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Qualitative analysis of propagation of isothermal and adiabatic acceleration waves in the range of finite deformations

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ON THE BASIS of constitutive equations of metallic isotropic elastic-plastic bodies, motion of a second order discontinuity surface in an infinite space is subject to a qualitative analysis in a local approach. Propagation velocities of isothermal and adiabatic waves (plastic, loading and unloading wave fronts) are compared with the corresponding elastic wave velocities. In this manner, the analysis of acceleration waves given by Mandel is generalized to the case of finite deformations.

Na podstawie równań konstytutywnych dla metalicznych izotropowych ciał sprężysto-plastycznych przedstawiono badania jakościowe ruchu powierzchni nieciągłości drugiego rzędu w nieskończonej przestrzeni, w aspekcie lokalnym. Porównano prędkości rozprzestrzeniania się fal izotermicznych i adiabatycznych (fal plastycznych, frontów fal obciążenia i odciążenia), z prędkościami odpowiednich fal sprężystych. Uogólniono w ten sposób rozważania nad analizą fal przyspieszenia podane przez J. Mandela na przypadek skończonych deformacji.

На основе определяющих уравнений для металлических изотропных упруго-пластических тел представлены качественные исследования движения поверхности разрыва второго порядка в бесконечном пространстве, в локальном аспекте. Сравнены скорости распространения изотермических и адиабатических волн (пластических волн, фронтов волн нагрузки и разгрузки) со скоростями соответствующих упругих волн. Таким образом обобщены рассуждения по анализу волн ускорения, приведенные Дж. Манделем, наслучайконечных деформаций.

Notations

$\mathbf{A}\otimes\mathbf{B}$	$A_i B_j$ or $A_{ij}B_{mn}$,
$\mathbf{A}\cdot\mathbf{B}$	$A_i B_i$ or $A_{ij}B_{ij}$,
$\mathbf{AB}$	$A_{ij}B_j$ or $A_{ijkl}B_{kl}$,
$\operatorname{tr}\mathbf{A}$	A_{kk},
$\mathbf{1}$	unit tensor,
$\overline{\mathbf{A}}$	$\mathbf{A}-\frac{1}{3}(\operatorname{tr}\mathbf{A})\mathbf{1}$,
$\overset{\mathrm{T}}{\mathbf{A}}$	transpose of a tensor

1. Introduction

A MOVING surface of discontinuity of field functions, its propagation velocity depending on the nonlinear properties of the material, is called a wave. Let us consider the acceleration waves.

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Propagation of the second order discontinuity in a three-dimensional, unbounded, elastic-plastic medium was considered originally in a paper by MANDEL [5], who investigated the motion of the discontinuity surface in the local sense: the speed was determined at an arbitrary point and in a given direction normal to the surface, on the basis of the HADAMARD Compatibility equations [4]. The paper was generalized by TING [13] who determined the character of the discontinuity vector and the form of the transport equation. Assuming certain particular forms of the constitutive equations, RANIECKI [11] and W. K. NOWACKI [8] investigated the problem of propagation of acceleration waves in metals and soils. All the approaches mentioned above apply to small deformations.

The first papers dealing with the problem of wave propagation at finite deformations appeared in the seventies: BALABAN *et al.* [1], D'ESCATHA [2], PIAU [9, 10]. Papers of this kind are usually aimed at determining the relations between the velocities of elastic and plastic waves, waves of loading and unloading. In some of the papers, variations in amplitudes of such waves were considered. GUELIN and NOWACKI [3] studied the propagation velocity of acceleration waves in an elastic-plastic medium with a perfect material hysteresis.

In the present paper, velocities of such waves in isothermal and adiabatic processes will be studied on the basis of the constitutive relations presented in [12, 7]; the relations are derived from Mandel's theory of elastic-plastic materials [6], logarithmic elastic strain measure being assumed as one of the fundamental state parameters. They describe the finite deformations of the medium. Let us start with a short presentation of the fundamental equations.

2. Fundamental equations

In the case of metallic, isotropic, elastic-plastic materials, the following equations in Eulerian description are used under certain well-grounded assumptions [12, 7]:

$$\overset{\nabla}{\boldsymbol{\tau}} = \mathbf{LD} - \frac{j}{H}(\overline{\mathbf{m}} \cdot \overline{\mathbf{D}})\overline{\mathbf{m}}, \tag{2.1}$$

where

$$L_{ijkl} = \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \left(K^r - \frac{2\mu}{3}\right)\delta_{ij}\delta_{kl},$$

$$K^r = \beta(K-p); \quad p = -\frac{\sigma_{kk}}{3} \quad \text{mean pressure},$$

K — bulk modulus, μ — Lamé constant, $\overset{\nabla}{\boldsymbol{\tau}} = \dot{\boldsymbol{\tau}} - \boldsymbol{\omega}\boldsymbol{\tau} + \boldsymbol{\tau}\boldsymbol{\omega}$ — Jaumann derivative of the stress tensor $\boldsymbol{\tau}$, $\boldsymbol{\tau} = \beta\boldsymbol{\sigma}\boldsymbol{\sigma}$ — Cauchy stress tensor, $\beta = \varrho_0/\varrho$, ϱ_0 and ϱ are the respective densities in the reference configuration and the actual configuration, according to the theory of MANDEL [6].

$$\mathbf{D} = \frac{1}{2}(\operatorname{grad}\mathbf{v} + \overset{T}{\operatorname{grad}}\mathbf{v}), \qquad v \text{ — velocity},$$

$$\boldsymbol{\omega} = \frac{1}{2}(\operatorname{grad}\mathbf{v} - \overset{T}{\operatorname{grad}}\mathbf{v}).$$

The yield condition is assumed in the following form

$$f(\bar{\boldsymbol{\tau}}, \theta, \alpha) = 0, \tag{2.2}$$

where f is a normed function, so that

$$\frac{\partial f}{\partial \bar{\boldsymbol{\tau}}} = \bar{\mathbf{m}}, \quad \mathrm{tr}\,\bar{\mathbf{m}} = 0, \quad \bar{\mathbf{m}} \cdot \bar{\mathbf{m}} = 1 \tag{2.3}$$

θ is the absolute temperature, α is the hardening parameter (power of plastic strain in this case).

The hardening function H for an isothermal process has the form

$$H^i = \frac{1}{2\mu}\left(\frac{h}{2\mu}+1\right) \quad \text{where} \quad h = -\frac{\partial f}{\partial \alpha}(\bar{\boldsymbol{\tau}} \cdot \bar{\mathbf{m}}) \tag{2.4}$$

and for an adiabatic process

$$H^a = \frac{1}{2\mu}\left(\frac{h}{2\mu}+1+q_d m_\theta\right). \tag{2.5}$$

Here q_d is the thermal coefficient of energy dissipation

$$q_d = \frac{1-\pi}{\varrho_0 C_e}(\bar{m} \cdot \bar{\boldsymbol{\tau}}), \quad \pi = \varrho_0 \frac{d\varphi(\alpha)}{d\alpha}, \tag{2.6}$$

φ stored energy per unit mass (which may be measured experimentally). In most metals π takes the values between 0.02 and 0.1. C_e is the specific heat at constant deformation, $m_\theta = -\frac{1}{2\mu}\frac{\partial f}{\partial \theta}$ — thermal coefficient of softening.

Equation (2.1) describes both the isothermal and adiabatic processes; the difference between them consists in the fact that $H^i \neq H^a$, if certain minor coupling effects are disregarded (heat of elastic deformation and thermal expansion due to the dissipation energy).

$$j = \begin{cases} 1 & \text{if} \quad f = 0 \quad \text{and} \quad \bar{\mathbf{m}} \cdot \bar{\mathbf{D}} \geqslant 0, \\ 0 & \text{if} \quad f < 0 \quad \text{or} \quad f = 0 \quad \text{and} \quad \bar{\mathbf{m}} \cdot \bar{\mathbf{D}} < 0. \end{cases} \tag{2.7}$$

These equations, together with the equations of continuity, motion, temperature and evolution, represent a closed system of equations for the following unknowns: β, τ_{ij}, $\mathbf{v}$, θ, α (cf. [7]); they all are functions of $\mathbf{x}$ and t.

3. Qualitative analysis of acceleration wave velocity

In elastic-plastic media the four types of acceleration waves mentioned above may propagate, depending on the state of the medium in front of and behind the wave. The propagation of waves may be analyzed in the space, that is in Eulerian coordinates, or with respect to the material, that is in Lagrangean coordinates. However, in order to obtain the simplest results, let us assume the material configuration at instant t as the reference configuration. It means that the motion of the wave in the time interval $(t, t+dt)$

is referred to the material particles at instant t. The following relation between the velocities holds true

$$W = \Omega + \mathbf{v}\cdot\mathbf{\nu}. \tag{3.1}$$

Here W is the wave velocity (in space), Ω is the local velocity (with respect to the material at time t), $\mathbf{\nu}$ is the unit vector normal to the discontinuity surface S.

In Eulerian coordinates, if $\gamma(\mathrm{x}, t)$ remains continuous at passing through S but its derivatives are discontinuous, there exists such Γ that

$$[\gamma_{,i}] = \Gamma\nu_i, \quad [\gamma_{,t}] = -\Gamma W, \quad [\dot{\gamma}] = -\Gamma\Omega, \tag{3.2}$$

where $[A]$ denotes the jump of A, $\gamma_{,i} = \dfrac{\partial\gamma}{\partial x_i}$, and

$$\gamma_{,t} = \frac{\partial\gamma}{\partial t}, \quad \dot{\gamma} = \gamma_{,t} + \gamma_{,i} v_i.$$

Owing to the compatibility conditions, we obtain

$$[\gamma_{,i}] = -\frac{\nu_i}{\Omega}[\dot{\gamma}]. \tag{3.3}$$

a) Isothermal wave

In the isothermal plasticity it is assumed that the temperature is constant and equal to θ_0. Isothermal waves represent a certain idealisation of the actual wave processes produced by the impact at the surface of elastic-plastic bodies. Let us now consider the propagation of isothermal acceleration waves in an infinite three-dimensional elastic-plastic material, and apply the method proposed by MANDEL [5].

In the case of acceleration waves, at passing across S, the functions β, τ_{ij}, v_i are continuous, but their first derivatives with respect to time and $\mathbf{x}$ suffer jump discontinuities.

The equations of continuity and motion assume now the forms

$$\dot{\beta} = \beta v_{k,k}, \tag{3.4}$$

$$\tau_{ij,i} - \frac{1}{\beta}\tau_{ij}\beta_{,i} = \varrho_0 \dot{v}_j. \tag{3.5}$$

Taking into account Eqs. (3.2), (3.3) and the formula for derivatives $\overset{\nabla}{\tau}$, Eqs. (3.4), (3.5) are reduced to the single equation

$$\Omega[\overset{\nabla}{\tau}_{ij}]\nu_i + \frac{1}{2}[\tau_i]\tau_{mj}\nu_i\nu_m + \frac{1}{2}[\dot{v}_m]\tau_{mj}\nu_i\nu_i + \frac{1}{2}[\dot{v}_m]\nu_j\nu_i\tau_{im} - \frac{1}{2}[\dot{v}_j]\tau_{im}\nu_i\nu_m + \varrho_0\Omega^2[\dot{v}_j] = 0. \tag{3.6}$$

(i) Elastic waves. Let the regions 1 (in front of the wave) and 2 (behind the wave) be elastic. From Eq. (2.1) it follows that

$$\overset{\nabla}{\tau}_{ij} = L_{ijkl}D_{kl}. \tag{3.7}$$

Substitution of Eq. (3.7) into (3.6) yields

$$(Q^{(e)}_{jk}(\nu) - \varrho\Omega^2\delta_{jk})[\dot{v}_k] = 0, \tag{3.8}$$

where $Q_{jk}^{(e)}$ is the so-called elastic acoustic tensor

(3.9) $$Q_{jk}^{(e)} = \frac{\varrho}{\varrho_0}\left(L_{ijkl}\nu_i\nu_l - \frac{1}{2}\tau_{mj}\nu_k\nu_m - \frac{1}{2}\tau_{kj}\nu_i\nu_i - \frac{1}{2}\tau_{ik}\nu_i\nu_j + \frac{1}{2}\tau_{im}\nu_i\nu_m\delta_{jk}\right).$$

Under the assumption that the elastic distortions were small, it was shown in [7] that tensor $Q_{jk}^{(e)}$ was symmetric and positive definite. Hence, for each direction ν normal to surface S, there exist three possible acceleration wave velocities; they fulfill the following relations

(3.10) $$\varrho\Omega_1^{e^2} = Q_{\mathrm{I}}^e, \quad \varrho\Omega_2^{e^2} = Q_{\mathrm{II}}^e, \quad \varrho\Omega_3^{e^2} = Q_{\mathrm{III}}^e.$$

Here $Q_{\mathrm{I}}^e \geqslant Q_{\mathrm{II}}^e \geqslant Q_{\mathrm{III}}^e$ are the eigenvalues of tensor $Q^{(e)}$, and

(3.11) $$\Omega_1^e \geqslant \Omega_2^e \geqslant \Omega_3^e,$$

where Ω_i^e are the elastic acceleration wave velocities.

(ii) Plastic waves. In this case, plastic state prevails at both sides of S. Combining Eqs. (2.1) and (3.6) we obtain

(3.12) $$(Q_{jk}^{(p)} - \varrho\Omega^2\delta_{jk})[\dot{v}_k] = 0,$$

where

(3.13) $$\mathbf{Q}^{(p)} = \mathbf{Q}^{(e)} - \frac{1}{\beta H^i}(\overline{\mathbf{m}}\boldsymbol{\nu})\otimes(\overline{\mathbf{m}}\boldsymbol{\nu}) = \mathbf{Q}^{(e)} - r\mathbf{a}\otimes\mathbf{a},$$
$$r = \frac{1}{\beta H^i} > 0, \quad \mathbf{a} = \overline{\mathbf{m}}\boldsymbol{\nu}.$$

It is seen from Eq. (3.12) that $\varrho\Omega^2$ and $[\dot{\mathbf{v}}]$ are the eigenvalues and eigenvectors of the tensor $\mathbf{Q}^{(p)}$, respectively. Non-trivial roots of the system (3.12) may exist, provided

(3.14) $$F^p(X) = \det(\mathbf{Q}^{(p)} - X\mathbf{1}) = \det(\mathbf{Q}^{(e)} - X\mathbf{1} - r\mathbf{a}\otimes\mathbf{a}) = 0,$$

where $\varrho\Omega^2 = X$.

Let us assume the coordinate axes to coincide with the principal axes of the tensor $\mathbf{Q}^{(e)}$; then it follows from Eq. (3.14) that

(3.15) $$EP(X) = (Q_{\mathrm{I}}^e - X)(Q_{\mathrm{II}}^e - X)(Q_{\mathrm{III}}^e - X) - r[(Q_{\mathrm{II}}^e - X)(Q_{\mathrm{III}}^e - X)a_1^2 + (Q_{\mathrm{III}}^e - X)(Q_{\mathrm{I}}^e - X)a_2^2 + (Q_{\mathrm{I}}^e - X)(Q_{\mathrm{II}}^e - X)a_3^2] = 0.$$

In view of the fact $r > 0$, Eq. (3.15) yields

(3.16) $$F^p(Q^e) \leqslant 0, \quad F^p(Q_{\mathrm{II}}^e) \geqslant 0, \quad F^p(Q_{\mathrm{III}}^e) \leqslant 0, \quad F^p(-\infty) > 0.$$

Figure 1 presents the diagram of function $FP(X)$ satisfying the conditions (3.16). It is seen that the eigenvalues of tensor $\mathbf{Q}^{(p)}$ satisfy the inequality

(3.17) $$Q_{\mathrm{III}}^p \leqslant Q_{\mathrm{III}}^e \leqslant Q_{\mathrm{II}}^p \leqslant Q_{\mathrm{II}}^e \leqslant Q_{\mathrm{I}}^p \leqslant Q_{\mathrm{I}}^e,$$

what implies that

(3.18) $$\Omega_3^e \leqslant \Omega_2^p \leqslant \Omega_2^e \leqslant \Omega_1^p \leqslant \Omega_1^e, \quad Q_{\mathrm{III}}^p \leqslant \varrho\Omega_3^{e^2},$$

where $\Omega_i^p (i = 1, 2, 3)$ denote the plastic wave velocities. It cannot be seen from Eq. (3.18)

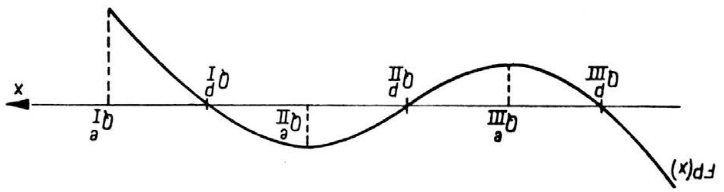

FIG. 1.

whether the velocity Ω^p_3 is real or not; in the case when the tensor $\mathbf{Q}^{(p)}$ is positive definite, we have $\Omega^p_3 > 0$ and

$$0 < \Omega^p_3 \leqslant \Omega^e_3 \leqslant \Omega^p_2 \leqslant \Omega^e_2 \leqslant \Omega^p_1 \leqslant \Omega^e_1. \tag{3.19}$$

The following conclusion may be drawn:

For each direction $\mathbf{v}$, *the plastic wave velocities are not greater than the corresponding elastic wave velocities.*

(iii) Unloading waves. This is the case when region 1 remains plastic, while region 2 is elastic, hence from Eq. (2.1) it follows that

$$[\overset{\nabla}{\tau}_{ij}] = \overset{\nabla}{\tau}{}^{(1)}_{ij} - \overset{\nabla}{\tau}{}^{(2)}_{ij} = L_{ijkl}[v_{k,l}] - \frac{1}{H^i}\bar{m}_{ij}\bar{m}_{kl}v^{(1)}_{k,l}, \tag{3.20}$$

where superscripts (1) and (2) denote the respective values of the functions at both sides of S. Introducing the notation

$$\Sigma = \bar{\mathbf{m}}\cdot\bar{\mathbf{D}} = \bar{\mathbf{m}}\cdot\mathbf{D} = \bar{m}_{ij}v_{i,j} \tag{3.21}$$

we obtain, after simple transformations of Eqs. (3.20), (3.6) (3.21),

$$(Q^{(e)}_{ij} - \varrho\Omega^2_{\mathrm{ul}}\delta_{jk})[\dot{v}_K] = -\frac{\Omega}{\beta H^i}\bar{m}_{ij}\nu_i\Sigma^{(1)} \tag{3.22}$$

if we assume (cf. MANDEL [5]) $\Sigma^{(2)} = \xi\Sigma^{(1)}$, so that

$$[\Sigma] = \Sigma^{(1)}(1-\xi), \qquad \xi \leqslant 0. \tag{3.23}$$

Consequently, Eqs. (3.21), (3.23) yield

$$\Sigma^{(1)} = -\frac{\bar{m}_{kl}\nu_l[\dot{v}_k]}{\Omega(1-\xi)}. \tag{3.24}$$

Substituting expression (3.24) into Eq. (3.22) we obtain

$$\begin{gathered}(Q^{\mathrm{ul}}_{jk} - \varrho\Omega^2_{\mathrm{ul}}\delta_{jk})[\dot{v}_k] = 0,\\ Q^{\mathrm{ul}}_{jk} = Q^{(e)}_{jk} - r^{\mathrm{ul}})a_ja_k,\end{gathered} \tag{3.25}$$

where

$$r^{\mathrm{ul}} = \frac{1}{\beta H^i(1-\xi)} = Q^{(p)}_{jk} + (r - r^{\mathrm{ul}})a_ja_k.$$

In view of $\xi \leqslant 0$, the inequality holds $0 < r^{\mathrm{ul}} \leqslant r$. Comparison of Eqs. (3.25) and (3.13) reduces the considered case of unloading waves to the case of plastic waves. Mandel's procedure allows for drawing from Eq. (3.25) the conclusion that three possible unloading

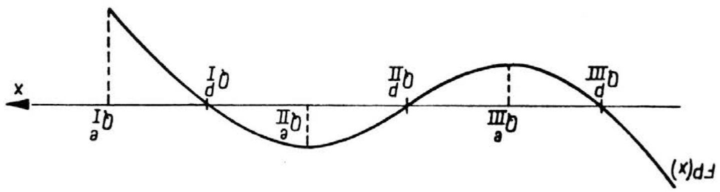

FIG. 1.

whether the velocity Ω_3^p is real or not; in the case when the tensor $\mathbf{Q}^{(p)}$ is positive definite, we have $\Omega_3^p > 0$ and

$$0 < \Omega_3^p \leqslant \Omega_3^e \leqslant \Omega_2^p \leqslant \Omega_2^e \leqslant \Omega_1^p \leqslant \Omega_1^e. \tag{3.19}$$

The following conclusion may be drawn:

For each direction $\mathbf{v}$, *the plastic wave velocities are not greater than the corresponding elastic wave velocities.*

(iii) Unloading waves. This is the case when region 1 remains plastic, while region 2 is elastic, hence from Eq. (2.1) it follows that

$$[\overset{\nabla}{\tau}_{ij}] = \overset{\nabla}{\tau}{}_{ij}^{(1)} - \overset{\nabla}{\tau}{}_{ij}^{(2)} = L_{ijkl}[v_{k,l}] - \frac{1}{H^i}\overline{m}_{ij}\overline{m}_{kl}v_{k,l}^{(1)}, \tag{3.20}$$

where superscripts (1) and (2) denote the respective values of the functions at both sides of S. Introducing the notation

$$\Sigma = \overline{\mathbf{m}}\cdot\overline{\mathbf{D}} = \overline{\mathbf{m}}\cdot\overline{\mathbf{D}} = \overline{m}_{ij}v_{i,j} \tag{3.21}$$

we obtain, after simple transformations of Eqs. (3.20), (3.6) (3.21),

$$(Q_{ij}^{(e)} - \varrho\Omega_{\mathrm{ul}}^2\delta_{jk})[\dot{v}_K] = -\frac{\Omega}{\beta H^i}\overline{m}_{ij}\nu_i\Sigma^{(1)} \tag{3.22}$$

if we assume (cf. MANDEL [5]) $\Sigma^{(2)} = \xi\Sigma^{(1)}$, so that

$$[\Sigma] = \Sigma^{(1)}(1-\xi), \qquad \xi \leqslant 0. \tag{3.23}$$

Consequently, Eqs. (3.21), (3.23) yield

$$\Sigma^{(1)} = -\frac{\overline{m}_{kl}\nu_l[\dot{v}_k]}{\Omega(1-\xi)}. \tag{3.24}$$

Substituting expression (3.24) into Eq. (3.22) we obtain

$$\begin{aligned}&(Q_{jk}^{\mathrm{ul}} - \varrho\Omega_{\mathrm{ul}}^2\delta_{jk})[\dot{v}_k] = 0,\\ &Q_{jk}^{\mathrm{ul}} = Q_{jk}^{(e)} - r^{\mathrm{ul}})a_j a_k,\end{aligned} \tag{3.25}$$

where

$$r^{\mathrm{ul}} = \frac{1}{\beta H^i(1-\xi)} = Q_{jk}^{(p)} + (r - r^{\mathrm{ul}})a_j a_k.$$

In view of $\xi \leqslant 0$, the inequality holds $0 < r^{\mathrm{ul}} \leqslant r$. Comparison of Eqs. (3.25) and (3.13) reduces the considered case of unloading waves to the case of plastic waves. Mandel's procedure allows for drawing from Eq. (3.25) the conclusion that three possible unloading

Analysis of these cases yields the final scheme of wave propagation shown in Fig. 3. The loading wave is now seen to propagate at the velocities lying in the intervals shown in Fig. 3. In this manner, all Mandel's results are generalized to the case of metallic isotropic elastic-plastic bodies in the range of finite deformations. The preceding results may also be proved under the assumption that the differences $\Sigma^{(1)}-\Sigma^{(2)}$ are known, as in the paper [10] by PIAU, and not the ratio $\xi = \Sigma^{(2)}/\Sigma^{(1)}$.

Let us now pass to adiabatic waves; the corresponding results concerning small deformations are given in [11].

(b) Adiabatic waves

Adiabatic waves represent also an extreme idealisation of actual wave processes. As mentioned before, once some minor coupling effects are disregarded, the difference between the isothermal and adiabatic processes consists merely in the fact that the hardening function H^i must be replaced by the adiabatic function H^a. Thus, the general scheme of propagation of adiabatic waves is similar to that found before. However, for given values of $\mathbf{v}$ and ξ, the corresponding velocities are different. In addition, adiabatic waves are not homothermal and the temperature is propagated at a finite velocity. Let us now compare the isothermal and adiabatic wave velocities.

Let us recall that in an isothermal process (cf. [20])

$$\mathbf{Q}^{(p)} = \mathbf{Q}^{(e(}-r\mathbf{a}\otimes\mathbf{a} \quad \text{where} \quad r = \frac{1}{\beta H^i},$$

while in the adiabatic case we obtain, using a similar approach,

$$\overset{A}{\mathbf{Q}}{}^{(p)} = \mathbf{Q}^{(e)}-\overset{A}{r}\mathbf{a}\otimes\mathbf{a}, \tag{3.30}$$

where

$$\overset{A}{r} = \frac{1}{\beta H_a} = \frac{1}{\beta\left(H^i+\dfrac{1}{2\mu}q_d m_\theta\right)}. \tag{3.31}$$

The yield limit in metals is known to decrease with increasing temperature, so that $\partial f/\partial\theta > 0$, and we have $m_\theta = -\dfrac{1}{2\mu}\dfrac{\partial f}{\partial\theta} < 0$, what implies that

$$\overset{A}{r} > r. \tag{3.32}$$

Taking into account Eqs. (3.13), (3.30) we conclude that the elastic wave velocities are identical in both processes. Let us now pass to the remaining waves. The procedure is analogous with that applied in (*a*) and will not be considered in detail. Superscripts A refer to the adiabatic process.

The propagation tensor for plastic, unloading and loading waves have the form:

$$\overset{A}{\mathbf{Q}}{}^{(p)} = \mathbf{Q}^{(p)}-(\overset{A}{r}-r)\mathbf{a}\otimes\mathbf{a}, \tag{3.33}$$

$$\overset{A}{\mathbf{Q}}{}^{(\mathrm{ul})} = \mathbf{Q}^{(\mathrm{ul})}-(\overset{A}{r}{}^{\mathrm{ul}}-r^{\mathrm{ul}})\mathbf{a}\otimes\mathbf{a}, \tag{3.34}$$

where

$$\overset{A}{r}{}^{ul} = \overset{A}{r}\frac{1}{1-\xi}, \quad \xi \leqslant 0, \tag{3.35}$$

$$\overset{A}{\mathbf{Q}}{}^{(1d)} = \mathbf{Q}^{(1d)} - (\overset{A}{r}{}^{1d} - r^{1d})\mathbf{a}\otimes\mathbf{a}, \tag{3.36}$$

where

$$\overset{A}{r}{}^{1d} = \overset{A}{r}\frac{\xi}{\xi-1}, \quad \zeta \geqslant 0. \tag{3.37}$$

Since $\overset{A}{r} > r$, we have $\overset{A}{r}{}^{ul} > r^{ul}$ and $\overset{A}{r}{}^{1d} > r^{1d}$ (cf. Fig. 2); hence the adiabatic waves of other types are propagated at velocities smaller than the corresponding isothermal waves,

$$\overset{A}{\Omega}_i \leqslant \Omega_i \quad (i = 1, 2, 3). \tag{3.38}$$

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