# 9.

## ON THE NUMBER OF FRACTIONS CONTAINED IN ANY "FAREY SERIES" OF WHICH THE LIMITING NUMBER IS GIVEN.

[*Philosophical Magazine*, xv. (1883), pp. 251—257; xvi. (1883), pp. 230—233.]

A *Farey series* ("suite de Farey") is a system of all the unequal vulgar fractions arranged in order of magnitude, the numerator and denominator of which do not exceed a given number.

The first scientific notice of these series appeared in the *Philosophical Magazine*, Vol. XLVII. (1816), pp. 385, 386. In 1879 Mr Glaisher published in the *Philosophical Magazine* (pp. 321—336) a paper on the same subject containing a proof of their known properties, an important extension of the subject to series in which the numerators and denominators are subject to distinct limits, and a bibliography of Mr Goodwyn's tables of such series. Finally, in 1881 Sir George Airy contributed a paper also to the *Philosophical Magazine* of that year, in which he refers to a table calculated by him "some years ago," and printed in the Selected Papers of the *Transactions* of the Institution of Civil Engineers, which is in fact a Farey table with the logarithms of the fractions appended to each of them. Previous tables had only given the decimal values of such fractions. The drift of this paper is to point out a caution which it is necessary to observe in the use of such tables, and which limits their practical utility: this arises from the fact of the differences receiving a very large augmentation in the immediate neighbourhood of the fractions which are a small aliquot part of unity—a fact which may be inferred *à priori* from the well-known law discovered by Farey applicable to those differences, but to which the author of the paper makes no allusion.

In addition to the tables of Farey series by Goodwyn, Wucherer, an anonymous author mentioned in the Babbage Catalogue, and Gauss, referred to by Mr Glaisher in his Report to the Bradford Meeting of the British Association (1873), may be mentioned one contained in Herzer's *Tabellen*

(Basle, 1864) with the limit 57, and another in Hrabak's *Tabellen-Werk* (Leipsic, 1876), in which the limit is taken at 50.

The writers on the theory are:—Cauchy (as mentioned by Mr Glaisher), who inserted a communication relating to it in the *Bulletin des Sciences par la Société Philomathique de Paris*, republished in his *Exercices de Mathématiques*; Mr Glaisher himself (*loc. cit.*); M. Halphen, in a recent volume of the Proceedings of the Mathematical Society of France; and M. Lucas, in the next following volume of the same collection. I am indebted to my friend and associate Dr Story for these later references.

For theoretical purposes it is desirable to count $\frac{1}{1}$ as one of the fractions in a Farey series. The number of such fractions for the limit $j$ then becomes identical with the sum of the *totients* of all the natural numbers up to $j$ inclusive—a totient to $x$ (which I denote by $\tau x$) meaning the number of numbers less than $x$ and prime to it. Such sum, that is, $\overset{x=1}{\underset{x=j}{\Sigma}} \tau x$, I denote by $Tj$. My attention was called to the subject by this number $Tj$ expressing the number of terms in a function whose residue (in Cauchy's sense) is the generating function to any given simple denumerant (see *American Journal of Mathematics*, [Vol. III. of this Reprint, p. 605]); and I became curious to know something about the value of $Tj$. I had no difficulty in finding a functional equation which serves to determine its limits (see *Johns Hopkins University Circular*, Jan. and Feb. 1883*). The most simple form of that equation (omitted to be given in the *Circular*) is

$$Tj + T\frac{j}{2} + T\frac{j}{3} + T\frac{j}{4} + T\frac{j}{5} + \ldots = \frac{j^2+j}{2},$$

(where, when $x$ is a fraction, $Tx$ is to be understood to mean $Tj$, $j$ being the integer next below $x$); and from this it is not difficult to deduce by strict demonstration that $Tj/j^2$, when $j$ increases indefinitely, approximates indefinitely near to $3/\pi^2$.

I have subsequently found that if $ux$ be used to denote the sum of all the numbers inferior and prime to $x$, and $Uj = \overset{x=1}{\underset{x=j}{\Sigma}} ux$, then †

$$Uj + 2U\frac{j}{2} + 3U\frac{j}{3} + 4U\frac{j}{4} + \ldots = \frac{j(j+1)(j+2)}{3}$$

(where $Ux$, when $x$ is a fraction, means the $U$ of the integer next inferior to $x$). From this equation it is also possible to prove that $Uj/j^3$, when $j$ becomes indefinitely great, approximates to $1/\pi^2$. $Uj$, it may be well to notice, is the sum of all the numerators of the fractions in a Farey series whose limit is $j$, just as $Tj$ is the number of these fractions.

In the annexed Table the value of $\tau x$ (the totient), of $Tx$ (the sum-totient), and of $3/\pi^2 . x^2$ is calculated for all the values of $x$ from 1 to 1000; and the

[* See pp. 84, 89 above.] [† The right side should be $\frac{1}{12}j(j+1)(2j+1)$.]

remarkable fact is brought to light that $Tx$ is always greater than $3/\pi^2 . x^2$ (the number opposite to it), and less than $3/\pi^2 . (x+1)^2$, the number which comes after the following one in the same table.

I have calculated in my head the first few values of $Ux$, and find (if I have made no mistake) that it obeys an analogous law, namely is always intermediate between $1/\pi^2 . x^3$ and $1/\pi^2 . (x+1)^3$.

It may also be noticed that when $n$ is a prime number, $Tn$ is always nearer, and usually very much nearer, to the superior than to the inferior limit—as might have been anticipated from the circumstance that, when this is the case, in passing from $n-1$ to $n$ the $T$ receives an augmentation of $n-1$, whereas its average augmentation is only $\frac{3}{\pi^2}(2n-1)$.

In like manner and for a similar reason, when $n$ contains several small factors $Tn$ is nearer to the inferior than to the superior limit. For instance, when $n = 210$, $Tn = 13414$ and $3/\pi^2 . n^2 = 13404{\cdot}79$.

TABLE *of Totients, of Sum-totients, and of* $3/\pi^2$ *into the Squares of all the Numbers from* 1 *to* 1000 *inclusive.*

$$\left[\frac{3}{\pi^2} = {\cdot}30396355\right].$$

| $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | ·30 | 27 | 18 | 230 | 221·59 | 53 | 52 | 882 | 853·83 |
| 2 | 1 | 2 | 1·22 | 28 | 12 | 242 | 238·31 | 54 | 18 | 900 | 886·36 |
| 3 | 2 | 4 | 2·74 | 29 | 28 | 270 | 255·63 | 55 | 40 | 940 | 919·49 |
| 4 | 2 | 6 | 4·86 | 30 | 8 | 278 | 273·56 | 56 | 24 | 964 | 953·23 |
| 5 | 4 | 10 | 7·60 | 31 | 30 | 308 | 292·11 | 57 | 36 | 1000 | 987·58 |
| 6 | 2 | 12 | 10·94 | 32 | 16 | 324 | 311·26 | 58 | 28 | 1028 | 1022·54 |
| 7 | 6 | 18 | 14·90 | 33 | 20 | 344 | 331·01 | 59 | 58 | 1086 | 1058·10 |
| 8 | 4 | 22 | 19·46 | 34 | 16 | 360 | 351·38 | 60 | 16 | 1102 | 1094·27 |
| 9 | 6 | 28 | 24·62 | 35 | 24 | 384 | 372·35 | 61 | 60 | 1162 | 1131·05 |
| 10 | 4 | 32 | 30·40 | 36 | 12 | 396 | 393·93 | 62 | 30 | 1192 | 1168·44 |
| 11 | 10 | 42 | 36·78 | 37 | 36 | 432 | 416·12 | 63 | 36 | 1228 | 1206·43 |
| 12 | 4 | 46 | 43·77 | 38 | 18 | 450 | 438·92 | 64 | 32 | 1260 | 1245·03 |
| 13 | 12 | 58 | 51·37 | 39 | 24 | 474 | 462·32 | 65 | 48 | 1308 | 1284·25 |
| 14 | 6 | 64 | 59·58 | 40 | 16 | 490 | 486·34 | 66 | 20 | 1328 | 1324·07 |
| 15 | 8 | 72 | 68·39 | 41 | 40 | 530 | 510·96 | 67 | 66 | 1394 | 1364·49 |
| 16 | 8 | 80 | 77·81 | 42 | 12 | 542 | 536·19 | 68 | 32 | 1426 | 1405·53 |
| 17 | 16 | 96 | 87·84 | 43 | 42 | 584 | 562·02 | 69 | 44 | 1470 | 1447·17 |
| 18 | 6 | 102 | 98·48 | 44 | 20 | 604 | 588·47 | 70 | 24 | 1494 | 1489·42 |
| 19 | 18 | 120 | 109·73 | 45 | 24 | 628 | 615·52 | 71 | 70 | 1564 | 1532·28 |
| 20 | 8 | 128 | 121·58 | 46 | 22 | 650 | 643·19 | 72 | 24 | 1588 | 1575·75 |
| 21 | 12 | 140 | 134·05 | 47 | 46 | 696 | 671·45 | 73 | 72 | 1660 | 1619·82 |
| 22 | 10 | 150 | 147·12 | 48 | 16 | 712 | 700·33 | 74 | 36 | 1696 | 1664·51 |
| 23 | 22 | 172 | 160·79 | 49 | 42 | 754 | 729·82 | 75 | 40 | 1736 | 1709·80 |
| 24 | 8 | 180 | 175·08 | 50 | 20 | 774 | 759·91 | 76 | 36 | 1772 | 1755·69 |
| 25 | 20 | 200 | 189·98 | 51 | 32 | 806 | 790·61 | 77 | 60 | 1832 | 1802·20 |
| 26 | 12 | 212 | 205·48 | 52 | 24 | 830 | 821·92 | 78 | 24 | 1856 | 1849·31 |

TABLE (*continued*).

| $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 79 | 78 | 1934 | 1897·04 | 134 | 66 | 5498 | 5457·97 | 189 | 108 | 10904 | 10857·88 |
| 80 | 32 | 1966 | 1945·37 | 135 | 72 | 5570 | 5539·74 | 190 | 72 | 10976 | 10973·09 |
| 81 | 54 | 2020 | 1994·31 | 136 | 64 | 5634 | 5622·11 | 191 | 190 | 11166 | 11088·90 |
| 82 | 40 | 2060 | 2043·85 | 137 | 136 | 5770 | 5705·09 | 192 | 64 | 11230 | 11205·31 |
| 83 | 82 | 2142 | 2094·01 | 138 | 44 | 5814 | 5788·68 | 193 | 192 | 11422 | 11322·34 |
| 84 | 24 | 2166 | 2144·77 | 139 | 138 | 5952 | 5872·88 | 194 | 96 | 11518 | 11439·97 |
| 85 | 64 | 2230 | 2196·14 | 140 | 48 | 6000 | 5957·69 | 195 | 96 | 11614 | 11558·21 |
| 86 | 42 | 2272 | 2248·12 | 141 | 92 | 6092 | 6043·10 | 196 | 84 | 11698 | 11677·06 |
| 87 | 56 | 2328 | 2300·70 | 142 | 70 | 6162 | 6129·12 | 197 | 196 | 11894 | 11796·52 |
| 88 | 40 | 2368 | 2353·90 | 143 | 120 | 6282 | 6215·75 | 198 | 60 | 11954 | 11916·59 |
| 89 | 88 | 2456 | 2407·70 | 144 | 48 | 6330 | 6302·99 | 199 | 198 | 12152 | 12037·26 |
| 90 | 24 | 2480 | 2462·10 | 145 | 112 | 6442 | 6390·83 | 200 | 80 | 12232 | 12158·54 |
| 91 | 72 | 2552 | 2517·12 | 146 | 72 | 6514 | 6479·29 | 201 | 132 | 12364 | 12280·43 |
| 92 | 44 | 2596 | 2572·75 | 147 | 84 | 6598 | 6568·35 | 202 | 100 | 12464 | 12402·93 |
| 93 | 60 | 2656 | 2628·98 | 148 | 72 | 6670 | 6658·02 | 203 | 168 | 12632 | 12526·03 |
| 94 | 46 | 2702 | 2685·82 | 149 | 148 | 6818 | 6748·29 | 204 | 64 | 12696 | 12649·75 |
| 95 | 72 | 2774 | 2743·27 | 150 | 40 | 6858 | 6839·18 | 205 | 160 | 12856 | 12774·07 |
| 96 | 32 | 2806 | 2801·33 | 151 | 150 | 7008 | 6930·67 | 206 | 102 | 12958 | 12899·00 |
| 97 | 96 | 2902 | 2860·00 | 152 | 72 | 7080 | 7022·77 | 207 | 132 | 13090 | 13024·54 |
| 98 | 42 | 2944 | 2919·27 | 153 | 96 | 7176 | 7115·48 | 208 | 96 | 13186 | 13150·68 |
| 99 | 60 | 3004 | 2979·15 | 154 | 60 | 7236 | 7208·80 | 209 | 180 | 13366 | 13277·43 |
| 100 | 40 | 3044 | 3039·64 | 155 | 120 | 7356 | 7302·72 | 210 | 48 | 13414 | 13404·79 |
| 101 | 100 | 3144 | 3100·73 | 156 | 48 | 7404 | 7397·26 | 211 | 210 | 13624 | 13532·76 |
| 102 | 32 | 3176 | 3162·44 | 157 | 156 | 7560 | 7492·40 | 212 | 104 | 13728 | 13661·34 |
| 103 | 102 | 3278 | 3224·75 | 158 | 78 | 7638 | 7588·15 | 213 | 140 | 13868 | 13790·52 |
| 104 | 48 | 3326 | 3287·67 | 159 | 104 | 7742 | 7684·51 | 214 | 106 | 13974 | 13920·32 |
| 105 | 48 | 3374 | 3351·20 | 160 | 64 | 7806 | 7781·47 | 215 | 168 | 14142 | 14050·72 |
| 106 | 52 | 3426 | 3415·34 | 161 | 132 | 7938 | 7879·04 | 216 | 72 | 14214 | 14181·73 |
| 107 | 106 | 3532 | 3480·08 | 162 | 54 | 7992 | 7977·22 | 217 | 180 | 14394 | 14313·34 |
| 108 | 36 | 3568 | 3545·44 | 163 | 162 | 8154 | 8076·01 | 218 | 108 | 14502 | 14445·57 |
| 109 | 108 | 3676 | 3611·40 | 164 | 80 | 8234 | 8175·41 | 219 | 144 | 14646 | 14578·40 |
| 110 | 40 | 3716 | 3677·96 | 165 | 80 | 8314 | 8275·41 | 220 | 80 | 14726 | 14711·84 |
| 111 | 72 | 3788 | 3745·14 | 166 | 82 | 8396 | 8376·02 | 221 | 192 | 14918 | 14845·89 |
| 112 | 48 | 3836 | 3812·92 | 167 | 166 | 8562 | 8477·24 | 222 | 72 | 14990 | 14980·54 |
| 113 | 112 | 3948 | 3881·31 | 168 | 48 | 8610 | 8579·07 | 223 | 222 | 15212 | 15115·81 |
| 114 | 36 | 3984 | 3950·31 | 169 | 156 | 8766 | 8681·50 | 224 | 96 | 15308 | 15251·68 |
| 115 | 88 | 4072 | 4019·92 | 170 | 64 | 8830 | 8784·55 | 225 | 120 | 15428 | 15388·16 |
| 116 | 56 | 4128 | 4090·14 | 171 | 108 | 8938 | 8888·20 | 226 | 112 | 15540 | 15525·25 |
| 117 | 72 | 4200 | 4160·96 | 172 | 84 | 9022 | 8992·46 | 227 | 226 | 15766 | 15662·94 |
| 118 | 58 | 4258 | 4232·39 | 173 | 172 | 9194 | 9097·33 | 228 | 72 | 15838 | 15801·24 |
| 119 | 96 | 4354 | 4304·43 | 174 | 56 | 9250 | 9202·80 | 229 | 228 | 16066 | 15940·15 |
| 120 | 32 | 4386 | 4377·08 | 175 | 120 | 9370 | 9308·88 | 230 | 88 | 16154 | 16079·67 |
| 121 | 110 | 4496 | 4450·33 | 176 | 80 | 9450 | 9415·57 | 231 | 120 | 16274 | 16219·80 |
| 122 | 60 | 4556 | 4524·19 | 177 | 116 | 9566 | 9522·87 | 232 | 112 | 16386 | 16360·53 |
| 123 | 80 | 4636 | 4598·66 | 178 | 88 | 9654 | 9630·78 | 233 | 232 | 16618 | 16501·87 |
| 124 | 60 | 4696 | 4673·74 | 179 | 178 | 9832 | 9739·29 | 234 | 72 | 16690 | 16643·82 |
| 125 | 100 | 4796 | 4794·43 | 180 | 48 | 9880 | 9848·42 | 235 | 184 | 16874 | 16786·38 |
| 126 | 36 | 4832 | 4825·72 | 181 | 180 | 10060 | 9958·15 | 236 | 116 | 16990 | 16929·55 |
| 127 | 126 | 4958 | 4902·63 | 182 | 72 | 10132 | 10068·49 | 237 | 156 | 17146 | 17073·32 |
| 128 | 64 | 5022 | 4980·14 | 183 | 120 | 10252 | 10179·44 | 238 | 96 | 17242 | 17217·70 |
| 129 | 84 | 5106 | 5058·26 | 184 | 88 | 10340 | 10290·99 | 239 | 238 | 17480 | 17362·70 |
| 130 | 48 | 5154 | 5136·98 | 185 | 144 | 10484 | 10403·15 | 240 | 64 | 17544 | 17508·30 |
| 131 | 130 | 5284 | 5216·32 | 186 | 60 | 10544 | 10515·92 | 241 | 240 | 17784 | 17654·51 |
| 132 | 40 | 5324 | 5296·26 | 187 | 160 | 10704 | 10629·30 | 242 | 110 | 17894 | 17801·32 |
| 133 | 108 | 5432 | 5376·81 | 188 | 92 | 10796 | 10743·29 | 243 | 162 | 18056 | 17948·74 |

TABLE (*continued*).

| $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 244 | 120 | 18176 | 18096·77 | 299 | 264 | 27318 | 27174·65 | 354 | 116 | 38174 | 38091·50 |
| 245 | 168 | 18344 | 18245·41 | 300 | 80 | 27398 | 27356·72 | 355 | 280 | 38454 | 38307·01 |
| 246 | 80 | 18424 | 18394·66 | 301 | 252 | 27650 | 27539·40 | 356 | 176 | 38630 | 38523·12 |
| 247 | 216 | 18640 | 18544·51 | 302 | 150 | 27800 | 27722·69 | 357 | 192 | 38822 | 38739·85 |
| 248 | 120 | 18760 | 18694·97 | 303 | 200 | 28000 | 27906·59 | 358 | 178 | 39000 | 38957·18 |
| 249 | 164 | 18924 | 18846·04 | 304 | 144 | 28144 | 28091·10 | 359 | 358 | 39358 | 39175·13 |
| 250 | 100 | 19024 | 18997·72 | 305 | 240 | 28384 | 28276·21 | 360 | 96 | 39454 | 39393·68 |
| 251 | 250 | 19274 | 19150·01 | 306 | 96 | 28480 | 28461·93 | 361 | 342 | 39796 | 39612·83 |
| 252 | 72 | 19346 | 19302·90 | 307 | 306 | 28786 | 28648·26 | 362 | 180 | 39976 | 39832·60 |
| 253 | 220 | 19566 | 19456·40 | 308 | 120 | 28906 | 28835·20 | 363 | 220 | 40196 | 40052·97 |
| 254 | 126 | 19692 | 19610·51 | 309 | 204 | 29110 | 29022·75 | 364 | 144 | 40340 | 40273·95 |
| 255 | 128 | 19820 | 19765·23 | 310 | 120 | 29230 | 29210·90 | 365 | 288 | 40628 | 40495·54 |
| 256 | 128 | 19948 | 19920·56 | 311 | 310 | 29540 | 29399·66 | 366 | 120 | 40748 | 40717·74 |
| 257 | 256 | 20204 | 20076·49 | 312 | 96 | 29636 | 29589·03 | 367 | 366 | 41114 | 40940·55 |
| 258 | 84 | 20288 | 20233·03 | 313 | 312 | 29948 | 29779·01 | 368 | 176 | 41290 | 41163·96 |
| 259 | 216 | 20504 | 20390·18 | 314 | 156 | 30104 | 29969·59 | 369 | 240 | 41530 | 41387·98 |
| 260 | 96 | 20600 | 20547·94 | 315 | 144 | 30248 | 30160·79 | 370 | 144 | 41674 | 41612·61 |
| 261 | 168 | 20768 | 20706·30 | 316 | 156 | 30404 | 30352·59 | 371 | 312 | 41986 | 41837·85 |
| 262 | 130 | 20898 | 20865·28 | 317 | 316 | 30720 | 30545·00 | 372 | 120 | 42106 | 42063·69 |
| 263 | 262 | 21160 | 21024·86 | 318 | 104 | 30824 | 30738·01 | 373 | 372 | 42478 | 42290·15 |
| 264 | 80 | 21240 | 21185·05 | 319 | 280 | 31104 | 30931·64 | 374 | 160 | 42638 | 42517·21 |
| 265 | 208 | 21448 | 21345·84 | 320 | 128 | 31232 | 31125·87 | 375 | 200 | 42838 | 42744·87 |
| 266 | 108 | 21556 | 21507·25 | 321 | 212 | 31444 | 31320·71 | 376 | 184 | 43022 | 42973·15 |
| 267 | 176 | 21732 | 21669·26 | 322 | 132 | 31576 | 31516·16 | 377 | 336 | 43358 | 43202·04 |
| 268 | 132 | 21864 | 21831·88 | 323 | 288 | 31864 | 31712·22 | 378 | 108 | 43466 | 43431·53 |
| 269 | 268 | 22132 | 21995·11 | 324 | 108 | 31972 | 31908·88 | 379 | 378 | 43844 | 43661·63 |
| 270 | 72 | 22204 | 22158·95 | 325 | 240 | 32212 | 32106·15 | 380 | 144 | 43988 | 43892·34 |
| 271 | 270 | 22474 | 22323·39 | 326 | 162 | 32374 | 32304·03 | 381 | 252 | 44240 | 44123·65 |
| 272 | 128 | 22602 | 22488·44 | 327 | 216 | 32590 | 32502·52 | 382 | 190 | 44430 | 44355·58 |
| 273 | 144 | 22746 | 22654·10 | 328 | 160 | 32750 | 32701·62 | 383 | 382 | 44812 | 44588·11 |
| 274 | 136 | 22882 | 22820·37 | 329 | 276 | 33026 | 32901·32 | 384 | 128 | 44940 | 44821·25 |
| 275 | 200 | 23082 | 22987·25 | 330 | 80 | 33106 | 33101·63 | 385 | 240 | 45180 | 45055·00 |
| 276 | 88 | 23170 | 23154·73 | 331 | 330 | 33436 | 33302·55 | 386 | 192 | 45372 | 45289·35 |
| 277 | 276 | 23446 | 23322·82 | 332 | 164 | 33600 | 33504·08 | 387 | 252 | 45624 | 45524·32 |
| 278 | 138 | 23584 | 23491·52 | 333 | 216 | 33816 | 33706·22 | 388 | 192 | 45816 | 45759·89 |
| 279 | 180 | 23764 | 23660·83 | 334 | 166 | 33982 | 33908·96 | 389 | 388 | 46204 | 45996·07 |
| 280 | 96 | 23860 | 23830·75 | 335 | 264 | 34246 | 34112·31 | 390 | 96 | 46300 | 46232·86 |
| 281 | 280 | 24140 | 24001·27 | 336 | 96 | 34342 | 34316·27 | 391 | 352 | 46652 | 46470·25 |
| 282 | 92 | 24232 | 24172·40 | 337 | 336 | 34678 | 34520·84 | 392 | 168 | 46820 | 46708·25 |
| 283 | 282 | 24514 | 24344·14 | 338 | 156 | 34834 | 34726·01 | 393 | 260 | 47080 | 46946·87 |
| 284 | 140 | 24654 | 24516·49 | 339 | 224 | 35058 | 34931·80 | 394 | 196 | 47276 | 47186·09 |
| 285 | 144 | 24798 | 24689·44 | 340 | 128 | 35186 | 35138·19 | 395 | 312 | 47588 | 47425·91 |
| 286 | 120 | 24918 | 24863·00 | 341 | 300 | 35486 | 35345·19 | 396 | 120 | 47708 | 47666·35 |
| 287 | 240 | 25158 | 25037·18 | 342 | 108 | 35594 | 35552·80 | 397 | 396 | 48104 | 47907·39 |
| 288 | 96 | 25254 | 25211·96 | 343 | 294 | 35888 | 35761·01 | 398 | 198 | 48302 | 48149·04 |
| 289 | 272 | 25526 | 25387·34 | 344 | 168 | 36056 | 35969·83 | 399 | 216 | 48518 | 48391·30 |
| 290 | 112 | 25638 | 25563·34 | 345 | 176 | 36232 | 36179·26 | 400 | 160 | 48678 | 48634·17 |
| 291 | 192 | 25830 | 25739·94 | 346 | 172 | 36404 | 36389·30 | 401 | 400 | 49078 | 48877·64 |
| 292 | 144 | 25974 | 25917·15 | 347 | 346 | 36750 | 36599·95 | 402 | 132 | 49210 | 49121·73 |
| 293 | 292 | 26266 | 26094·97 | 348 | 112 | 36862 | 36811·21 | 403 | 360 | 49570 | 49366·42 |
| 294 | 84 | 26350 | 26273·40 | 349 | 348 | 37210 | 37023·07 | 404 | 200 | 49770 | 49611·72 |
| 295 | 232 | 26582 | 26452·43 | 350 | 120 | 37330 | 37235·54 | 405 | 216 | 49986 | 49857·62 |
| 296 | 144 | 26726 | 26632·07 | 351 | 216 | 37546 | 37448·61 | 406 | 168 | 50154 | 50104·14 |
| 297 | 180 | 26906 | 26812·32 | 352 | 160 | 37706 | 37662·30 | 407 | 360 | 50514 | 50351·26 |
| 298 | 148 | 27054 | 26993·18 | 353 | 352 | 38058 | 37876·59 | 408 | 128 | 50642 | 50598·99 |

TABLE (*continued*).

| $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 409 | 408 | 51050 | 50847·33 | 464 | 224 | 65630 | 65442·14 | 519 | 344 | 82028 | 81875·93 |
| 410 | 160 | 51210 | 51096·27 | 465 | 240 | 65870 | 65724·52 | 520 | 192 | 82220 | 82191·75 |
| 411 | 272 | 51482 | 51345·83 | 466 | 232 | 66102 | 66007·51 | 521 | 520 | 82740 | 82508·18 |
| 412 | 204 | 51686 | 51595·99 | 467 | 466 | 66568 | 66291·11 | 522 | 168 | 82908 | 82825·21 |
| 413 | 348 | 52034 | 51846·76 | 468 | 144 | 66712 | 66575·31 | 523 | 522 | 83430 | 83142·85 |
| 414 | 132 | 52166 | 52098·14 | 469 | 396 | 67108 | 66860·13 | 524 | 260 | 83690 | 83461·10 |
| 415 | 328 | 52494 | 52350·12 | 470 | 184 | 67292 | 67145·55 | 525 | 240 | 83930 | 83779·95 |
| 416 | 192 | 52686 | 52602·72 | 471 | 312 | 67604 | 67431·58 | 526 | 262 | 84192 | 84099·42 |
| 417 | 276 | 52962 | 52855·92 | 472 | 232 | 67836 | 67718·22 | 527 | 480 | 84672 | 84419·49 |
| 418 | 180 | 53142 | 53109·73 | 473 | 420 | 68256 | 68005·46 | 528 | 160 | 84832 | 84740·17 |
| 419 | 418 | 53560 | 53364·15 | 474 | 156 | 68412 | 68293·32 | 529 | 506 | 85338 | 85061·46 |
| 420 | 96 | 53656 | 53619·17 | 475 | 360 | 68772 | 68581·78 | 530 | 208 | 85546 | 85383·36 |
| 421 | 420 | 54076 | 53874·80 | 476 | 192 | 68964 | 68870·85 | 531 | 348 | 85894 | 85705·87 |
| 422 | 210 | 54286 | 54131·04 | 477 | 312 | 69276 | 69160·52 | 532 | 216 | 86110 | 86028·98 |
| 423 | 276 | 54562 | 54387·89 | 478 | 238 | 69514 | 69450·81 | 533 | 480 | 86590 | 86352·70 |
| 424 | 208 | 54770 | 54645·35 | 479 | 478 | 69992 | 69741·70 | 534 | 176 | 86766 | 86677·03 |
| 425 | 320 | 55090 | 54903·42 | 480 | 128 | 70120 | 70033·20 | 535 | 424 | 87190 | 87001·97 |
| 426 | 140 | 55230 | 55162·09 | 481 | 432 | 70552 | 70325·31 | 536 | 264 | 87454 | 87327·51 |
| 427 | 360 | 55590 | 55421·39 | 482 | 240 | 70792 | 70618·03 | 537 | 356 | 87810 | 87653·66 |
| 428 | 212 | 55802 | 55681·26 | 483 | 264 | 71056 | 70911·35 | 538 | 268 | 88078 | 87980·42 |
| 429 | 240 | 56042 | 55941·76 | 484 | 220 | 71276 | 71205·29 | 539 | 420 | 88498 | 88307·79 |
| 430 | 168 | 56210 | 56202·86 | 485 | 384 | 71660 | 71499·83 | 540 | 144 | 88642 | 88635·77 |
| 431 | 430 | 56640 | 56464·57 | 486 | 162 | 71822 | 71794·98 | 541 | 540 | 89182 | 88964·35 |
| 432 | 144 | 56784 | 56726·89 | 487 | 486 | 72308 | 72090·73 | 542 | 270 | 89452 | 89293·54 |
| 433 | 432 | 57216 | 56989·82 | 488 | 240 | 72548 | 72387·10 | 543 | 360 | 89812 | 89623·34 |
| 434 | 180 | 57396 | 57253·36 | 489 | 324 | 72872 | 72684·07 | 544 | 256 | 90068 | 89953·75 |
| 435 | 224 | 57620 | 57517·50 | 490 | 168 | 73040 | 72981·65 | 545 | 432 | 90500 | 90284·77 |
| 436 | 216 | 57836 | 57782·26 | 491 | 490 | 73530 | 73279·84 | 546 | 144 | 90644 | 90616·39 |
| 437 | 396 | 58232 | 58047·62 | 492 | 160 | 73690 | 73578·63 | 547 | 546 | 91190 | 90948·62 |
| 438 | 144 | 58376 | 58313·58 | 493 | 448 | 74138 | 73878·04 | 548 | 272 | 91462 | 91281·46 |
| 439 | 438 | 58814 | 58580·16 | 494 | 216 | 74354 | 74178·05 | 549 | 360 | 91822 | 91614·91 |
| 440 | 160 | 58974 | 58847·34 | 495 | 240 | 74594 | 74478·67 | 550 | 200 | 92022 | 91948·97 |
| 441 | 252 | 59226 | 59115·14 | 496 | 240 | 74834 | 74779·90 | 551 | 504 | 92526 | 92283·64 |
| 442 | 192 | 59418 | 59383·54 | 497 | 420 | 75254 | 75081·73 | 552 | 176 | 92702 | 92618·91 |
| 443 | 442 | 59860 | 59652·54 | 498 | 164 | 75418 | 75384·18 | 553 | 468 | 93170 | 92954·79 |
| 444 | 144 | 60004 | 59922·16 | 499 | 498 | 75916 | 75687·23 | 554 | 276 | 93446 | 93291·28 |
| 445 | 352 | 60356 | 60192·38 | 500 | 200 | 76116 | 75990·89 | 555 | 288 | 93734 | 93628·38 |
| 446 | 222 | 60578 | 60463·22 | 501 | 332 | 76448 | 76295·15 | 556 | 276 | 94010 | 93966·08 |
| 447 | 296 | 60874 | 60734·66 | 502 | 250 | 76698 | 76600·03 | 557 | 556 | 94566 | 94304·39 |
| 448 | 192 | 61066 | 61006·70 | 503 | 502 | 77200 | 76905·52 | 558 | 180 | 94746 | 94643·31 |
| 449 | 448 | 61514 | 61279·36 | 504 | 144 | 77344 | 77211·61 | 559 | 504 | 95250 | 94982·84 |
| 450 | 120 | 61634 | 61552·62 | 505 | 400 | 77744 | 77518·31 | 560 | 192 | 95442 | 95322·98 |
| 451 | 400 | 62034 | 61826·49 | 506 | 220 | 77964 | 77825·62 | 561 | 320 | 95762 | 95663·72 |
| 452 | 224 | 62258 | 62100·97 | 507 | 312 | 78276 | 78133·54 | 562 | 280 | 96042 | 96005·07 |
| 453 | 300 | 62558 | 62376·06 | 508 | 252 | 78528 | 78442·06 | 563 | 562 | 96604 | 96347·03 |
| 454 | 226 | 62784 | 62651·75 | 509 | 508 | 79036 | 78751·19 | 564 | 184 | 96788 | 96689·60 |
| 455 | 288 | 63072 | 62928·05 | 510 | 128 | 79164 | 79060·93 | 565 | 448 | 97236 | 97032·77 |
| 456 | 144 | 63216 | 63204·97 | 511 | 432 | 79596 | 79371·28 | 566 | 282 | 97518 | 97376·55 |
| 457 | 456 | 63672 | 63482·48 | 512 | 256 | 79852 | 79682·23 | 567 | 324 | 97842 | 97720·94 |
| 458 | 228 | 63900 | 63760·61 | 513 | 324 | 80176 | 79993·79 | 568 | 280 | 98122 | 98065·94 |
| 459 | 288 | 64188 | 64039·35 | 514 | 256 | 80432 | 80305·96 | 569 | 568 | 98690 | 98411·55 |
| 460 | 176 | 64364 | 64318·69 | 515 | 408 | 80840 | 80618·74 | 570 | 144 | 98834 | 98757·76 |
| 461 | 460 | 64824 | 64598·64 | 516 | 168 | 81008 | 80932·13 | 571 | 570 | 99404 | 99104·58 |
| 462 | 120 | 64944 | 64879·20 | 517 | 460 | 81468 | 81246·12 | 572 | 240 | 99644 | 99452·01 |
| 463 | 462 | 65406 | 65160·36 | 518 | 216 | 81684 | 81560·72 | 573 | 380 | 100024 | 99800·05 |

TABLE (*continued*).

| $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 574 | 240 | 100264 | 100148·70 | 629 | 576 | 120544 | 120260·45 | 684 | 216 | 142380 | 142211·17 |
| 575 | 440 | 100704 | 100497·95 | 630 | 144 | 120688 | 120643·14 | 685 | 544 | 142924 | 142627·30 |
| 576 | 192 | 100896 | 100847·81 | 631 | 630 | 121318 | 121026·44 | 686 | 294 | 143218 | 143044·03 |
| 577 | 576 | 101472 | 101198·28 | 632 | 312 | 121630 | 121410·35 | 687 | 456 | 143674 | 143461·37 |
| 578 | 272 | 101744 | 101549·36 | 633 | 420 | 122050 | 121794·86 | 688 | 536 | 144010 | 143879·32 |
| 579 | 384 | 102128 | 101901·05 | 634 | 316 | 122366 | 122179·98 | 689 | 624 | 144634 | 144297·88 |
| 580 | 224 | 102352 | 102253·34 | 635 | 504 | 122870 | 122565·71 | 690 | 176 | 144810 | 144717·05 |
| 581 | 492 | 102844 | 102606·24 | 636 | 208 | 123078 | 122952·05 | 691 | 690 | 145500 | 145136·82 |
| 582 | 192 | 103036 | 102959·75 | 637 | 504 | 123582 | 123338·00 | 692 | 344 | 145844 | 145557·20 |
| 583 | 520 | 103556 | 103313·87 | 638 | 280 | 123862 | 123726·55 | 693 | 360 | 146204 | 145978·19 |
| 584 | 288 | 103844 | 103668·60 | 639 | 420 | 124282 | 124114·71 | 694 | 346 | 146550 | 146399·79 |
| 585 | 288 | 104132 | 104023·93 | 640 | 256 | 124538 | 124503·48 | 695 | 552 | 147102 | 146821·99 |
| 586 | 292 | 104424 | 104379·87 | 641 | 640 | 125178 | 124892·86 | 696 | 224 | 147326 | 147244·80 |
| 587 | 586 | 105010 | 104736·42 | 642 | 212 | 125390 | 125282·85 | 697 | 640 | 147966 | 147668·22 |
| 588 | 168 | 105178 | 105093·58 | 643 | 642 | 126032 | 125673·44 | 698 | 348 | 148314 | 148092·25 |
| 589 | 540 | 105718 | 105451·35 | 644 | 264 | 126296 | 126064·64 | 699 | 464 | 148778 | 148516·89 |
| 590 | 232 | 105950 | 105809·72 | 645 | 336 | 126632 | 126456·45 | 700 | 240 | 149018 | 148942·14 |
| 591 | 392 | 106342 | 106168·70 | 646 | 288 | 126920 | 126848·87 | 701 | 700 | 149718 | 149367·99 |
| 592 | 288 | 106630 | 106528·29 | 647 | 646 | 127566 | 127241·89 | 702 | 216 | 149934 | 149794·45 |
| 593 | 592 | 107222 | 106888·49 | 648 | 216 | 127782 | 127635·52 | 703 | 648 | 150582 | 150221·52 |
| 594 | 180 | 107402 | 107249·29 | 649 | 580 | 128362 | 128029·76 | 704 | 320 | 150902 | 150649·20 |
| 595 | 384 | 107786 | 107610·70 | 650 | 240 | 128602 | 128424·60 | 705 | 368 | 151270 | 151077·48 |
| 596 | 296 | 108082 | 107972·72 | 651 | 360 | 128962 | 128820·06 | 706 | 352 | 151622 | 151506·37 |
| 597 | 396 | 108478 | 108335·35 | 652 | 324 | 129286 | 129216·12 | 707 | 600 | 152222 | 151935·87 |
| 598 | 264 | 108742 | 108698·59 | 653 | 652 | 129938 | 129612·79 | 708 | 232 | 152454 | 152365·98 |
| 599 | 598 | 109340 | 109062·43 | 654 | 216 | 130154 | 130010·07 | 709 | 708 | 153162 | 152796·70 |
| 600 | 160 | 109500 | 109426·88 | 655 | 520 | 130674 | 130407·96 | 710 | 280 | 153442 | 153228·02 |
| 601 | 600 | 110100 | 109791·94 | 656 | 320 | 130994 | 130806·46 | 711 | 468 | 153910 | 153659·95 |
| 602 | 252 | 110352 | 110157·61 | 657 | 432 | 131426 | 131205·56 | 712 | 352 | 154262 | 154092·49 |
| 603 | 396 | 110748 | 110523·89 | 658 | 276 | 131702 | 131605·27 | 713 | 660 | 154922 | 154525·64 |
| 604 | 300 | 111048 | 110890·77 | 659 | 658 | 132360 | 132005·59 | 714 | 192 | 155114 | 154959·40 |
| 605 | 440 | 111488 | 111258·26 | 660 | 160 | 132520 | 132406·52 | 715 | 480 | 155594 | 155393·76 |
| 606 | 200 | 111688 | 111626·36 | 661 | 660 | 133180 | 132808·06 | 716 | 356 | 155950 | 155828·73 |
| 607 | 606 | 112294 | 111995·07 | 662 | 330 | 133510 | 133210·20 | 717 | 476 | 156426 | 156264·31 |
| 608 | 288 | 112582 | 112364·39 | 663 | 384 | 133894 | 133612·95 | 718 | 358 | 156784 | 156700·50 |
| 609 | 336 | 112918 | 112734·31 | 664 | 328 | 134222 | 134016·31 | 719 | 718 | 157502 | 157137·30 |
| 610 | 240 | 113158 | 113104·84 | 665 | 432 | 134654 | 134420·28 | 720 | 192 | 157694 | 157574·70 |
| 611 | 552 | 113710 | 113475·98 | 666 | 216 | 134870 | 134824·86 | 721 | 612 | 158306 | 158012·71 |
| 612 | 192 | 113902 | 113847·73 | 667 | 616 | 135486 | 135230·04 | 722 | 342 | 158648 | 158451·33 |
| 613 | 612 | 114514 | 114220·09 | 668 | 332 | 135818 | 135635·83 | 723 | 480 | 159128 | 158890·56 |
| 614 | 306 | 114820 | 114593·05 | 669 | 444 | 136262 | 136042·23 | 724 | 360 | 159488 | 159330·40 |
| 615 | 320 | 115140 | 114966·62 | 670 | 264 | 136526 | 136449·24 | 725 | 560 | 160048 | 159770·84 |
| 616 | 240 | 115380 | 115340·80 | 671 | 600 | 137126 | 136856·86 | 726 | 220 | 160268 | 160211·89 |
| 617 | 616 | 115996 | 115715·59 | 672 | 192 | 137318 | 137265·08 | 727 | 726 | 160994 | 160653·55 |
| 618 | 204 | 116200 | 116090·99 | 673 | 672 | 137990 | 137673·91 | 728 | 288 | 161282 | 161095·82 |
| 619 | 618 | 116818 | 116466·99 | 674 | 336 | 138326 | 138083·35 | 729 | 486 | 161768 | 161538·69 |
| 620 | 240 | 117058 | 116843·60 | 675 | 360 | 138686 | 138493·40 | 730 | 288 | 162056 | 161982·17 |
| 621 | 396 | 117454 | 117220·82 | 676 | 312 | 138998 | 138904·05 | 731 | 672 | 162728 | 162426·26 |
| 622 | 310 | 117764 | 117598·65 | 677 | 676 | 139674 | 139315·31 | 732 | 240 | 162968 | 162870·96 |
| 623 | 528 | 118292 | 117977·08 | 678 | 224 | 139898 | 139727·18 | 733 | 732 | 163700 | 163316·27 |
| 624 | 192 | 118484 | 118356·12 | 679 | 576 | 140474 | 140139·66 | 734 | 366 | 164066 | 163762·18 |
| 625 | 500 | 118984 | 118735·77 | 680 | 256 | 140730 | 140552·75 | 735 | 336 | 164402 | 164208·70 |
| 626 | 312 | 119296 | 119116·03 | 681 | 452 | 141182 | 140966·44 | 736 | 352 | 164754 | 164655·83 |
| 627 | 360 | 119656 | 119496·90 | 682 | 300 | 141482 | 141380·74 | 737 | 660 | 165414 | 165103·57 |
| 628 | 312 | 119968 | 119878·37 | 683 | 682 | 142164 | 141795·65 | 738 | 240 | 165654 | 165551·92 |

TABLE (*continued*).

| $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 739 | 738 | 166392 | 166000·87 | 794 | 396 | 191870 | 191629·56 | 849 | 564 | 219340 | 219097·23 |
| 740 | 288 | 166680 | 166450·43 | 795 | 416 | 192286 | 192112·56 | 850 | 320 | 219660 | 219613·66 |
| 741 | 432 | 167112 | 166900·60 | 796 | 396 | 192682 | 192596·17 | 851 | 792 | 220452 | 220130·71 |
| 742 | 312 | 167424 | 167351·38 | 797 | 796 | 193478 | 193080·39 | 852 | 280 | 220732 | 220648·36 |
| 743 | 742 | 168166 | 167802·77 | 798 | 216 | 193694 | 193565·21 | 853 | 852 | 221584 | 221166·62 |
| 744 | 240 | 168406 | 168254·76 | 799 | 736 | 194430 | 194050·64 | 854 | 360 | 221944 | 221685·48 |
| 745 | 592 | 168998 | 168707·36 | 800 | 320 | 194750 | 194536·67 | 855 | 432 | 222376 | 222204·96 |
| 746 | 372 | 169370 | 169160·57 | 801 | 528 | 195278 | 195023·32 | 856 | 424 | 222800 | 222725·04 |
| 747 | 492 | 169862 | 169614·39 | 802 | 400 | 195678 | 195510·57 | 857 | 856 | 223656 | 223245·73 |
| 748 | 320 | 170182 | 170068·82 | 803 | 720 | 196398 | 195998·43 | 858 | 240 | 223896 | 223767·03 |
| 749 | 636 | 170818 | 170523·85 | 804 | 264 | 196662 | 196486·90 | 859 | 858 | 224754 | 224288·93 |
| 750 | 200 | 171018 | 170979·50 | 805 | 528 | 197190 | 196975·98 | 860 | 336 | 225090 | 224811·44 |
| 751 | 750 | 171768 | 171435·75 | 806 | 360 | 197550 | 197465·66 | 861 | 480 | 225570 | 225334·56 |
| 752 | 368 | 172136 | 171892·61 | 807 | 536 | 198086 | 197955·96 | 862 | 430 | 226000 | 225858·29 |
| 753 | 500 | 172636 | 172350·07 | 808 | 400 | 198486 | 198446·86 | 863 | 862 | 226862 | 226382·62 |
| 754 | 336 | 172972 | 172808·14 | 809 | 808 | 199294 | 198938·37 | 864 | 288 | 227150 | 226907·57 |
| 755 | 600 | 173572 | 173266·82 | 810 | 216 | 199510 | 199430·48 | 865 | 688 | 227838 | 227433·12 |
| 756 | 216 | 173788 | 173726·11 | 811 | 810 | 200320 | 199923·21 | 866 | 432 | 228270 | 227959·28 |
| 757 | 756 | 174544 | 174186·01 | 812 | 336 | 200656 | 200416·54 | 867 | 544 | 228814 | 228486·05 |
| 758 | 378 | 174922 | 174646·52 | 813 | 540 | 201196 | 200910·48 | 868 | 360 | 229174 | 229012·43 |
| 759 | 440 | 175362 | 175107·63 | 814 | 360 | 201556 | 201405·03 | 869 | 780 | 229954 | 229541·41 |
| 760 | 288 | 175650 | 175569·35 | 815 | 648 | 202204 | 201900·19 | 870 | 224 | 230178 | 230070·01 |
| 761 | 760 | 176410 | 176031·68 | 816 | 256 | 202460 | 202395·95 | 871 | 792 | 230970 | 230599·21 |
| 762 | 252 | 176662 | 176494·62 | 817 | 756 | 203216 | 202892·32 | 872 | 432 | 231402 | 231129·02 |
| 763 | 648 | 177310 | 176958·16 | 818 | 408 | 203624 | 203389·30 | 873 | 576 | 231978 | 231659·43 |
| 764 | 380 | 177690 | 177422·31 | 819 | 432 | 204056 | 203886·89 | 874 | 396 | 232374 | 232190·46 |
| 765 | 384 | 178074 | 177887·07 | 820 | 320 | 204376 | 204385·09 | 875 | 600 | 232974 | 232722·09 |
| 766 | 382 | 178456 | 178352·44 | 821 | 820 | 205196 | 204883·89 | 876 | 288 | 233262 | 233254·33 |
| 767 | 696 | 179152 | 178818·42 | 822 | 272 | 205468 | 205383·30 | 877 | 876 | 234138 | 232787·18 |
| 768 | 256 | 179408 | 179285·00 | 823 | 822 | 206290 | 205883·32 | 878 | 438 | 234576 | 234320·64 |
| 769 | 768 | 180176 | 179752·19 | 824 | 408 | 206698 | 206383·95 | 879 | 584 | 235160 | 234854·70 |
| 770 | 240 | 180416 | 180219·99 | 825 | 400 | 207098 | 206885·19 | 880 | 320 | 235480 | 235389·37 |
| 771 | 512 | 180928 | 180688·40 | 826 | 348 | 207446 | 207387·03 | 881 | 880 | 236360 | 235924·65 |
| 772 | 384 | 181312 | 181157·42 | 827 | 826 | 208272 | 207889·48 | 882 | 252 | 236612 | 236460·54 |
| 773 | 772 | 182084 | 181627·04 | 828 | 264 | 208536 | 208392·54 | 883 | 882 | 237494 | 236997·04 |
| 774 | 252 | 182336 | 182097·27 | 829 | 828 | 209364 | 208896·21 | 884 | 384 | 237878 | 237534·14 |
| 775 | 600 | 182936 | 182568·11 | 830 | 328 | 209692 | 206400·49 | 885 | 464 | 238342 | 238071·85 |
| 776 | 384 | 183320 | 183039·56 | 831 | 552 | 210244 | 209905·37 | 886 | 442 | 238784 | 238610·17 |
| 777 | 432 | 183752 | 183511·61 | 832 | 384 | 210628 | 210410·86 | 887 | 886 | 239670 | 239149·10 |
| 778 | 388 | 184140 | 183984·28 | 833 | 672 | 211300 | 210916·96 | 888 | 288 | 239958 | 239688·64 |
| 779 | 720 | 184860 | 184457·55 | 834 | 276 | 211576 | 211423·67 | 889 | 756 | 240714 | 240228·78 |
| 780 | 192 | 185052 | 184931·43 | 835 | 664 | 212240 | 211930·98 | 890 | 352 | 241066 | 240769·53 |
| 781 | 700 | 185752 | 185405·92 | 836 | 360 | 212600 | 212438·91 | 891 | 540 | 241606 | 241310·89 |
| 782 | 352 | 186104 | 185881·01 | 837 | 540 | 213140 | 212947·44 | 892 | 444 | 242050 | 241852·86 |
| 783 | 504 | 186608 | 186356·71 | 838 | 418 | 213558 | 213456·58 | 893 | 828 | 242878 | 242395·43 |
| 784 | 336 | 186944 | 186833·02 | 839 | 838 | 214396 | 213966·32 | 894 | 296 | 243174 | 242938·62 |
| 785 | 624 | 187568 | 187309·94 | 840 | 192 | 214588 | 214476·68 | 895 | 712 | 243886 | 243482·41 |
| 786 | 260 | 187828 | 187787·47 | 841 | 812 | 215400 | 214987·64 | 896 | 384 | 244270 | 244026·81 |
| 787 | 786 | 188614 | 188265·60 | 842 | 420 | 215820 | 215499·21 | 897 | 528 | 244798 | 244571·81 |
| 788 | 392 | 189006 | 188744·34 | 843 | 560 | 216380 | 216011·39 | 898 | 448 | 245246 | 245117·43 |
| 789 | 524 | 189530 | 189223·69 | 844 | 420 | 216800 | 216524·18 | 899 | 840 | 246086 | 245663·65 |
| 790 | 312 | 189842 | 189703·65 | 845 | 624 | 217424 | 217037·57 | 900 | 240 | 246326 | 246210·48 |
| 791 | 672 | 190514 | 190184·22 | 846 | 276 | 217700 | 217551·58 | 901 | 832 | 247158 | 246757·91 |
| 792 | 240 | 190754 | 190665·39 | 847 | 660 | 218360 | 218066·19 | 902 | 400 | 247558 | 247305·96 |
| 793 | 720 | 191474 | 191147·17 | 848 | 416 | 218776 | 218581·40 | 903 | 504 | 248062 | 247854·61 |

TABLE* (*continued*).

| $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ | $n$ | $\tau(n)$ | $T(n)$ | $\frac{3}{\pi^2}n^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 904 | 448 | 248510 | 248403·88 | 937 | 936 | 267256 | 266870·57 | 970 | 384 | 286076 | 285999·30 |
| 905 | 720 | 249230 | 248953·75 | 938 | 396 | 267652 | 267440·51 | 971 | 970 | 287046 | 286589·30 |
| 906 | 300 | 249530 | 249504·22 | 939 | 624 | 268276 | 268011·05 | 972 | 324 | 287370 | 287179·90 |
| 907 | 906 | 250436 | 250055·31 | 940 | 368 | 268644 | 268582·19 | 973 | 828 | 288198 | 287771·11 |
| 908 | 452 | 250888 | 250607·00 | 941 | 940 | 269584 | 269153·95 | 974 | 486 | 288684 | 288362·92 |
| 909 | 600 | 251488 | 251159·31 | 942 | 312 | 269896 | 269726·31 | 975 | 480 | 289164 | 288955·35 |
| 910 | 288 | 251776 | 251712·22 | 943 | 880 | 270776 | 270299·28 | 976 | 480 | 289644 | 289548·39 |
| 911 | 910 | 252686 | 252265·73 | 944 | 464 | 271240 | 270872·86 | 977 | 976 | 290620 | 290142·03 |
| 912 | 288 | 252974 | 252819·86 | 945 | 432 | 271672 | 271447·05 | 978 | 324 | 290944 | 290736·28 |
| 913 | 820 | 253794 | 253374·59 | 946 | 420 | 272092 | 272021·84 | 979 | 880 | 291824 | 291331·13 |
| 914 | 456 | 254250 | 253929·93 | 947 | 946 | 273038 | 272597·25 | 980 | 336 | 292160 | 291926·60 |
| 915 | 480 | 254730 | 254485·88 | 948 | 312 | 273350 | 273173·26 | 981 | 648 | 292808 | 292522·67 |
| 916 | 456 | 255186 | 255042·44 | 949 | 864 | 274214 | 273749·88 | 982 | 490 | 293298 | 293119·35 |
| 917 | 780 | 255966 | 255599·61 | 950 | 360 | 274574 | 274327·10 | 983 | 982 | 294280 | 293716·64 |
| 918 | 288 | 256254 | 256157·38 | 951 | 632 | 275206 | 274905·94 | 984 | 320 | 294600 | 294314·54 |
| 919 | 918 | 257172 | 256715·76 | 952 | 384 | 275590 | 275483·38 | 985 | 784 | 295384 | 294913·04 |
| 920 | 352 | 257524 | 257274·75 | 953 | 952 | 276542 | 276062·43 | 986 | 448 | 295832 | 295512·15 |
| 921 | 612 | 258136 | 257834·34 | 954 | 312 | 276854 | 276642·09 | 987 | 552 | 296384 | 296111·87 |
| 922 | 460 | 258596 | 258394·55 | 955 | 760 | 277614 | 277222·36 | 988 | 432 | 296816 | 296712·20 |
| 923 | 840 | 259436 | 258955·36 | 956 | 476 | 278090 | 277803·23 | 989 | 924 | 297740 | 297313·14 |
| 924 | 240 | 259676 | 259516·78 | 957 | 560 | 278650 | 278384·71 | 990 | 240 | 297980 | 297914·68 |
| 925 | 720 | 260396 | 260078·81 | 958 | 478 | 279128 | 278966·80 | 991 | 990 | 298970 | 298516·83 |
| 926 | 462 | 260858 | 260641·45 | 959 | 816 | 279944 | 279549·50 | 992 | 480 | 299450 | 299119·59 |
| 927 | 612 | 261470 | 261204·69 | 960 | 256 | 280200 | 280132·81 | 993 | 660 | 300110 | 299722·96 |
| 928 | 448 | 261918 | 261768·55 | 961 | 930 | 281130 | 280716·72 | 994 | 420 | 300530 | 300326·94 |
| 929 | 928 | 262846 | 262333·01 | 962 | 432 | 281562 | 281301·24 | 995 | 792 | 301322 | 300931·52 |
| 930 | 240 | 263086 | 262898·07 | 963 | 636 | 282198 | 281886·37 | 996 | 328 | 301650 | 301536·71 |
| 931 | 756 | 263842 | 263463·75 | 964 | 480 | 282678 | 282472·11 | 997 | 996 | 302646 | 302142·51 |
| 932 | 464 | 264306 | 264030·03 | 965 | 768 | 283446 | 283058·46 | 998 | 498 | 303144 | 302748·92 |
| 933 | 620 | 264926 | 264596·93 | 966 | 264 | 283710 | 283645·41 | 999 | 648 | 303792 | 303355·93 |
| 934 | 466 | 265392 | 265164·43 | 967 | 966 | 284676 | 284232·97 | 1000 | 400 | 304192 | 303963·55 |
| 935 | 640 | 266032 | 265732·53 | 968 | 440 | 285116 | 284821·14 | | | | |
| 936 | 288 | 266320 | 266301·25 | 969 | 576 | 285692 | 285409·92 | | | | |

* In the extended as well as in the original Table it will be seen that the sum-totient is always intermediate between $3/\pi^2 . n^2$ and $3/\pi^2 . (n+1)^2$.

The formula of verification applied at every tenth step to the $T$ column precludes the possibility of the existence of other than typographical errors or errors of transcription. Accumulative errors are rendered impossible.