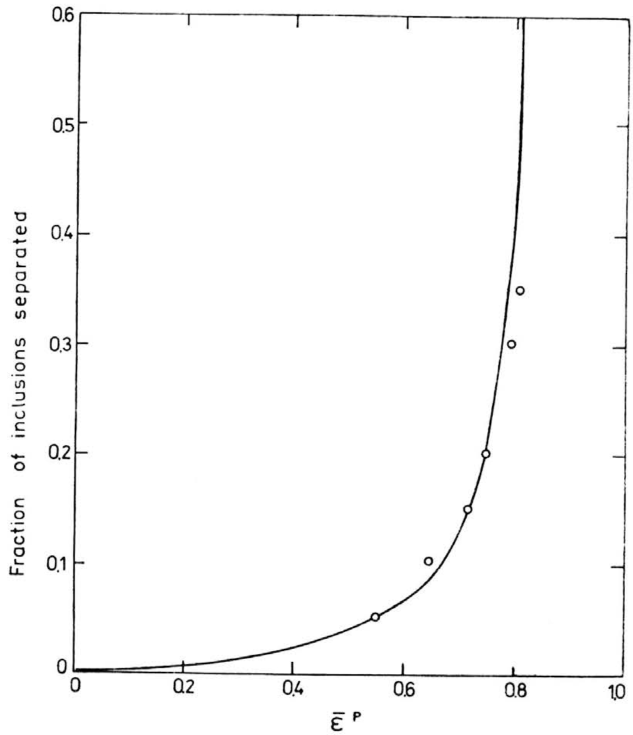

FIG. 5. Fraction of separated particles as a function of the equivalent plastic strain for 1045 steel. Dots are taken from experimental observations of A. S. ARGON and J. IM [1], the curve is obtained from the theoretical description of Ref. [19].

$$f(\cdot) = J_2'\left[1-(n_1+n_2\xi)\frac{J_3'^2}{J_2'^3}+n_3\xi\frac{J_1^2}{J_2'}\right],$$

$$\hat{\varkappa}(\xi) = \varkappa_0^2\left(1-n_4\xi^{\frac{1}{2}}\right)^2, \qquad \varphi = \varphi\left(\frac{I_2}{I_2^s}-1\right),$$

where $\overset{\nabla}{\boldsymbol{\sigma}}$ denotes the symmetric Zaremba–Jaumann rate of change of the Cauchy stress tensor $\boldsymbol{\sigma}$, G is the shear modulus, ν is the Poisson ratio, $\mathbf{I}$ denotes the unit tensor, $\mathbf{D}$ is the symmetric rate of the deformation tensor, γ is the viscosity constant, φ is introduced as the control function and is assumed to depend on $(I_2/I_2^s)-1$, where I_2 is the second invariant of the rate of the deformation tensor and I_2^s is its static value, Φ denotes the overstress viscoplastic function, f is the quasi-static yield function which is postulated to depend on the first invariant of the Cauchy stress tensor J_1, on the second and third invariants of the stress deviator J_2' and J_3' and on the imperfection internal state variable ξ interpreted as the void volume fraction (or porosity) parameter, $\hat{\varkappa}(\xi)$ is the material softening function, the symbol $\langle[\ \ ]\rangle$ is understood according to the definition

$$\langle[\ ]\rangle = \begin{cases} 0 & \text{if} \quad f \leqq \hat{\varkappa}(\xi), \\ [\ ] & \text{if} \quad f > \hat{\varkappa}(\xi), \end{cases} \tag{2.2}$$

h and Ξ are the nucleation and growth material functions, respectively, and n_1, n_2, n_3 and n_4 denote the material constants.

The control function φ satisfies the conditions

$$\lim_{I_2 \to I_2^s} \varphi = 0 \quad \text{and} \quad \varphi(\cdot) = 0 \quad \text{for} \quad I_2 < I_2^s \tag{2.3}$$

and is introduced to describe the properties of a material in a range of strain rates near the static value, $I_2 = I_2^s$.

The model proposed describes the postcritical phenomena in dissipative solids observed experimentally and satisfies the requirement that during the deformation process in which the effective strain rate is equal to the static value assumed, the response of a material becomes elastic-plastic.

2.3. Elastic-plastic response

For the limit case, when $I_2 \leqq I_2^s$, Eqs. (2.1) yield

$$\begin{aligned} &\frac{1}{2G}\left[\overset{\nabla}{\boldsymbol{\sigma}} - \frac{\nu}{1+\nu}\,\mathrm{tr}\overset{\nabla}{\boldsymbol{\sigma}}\mathbf{I}\right] = \mathbf{D} - \Lambda\partial_\sigma f, \\ &\dot{\xi} = \frac{h}{1-\xi}\,\mathrm{tr}(\boldsymbol{\sigma}\mathbf{D}^p) + (1-\xi)\Xi\,\mathrm{tr}\,\mathbf{D}^p, \end{aligned} \tag{2.4}$$

for

$$f(\cdot) = \varkappa, \tag{2.5}$$

i.e.,

$$J_2'\left[1 - (n_1 + n_2\xi)\frac{J_3'^2}{J_2'^3} + n_3\xi\frac{J_1^2}{J_2'}\right] = \varkappa_0^2\left(1 - n_4\xi^{\frac{1}{2}}\right)^2 \tag{2.6}$$

and

$$\mathrm{tr}(\partial_\sigma f\dot{\boldsymbol{\sigma}}) > 0. \tag{2.7}$$

The parameter Λ has to be determined from the continuity condition (4)

$$\dot{f} = \dot{\varkappa}. \tag{2.8}$$

Equations (2.4) describe an inviscid elastic-plastic model of solids with internal imperfections generated by the nucleation and growth of microvoids.

2.4. Determination of the material functions and constants

The identification procedure for all material functions and constants has to be based on the available experimental data.

As it has been shown in Refs. [16, 17, 18], two kinds of experimental tests can be utilized. First, the mechanical test data for a broad range of strain rate changes are used to determine the material functions and constants in the evolution equation for the inelastic deformation tensor. Second, the physical, metallurgical observations are assumed as a basis for determining material functions in the evolution equation for the void volume fraction parameter.

(4) For a thorough analysis of the elastic-plastic response as a limit case of an elastic-viscoplastic solid with internal imperfections see Refs. [16, 17, 18].

The evolution equation for the inelastic deformation tensor leads to the following relation:

$$(2.9)\qquad J_2'\left[1-(n_1+n_2\xi)\frac{J_3'^2}{J_2'^3}+n_3\xi\frac{J_1^2}{J_2'}\right] = \varkappa_0^2\left(1-n_4\xi^{\frac{1}{2}}\right)^2\left\{1+\Phi^{-1}\left[\frac{(\mathrm{tr}(\mathbf{D}^p)^2)^{\frac{1}{2}}}{\gamma}\varphi\left(\frac{I_2}{I_2^s}-1\right)(\mathrm{tr}(\partial_\sigma f)^2)^{\frac{1}{2}}\right]\right\},$$

which may be interpreted as the dynamical yield criterion. It describes the actual change of the yield surface during the inelastic deformation process. This change is caused by material softening induced by the nucleation and growth of microvoids and by the influence of the strain rate sensitivity effect.

Equation (2.9) constitutes a basis for the comparison of theoretical results with experimental investigation data.

The determination of the overstress viscoplastic material function Φ, the control function φ and the material constants $\varkappa_0$ and γ can be based on dynamical test data performed for different loading conditions.

Let us assume for simplicity $n_1 = n_2 = 0$ and consider unvoided solid, i.e. $\xi = 0$; then from Eq. (2.9) we have

$$(2.10)\qquad J_2' = \varkappa_0^2\left\{1+\Phi^{-1}\left[\frac{(\mathrm{tr}(\mathbf{D}^p)^2)^{\frac{1}{2}}}{\gamma}\varphi\left(\frac{I_2}{I_2^s}-1\right)\right]\right\}.$$

Now we can apply directly the procedure of identification developed for the elastic-perfectly viscoplastic material in Ref. [13] and generalized to the elastic work-hardening viscoplastic material in Ref. [22].

The constant n_3 in the yield function

$$(2.11)\qquad f(\cdot) = J_2'\left[1+n_3\xi\frac{J_1^2}{J_2'}\right]$$

for voided solid can be determined by the comparison of the Gurson solution (cf. A. L. GURSON [9]) for the inviscid plastic response with the present proposition

$$(2.12)\qquad \frac{J_2'}{\varkappa_0^2}+n_3\xi\frac{J_1^2}{\varkappa_0^2} = \left(1-n_4\xi^{\frac{1}{2}}\right)^2$$

under the assumption that n_4 is known from the fracture condition (see Sect. 3).

To determine the nucleation and growth material functions in the evolution equation for the imperfection parameter ξ we can use Fisher's metallurgical observation data (cf. J. R. FISHER [5]).

This procedure is not simple and it has some disadvantages. First, there is no unique way to determine the nucleation material function h independently of the growth material function Ξ. Second, the procedure is based on the assumption that the distribution of the stress in the neck of the specimen is known (5).

(5) A detailed discussion of the determination of the material functions h and Ξ can be found in Ref. [20].

3. Fracture criterion

3.1. Fundamental assumptions

The model proposed in Sect. 2 describes the postcritical phenomena in dissipative solids observed experimentally. To describe a final stage of the flow process, namely the mechanism of fracture, we need to introduce some important features of fracture phenomena directly to the model. However, there is no unique way to accomplish this task ([6]).

Some information may be introduced to the evolution equation for the void volume fraction parameter ξ by modifying the evolution equation proposed, cf. Eq. $(2.4)_2$. Some other properties can be described directly by the proper determination of the material constants in the yield criterion assumed. We shall here take advantage of both possibilities.

From the analysis of experimental observations (cf. J. R. FISHER [5] for two kinds of spheroidized plain carbon steels (0.17 and 0.44% C) and R. GRABER, I. M. BERNSTEIN and A. W. THOMPSON [6] for spheroidized plain carbon steel 1080) we can deduce for each kind of material investigated the critical plastic deformation ε_c^p which is understood in such a way that, during the inelastic flow process when the equivalent plastic deformation ε^p (assumed as the square root of the second invariant of the plastic deformation tensor) tends to ε_c^p, then the void volume fraction parameter ξ tends to its upper bound, i.e. $\xi \to 1$, see Fig. 6 (cf. Refs. [17, 18]).

The main problem concerns not only the question of the behaviour of ξ, but also how $\frac{d\xi}{d\varepsilon^p}$ behaves when the equivalent plastic deformation ε^p tends to its critical value ε_c^p.

Basing on experimental metallurgical observations and careful theoretical considerations we deduce that ξ achieves its upper bound and $\frac{d\xi}{d\varepsilon^p}$ is singular at the critical plastic deformation ε_c^p (cf. Ref. [19]).

It seems reasonable to introduce these important features directly to the evolution equation for the void volume fraction parameter ξ. We shall accomplished this by using the idea of the control function (cf. Ref. [17, 18])

$$\psi = \psi\left(1-\frac{\varepsilon^p}{\varepsilon_c^p}\right) \tag{3.1}$$

with the property: $\psi(0) = 0$. The evolution equation $(2.4)_2$ is postulated in the form as follows:

$$\dot{\xi} = \frac{h}{1-\xi}\,\mathrm{tr}(\boldsymbol{\sigma}\mathbf{D}^p)+\frac{\Xi}{\psi\left(1-\dfrac{\varepsilon^p}{\varepsilon_c^p}\right)}(1-\xi)\,\mathrm{tr}\mathbf{D}^p. \tag{3.2}$$

([6]) Recently V. TVERGAARD [25, 26] has suggested a concept of modification of the evolution equation for the void volume fraction parameter by adding an extra term responsible for the failure of a material. It seems, however, that Tvergaard's method of modification has no clear physical foundations. Very recently V. TVERGAARD and A. NEEDLMAN [27] have introduced a new method of fracture phenomenon basing only on the modification of the yield criterion.

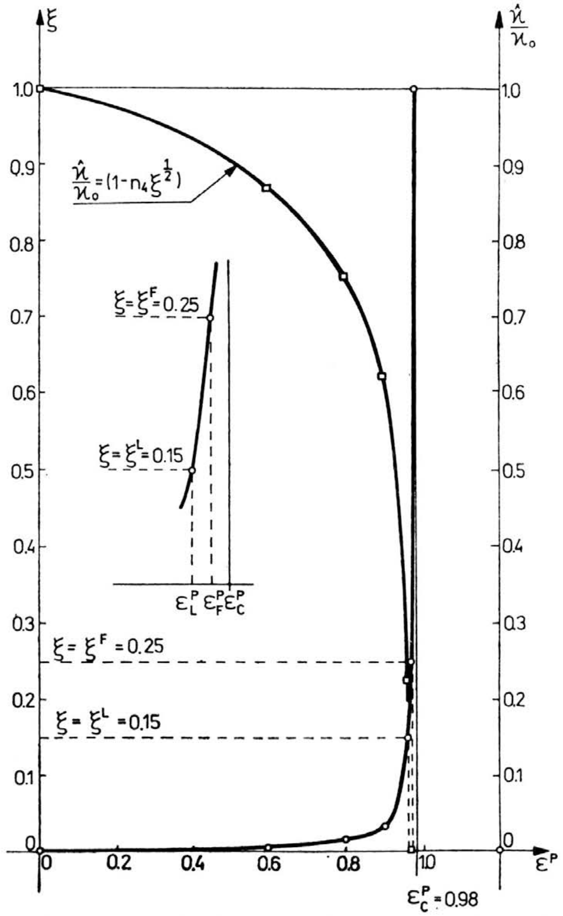

FIG. 6. Explanation of the fracture criterion based on the evolution of porosity for 0.44% C steel (cf. Ref. [17, 18]).

Experimental observations suggest that there exists the interval

$$[\xi^L, \xi^F] \subset [0, 1], \tag{3.3}$$

where [0, 1] is the domain of the ξ parameter, ξ^L is the value of the void volume fraction parameter which corresponds to the incipient linkage of voids, and ξ^F corresponds to the incipient global fracture of the specimen by the shearing mechanism.

For $\xi = \xi^F$ catastrophe takes place

$$\hat{\varkappa}(\xi)|_{\xi=\xi^F} = 0, \tag{3.4}$$

i.e. the material loses its stress carrying capacity.

A criterion of fracture is postulated in the form as follows (cf. Refs. [17, 18]):

During the tensile flow process the fracture phenomenon occurs when

$$\varepsilon^p = \varepsilon^p_F \Rightarrow \xi = \xi^F, \tag{3.5}$$

where ε^p_F is the value of the equivalent plastic deformation which corresponds to the fracture point and is called the fracture deformation.

The criterion of fracture (3.5) leads to the condition $\hat{\varkappa}(\xi)|_{\xi=\xi^F} = 0$, cf. Eq. (3.4).

For the particular choice of the softening function $\hat{\varkappa}$ as in Eq. $(2.1)_4$ the condition (3.4) implies the particular relation

$$\lim_{\xi\to\xi^F} \varkappa_0^2 \left(1-n_4\xi^{\frac{1}{2}}\right)^2 = 0, \tag{3.6}$$

hence we can obtain the material constant

$$n_4 = (\xi^F)^{-\frac{1}{2}}, \tag{3.7}$$

which is crucial for the description of fracture phenomena.

3.2. Influence of different effects

The criterion (3.5) describes the dependence of fracture phenomena upon the evolution of the constitutive structure. This means that the criterion (3.5) is sensitive to the particular choice of the yield condition as well as to the evolution equation proposed for the void volume fraction parameter.

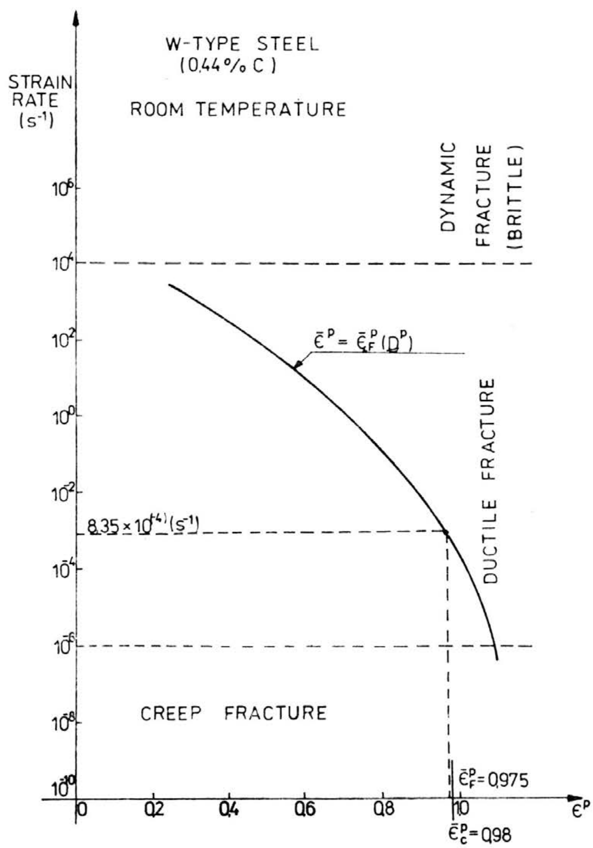

FIG. 7. Dependence of the fracture equivalent strain on the rate of deformation suggested by experimental results for 0.44% C steel at room temperature.

It has been pointed out in Refs. [17, 18] that the criterion of fracture in the form

$$\varepsilon^p = \varepsilon_F^p = \text{const} \tag{3.8}$$

can be valid only under the very strong assumption that the process under consideration is characterized by a small range of strain rate as well as temperature changes.

In the general case, when during the deformation process the strain rate can change in a large range and temperature varies strongly, the fracture equivalent plastic deformation depends on the rate of deformation as well as on the temperature, i.e.

$$\varepsilon^p = \varepsilon_F^p(\mathbf{D}^p, \vartheta). \tag{3.9}$$

The experimental investigations performed by P. J. WRAY [28–30] for steel and iron have shown that even for very small strain rates but for elevated temperature the criterion of fracture in the form (3.9) is valid.

The strain rate sensitivity effect for mild steel has mainly been investigated for post-critical behaviour. However, several authors have also studied this effect for fracture phenomena.

Available experimental results suggest for plain carbon steel (0.44% C) investigated

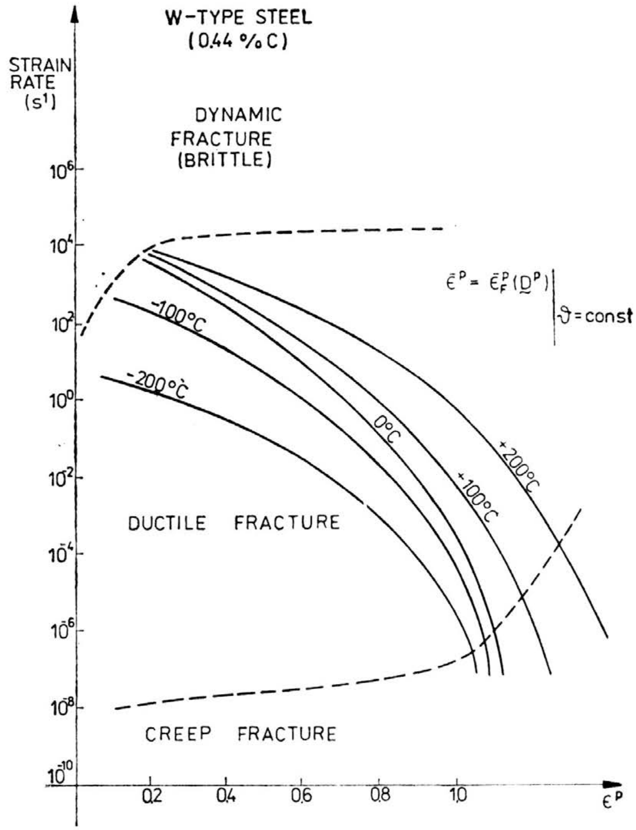

FIG. 8. Dependence of the fracture equivalent strain on the rate of deformation suggested by experimental results for 0,44 % C steel measured at different temperature

at room temperature the dependence of the fracture equivalent strain on the rate of deformation as it has been shown in Fig. 7.

It is noteworthy that at room temperature, in the entire range of strain rate investigated for 0.44% C steel, three general kinds of fracture can be observed. In the range of strain rate from 10^{-6} to 10^{4} s^{-1} ductile fracture is detected, for strain rate lower than 10^{-6} s^{-1} creep fracture occurs, and for very large strain rate (over 10^{4} s^{-1}) the dynamic fracture (brittle fracture) takes place.

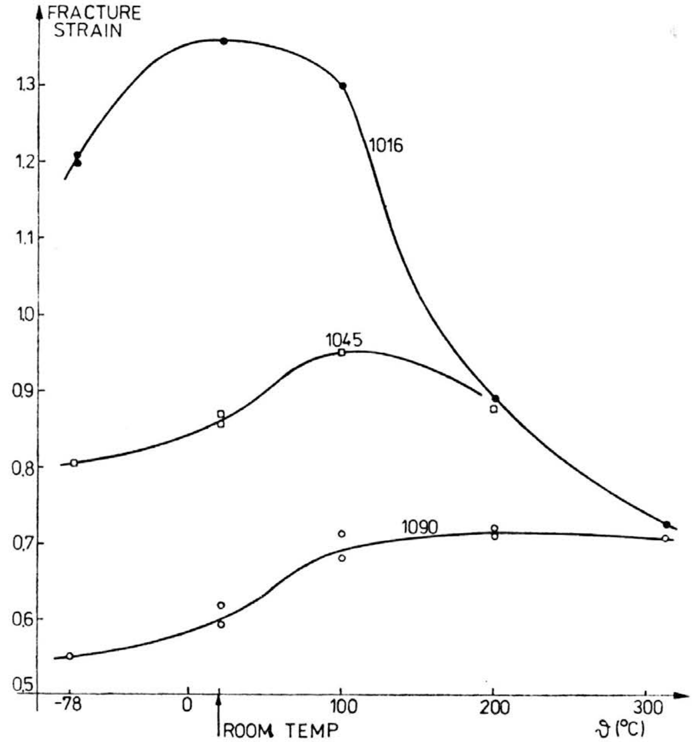

FIG. 9. The fracture equivalent plastic deformation as a function of temperature for 1016, 1045 and 1090 steels. After G. LE ROY, J. D. EMBURY, G. EDWARD and M. F. ASHBY [11].

When temperature is assumed as a parameter which can vary, then in the strain rate — equivalent plastic deformation plane we obtain a family of curves and the picture becomes more complicated as it has been shown in Fig. 8.

G. LE ROY, J. D. EMBURY, G. EDWARD and M. F. ASHBY [11] have investigated the influence of temperature on fracture phenomena. They measured the fracture equivalent plastic deformation as a function of temperature for several kinds of steel. The results for 1016, 1045 and 1090 steels are shown in Fig. 9. These results proved that the influence of temperature on fracture phenomena in a range from -78 to 300°C is more pronounced for low carbon steel than for medium carbon steel (cf. the results for 1016 and 1090 steels).

In the previous paper [21] the analysis of strain rate effects on the ductile fracture of metals in the plane state of stress was given. The authors have utilized the results of crack-speed measurements performed on cold worked steel foil obtained by M. F. KANNINEN, A. K. MUKHERJE, A. R. ROSENFIELD and G. T. HAHN [10] and the fundamental assumptions of the Dugdale model. It is a well-known fact that growth and unstable propagation of ductile crack in metals is accompanied by a local plastic zone characterized by a large gradient of strain and high strain rates from 10^3 to $10^5\ s^{-1}$, therefore the measurements of ductile crack propagation in foils can supply much valuable information upon strain rate effects on the ductile fracture of metals. This shows also the necessity of applying the viscoplasticity theory to a theoretical analysis of the deformation process in the plastic zone at the crack tip. The main result of the paper [21] has been the determination of the dependence of fracture strain on the strain rate.

This theoretical analysis of ductile fracture of plain carbon steel at room temperature proved that the assumption (3.8) is valid up to strain rate of the order $10^3\ s^{-1}$. If the strain rate is larger than $10^3\ s^{-1}$, then, for constant temperature, the criterion of fracture (3.8) has to be replaced by

$$\varepsilon^p = \varepsilon_F^p(D^p). \tag{3.10}$$

4. Discussion of the initial-boundary-value problem

In the previous papers of the author [16, 17, 18] the tensile, axisymmetric deformation problem of a circular bar was investigated, provided the deformations were symmetric about the mid-plane, constant velocity was applied to the ends of the specimen and, additionally, the ends were assumed to remain shear free. The material of the specimen is assumed to be elastic-plastic with internal imperfections, the yield condition has the form (2.11) and the evolution equation for the volume fraction parameter ξ has the modified form (3.2). The problem has been treated as quasi-static.

The numerical solution of the problem formulated has the property that the load tends to zero when the equivalent plastic deformation tends to the fracture value assumed.

So the theory proposed describes the fracture point as it has been suggested by experimental observations.

In the paper [16] the discussion of the influence of the strain rate effect on the onset of localization and the influence of imperfections on the postcritical behaviour was given.

The theoretical predictions are consistent with experimental observations that the load at the instability point is an increasing function of the strain rate while the strain at the same point is a decreasing function of the strain rate.

The imperfection effect leads to a very pronounced softening of the material in the postcritical region. It has been shown that the proper determination of the softening material function $\hat{\varkappa}(\xi)$ provides the realistic description.

The main results obtained by the numerical solution of the initial-boundary-value problem have proved that the model of an elastic viscoplastic solid with internal imperfections generated by the nucleation, growth and coalescence of microvoids can be applicable for practical purposes.

5. Final comments

The theory of postcritical behaviour and fracture has been inspired by experimental observations.

The criterion of fracture proposed for the tensile deformation problem does depend on the entire evolution of the constitutive structure of solids. This has been achieved mainly following a careful analysis of the evolution of the void volume fraction parameter (or porosity parameter) ξ during the tensile inelastic flow process ([7]).

The criterion of fracture is sensitive to the particular choice of the yield condition as well as to the evolution equation proposed for the void volume fraction parameter.

Thus the crucial idea in the theory proposed is the very efficient interpretation of the internal state variable ξ. The assumption that ξ is the void volume fraction (or porosity) permitted to base all considerations on good physical foundations and to use all available experimental observations.

A simple modification of the evolution equation assumed for the imperfection parameter provides the description of fracture according to experimental observations.

Basing on available experimental results for particular materials, the dependence of the fracture criterion on strain rate sensitivity and thermal effects has also been investigated. Of course, these investigations are preliminary and we need careful experimental observations of the effects discussed.

The simplifications assumed for practical purposes which have been carefully discussed and commented prove the practical character of the theory proposed.

In the discussion of the solution obtained by means of the numerical procedure, particular attention is given to the influence of the results by the information about fracture phenomena introduced directly through the evolution equation and the yield condition. This analysis has proved that the model proposed predicts the fracture point consistently with experimental observations.

We hope that the theory of postcritical behaviour and fracture proposed is sufficiently simple in its nature so that it can be applicable to the solution of initial-boundary-value problems.

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([7]) A qualitative analysis of the evolution for the void volume fraction parameter in the entire domain has been given in Ref. [19]. This analysis concerns the modification of the evolution equation by using three different methods. First, the control function is introduced and the modified evolution equation is reduced to Abel's equation of the second kind and its solution for a particular case is presented. Second, an auxiliary function representing the solution of the modified evolution equation as well as fitting experimental observation data is introduced. Third, the perturbation procedure is developed and the approximate solution of the modified evolution equation is presented.

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